Problem #1 (20 Points)

For each of the following equations (a) – (c) used frequently in this course, state whether or not each of the following restrictions (i) – (x) are necessary to derive or use these equations.

i. steady
ii. one-dimensional flow
iii. ideal gas
iv. constant specific heats
v. reversible
vi. adiabatic
vii. no work transfer
viii. negligible change in kinetic energy
ix. constant area
x. inviscid flow

a) Thrust equation: \( T = \dot{m} \left[ (1+FAR)u_9 - u_1 + (P_9 - P_1)A_9 \right] \)

b) Constant stagnation temperature between states 1 and 2: 
\[
T_1 \left( 1 + \frac{\gamma - 1}{2} M_1^2 \right) = T_2 \left( 1 + \frac{\gamma - 1}{2} M_2^2 \right)
\]

c) Compression/expansion law: 
\[
\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}}
\]

Problem #2 (gas turbine performance) (20 points)

The following performance parameters were measured in a turbojet-powered aircraft (no fan or afterburner):

- Flight velocity \( u_1 = 250 \text{ m/s} \); air mass flow \( \dot{m}_a = 10 \text{ kg/s} \); fuel mass flow \( \dot{m}_f = 0.3 \text{ kg/s} \)
- Compressor pressure ratio \( \pi_c = 30 \)
- Thrust = 10650 N
- Ambient pressure \( P_1 = \text{exit pressure } P_9 = 0.5 \text{ atm} = 5.07 \times 10^4 \text{ N/m}^2 \)
- Ambient temperature \( T_1 = 250 \text{K} \)
- Gas constant \( R = 300 \text{ J/kgK}, \text{gas specific heat ratio } \gamma = 1.35 \)
- Fuel heating value \( Q_R = 4.3 \times 10^7 \text{ J/kg} \)
From this information compute:

a) Flight Mach number ($M_1$) and recovery temperature ratio ($\tau_r$)
b) Specific Thrust
c) Exhaust velocity ($u_e$)
d) Overall efficiency ($\eta_o$)
e) Propulsive efficiency ($\eta_{prop}$)
f) Turbine inlet temperature $T_4$ (assuming an ideal cycle with no heat losses or irreversibilities)

Problem #3 (10 points)

Using the Breguet range equation, estimate the range of a bar-tailed godwit, a bird that stores 55 percent of its body weight as fat to fuel its journey. When estimating the heating value of bird fat, note that 1 diet calorie = 1000 thermodynamic calories. (This question always throws students for a loop. The point is not to get an exact answer, but to estimate each of the terms in the Breguet range equation, and see if the result is reasonable or not.) Compare your prediction to the actual range (see for example the Wikipedia article on the bar-tailed godwit.)

Problem #4 (T-s diagrams) (20 points)

In an ideal $\tau$-limited turbofan, how would the T-s diagrams be affected if the following changes were made? In all cases, the compressor and fan pressure ratios are the same for the baseline and modified cycle. When useful, add statements like “this $\Delta T = $ that $\Delta T$,” “this area = that area,” etc. In some cases there may be no change. Please make your modifications clear; cycles that look like random scribbles and have no explanations don’t get much credit!

<table>
<thead>
<tr>
<th>a)</th>
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<tr>
<td><img src="image" alt="Diagram" /></td>
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<td>the aircraft is taken to planet X whose atmospheric pressure is twice that of earth, but the ambient temperature and all other properties of the atmosphere, fuel and engine are the same as on earth.</td>
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b) The fan is removed and an afterburner with the same $\tau_{\text{limit}}$ as the turbine is added.

c) Half way through the standard (constant pressure) burn, abnormal combustion occurs which results in constant volume combustion. The same total amount of fuel is burned as in the baseline cycle ($\tau_{\text{limit}}$ cannot be honored in this case). All other components operate normally.

d) A new fuel is used that has 10% higher heating value.
Problem #5 (15 points total)

The following 5 changes to a $\tau_{\lambda}$-limited turbofan engine flying at subsonic conditions ($M_1 = 0.8$) are being considered:

1) Use a new fuel with twice the heating value per unit mass ($Q_R$)
2) Increase ambient pressure ($P_1$) by a factor of 2 (ambient temperature $T_1$ not changed)
3) Increase ambient temperature ($T_1$) by a factor of 2 (ambient pressure $P_1$ not changed)
4) Increase the flight Mach number $M_1$ from 0.8 to 1.6
5) Use a new wing with twice the lift to drag ratio (L/D) of the original wing

All other properties of the engine, e.g. bypass ratio ($\alpha$), compressor pressure ratio ($\pi_c$), fan pressure ratio ($\pi_c'$), engine size, turbine inlet temperature limit ($\tau_{\lambda}$), etc. are held constant.

Briefly answer the following questions (no credit without explanation!) In some cases there might be more than one potentially “correct” answer; if so, any one of those answers are acceptable. Do not list more than one answer for each part.

If only one of these 5 changes were implemented:

a) Which change would increase specific thrust ($\text{Thrust}/m_a c_i$) the most?
b) Which change would decrease thrust (not specific thrust) the most?
c) Which change would increase Thrust Specific Fuel Consumption (TSFC) the most?
d) Which change would increase propulsive efficiency the most?
e) Which change would increase aircraft range the most?

Problem #6 (15 points)

For turbofan example at the end of Lecture 13, using aircycles4propulsion.xls, determine what combination of bypass ratio ($\alpha$) and fan pressure ratio ($\pi_c'$) (changing nothing else) gives the minimum thrust specific fuel consumption under the following 3 conditions:

a) Ideal cycle (all component efficiencies = 1) as in Lecture 13
b) Ideal cycle (all component efficiencies = 1) but with drag coefficient = 0.1

You don’t have to show any calculations, just use the spreadsheet to find the optima under these conditions, but answer the following questions:

1) Why was the answer to (a) $\alpha \to \infty$, $\pi_c' \to 1$?
2) Why was the optimal $\alpha$ smaller for part (b) than (a)?