Cite as: Ronney, P. D., "Heat-Recirculating Combustors," Chapter 10 in *Microscale Combustion and Power Generation* (Y. Ju, C. Cadou and K. Maruta, Eds.), Momentum Press LLC, New York, 2015, pp. 287 – 320.

Heat-recirculating combustors

Paul D. Ronney

Department of Aerospace and Mechanical Engineering, 3650 McClintock Ave., OHE 430J University of Southern California, Los Angeles, CA, USA 90089-1453 1- 213-740-0490 ronney@usc.edu

A major challenge for any small-scale combustion device is to avoid flame extinction via heat losses. These losses are more significant at small scales than larger ones due to the larger surface area to volume ratio and thus larger heat loss to heat generation ratio as the device scale decreases. For this reason, many researchers have considered the use of heat-recirculating combustors to minimize the detrimental effects of such losses. In this chapter, a simple analysis of linear and spiral counter-current heat-recirculating combustors is conducted to identify the dimensionless scaling parameters expected to quantify the performance of such heat-recirculating combustors. The predictions of this simple analysis are compared to 3D numerical models. By adjustment of property values, it was confirmed that four dimensionless parameters were sufficient to characterize combustor performance at all scales: the Reynolds number, a heat loss coefficient, a Damköhler number and a radiative transfer number. The key (and detrimental) role of streamwise heat conduction along the heat exchange dividing wall is discussed and characterized through a Biot number. Substantial differences between the performance of linear and spiral combustors are found and can explained in terms of the effects of the area exposed to heat loss to ambient and the sometimes detrimental effect of increasing heat transfer to adjacent outlet turns of the spiral exchanger. Practical aspects of the performance of heat-recirculating combustors, including the effects of the number of turns, height, wall thermal conductivity, turbulence and catalysis, are discussed.

Nomenclature

a	Exponent in scaling relation for turbulent flow (Nu $\sim \text{Re}^n$)			
A _T	Heat exchange area (m ²)			
A	Heat loss area (m ²)			
A _x	Cross-section area of heat exchanger channel (m ²)			
Bi	Biot number			
Ср	Gas heat capacity (J/kgK)			
d	Channel width (m)			
D	Mass diffusivity			
Da	Damköhler number			
Е	Dimensionless excess enthalpy			
Ea	Activation energy (J/mole)			
h _T	Convective heat transfer coefficient inside channel (W/m^2K)			
h _L	Convective heat loss coefficient (W/m^2K)			
k _g	Gas thermal conductivity (W/mK)			
k _w	Wall thermal conductivity (W/mK)			
L	Channel length (m)			
m	Mass flow rate (kg/s)			
М	Fuel molecular weight (kg/mole)			
n	Number of turns of spiral heat exchanger			
Ν	Number of Transfer Units			
Nu	Nusselt number inside channel			
Pr	Prandtl number = $\mu/\rho\kappa$			
Q	Fuel heating value (J/kg)			
Q _{L,i}	Heat loss from inlet channel (W)			
Q _{L,o}	Heat loss from outlet channel (W)			
Q _T	Heat exchange rate from products to reactants (W)			
Q_{w}	Wall-to-wall radiative transfer rate (W)			
R	Internal radiation coefficient			
\mathcal{R}	Gas constant (J/moleK)			
Re	Reynolds number of flow inside channel			
Т	Temperature (K)			
UT	Overall heat transfer coefficient inside the channel (W/m ² K)			
UL	Overall heat transfer coefficient to environment (W/m^2K)			
U _R	Heat transfer coefficient for internal wall-to-wall radiation (W/m ² K)			
v	Gas flow velocity inside channel			
V	Reaction zone volume (m ³)			
Y	Fuel mass fraction			
Y_{∞}	Fuel mass fraction in the fresh reactants (= Y_1)			
Ζ	Pre-exponential factor in Arrhenius reaction rate (1/s)			
α	Heat loss coefficient			
ε	Internal wall emissivity			
ε _L	External wall emissivity			
к	Gas thermal diffusivity			
u	Gas dynamic viscosity			
0	Gas density			

σ	Stefan-Boltzmann constant (W/m^2K^4)
τ	Heat exchanger wall thickness

1. Introduction

It is well known [1, 2, 3, 4, 5] that hydrocarbon fuels store at least 50 times more energy per unit mass than the state-of-the-art batteries. As a consequence, many researchers have attempted to develop devices that convert hydrocarbon fuels to electrical power at small scales in applications where traditionally batteries are employed. Although conversion of hydrocarbons to electricity at large scales using internal combustion engines is routine, because of issues associated with heat and friction losses at small scales it has proven difficult to employ the same technologies at small scales. An alternative approach frequently studied is that of minimizing the impact of heat losses and avoiding moving parts using heat-recirculating reactors and coupling these reactors to thermoelectric devices [6] or fuel cells [7] to generate electrical power. This chapter discusses the behavior of heat-recirculating combustors with an emphasis on scaling, particularly how size affects performance as characterized by reactor temperature and extinction limits, as a function of mass flow or Reynolds number (Re). This information is essential to the development of practical micropower generation systems. Because of fabrication limitations it is difficult to build and test geometrically similar devices of widely varying scales; instead, it is often preferable to test scaled-up (i.e., laboratory-scale) devices that are easily built and instrumented and use appropriate dimensionless parameters to predict the performance at smaller scales.

In this chapter, first an approximate mixing-cup analysis of a simple linear countercurrent combustion is used to identify the governing dimensionless parameters and general performance characteristics of heat recirculating combustors. These results are then extended to more detailed analyses and computations to understand the limitation of the simplified analyses. Finally the practical limitations on the performance of heat-recirculating combustors due to manufacturing, materials and catalysis are discussed. Emphasis is placed on spiral countercurrent combustors because, as the results will show, for practical conditions the performance (in terms of the range of mass flow rates, extinction limits, excess enthalpy, etc.) attainable from spiral combustors greatly exceeds that of combustors in linear or other geometries.

2. Simplified analysis

2.1 Linear exchanger

An elementary model of a linear counter-current heat-recirculating combustor (Figure 1) is used to identify the relevant non-dimensional parameters, using approximations enabling simple closed-form solutions to be obtained. The reactants are preheated from ambient temperature T_1 to the preheat temperature T_2 in the inlet arm of the heat exchanger. The reactants are presumed to be inert in this region. The temperature then increases from T_2 to T_3 due chemical reaction in the combustor, which initially will be presumed to be a well-stirred reactor (WSR). Finally the temperature of the products of combustion decreases from T_3 to T_4 in the outlet arm of the heat exchanger due to heat transfer to the reactants. The products are presumed to be inert in this region as well. Assuming equal heat capacity (C_p) of the reactant and product streams, the energy balances for the inlet and outlet arms of the heat exchanger and the WSR are, respectively,

$$\dot{m}C_{P}(T_{2}-T_{1}) = Q_{T} - Q_{L,i}$$
(1a)

$$\dot{m}C_{P}(T_{4}-T_{3}) = Q_{T} - Q_{L,e}$$
(1b)

$$\dot{m}C_P(T_3 - T_2) = \dot{m}Q(Y_3 - Y_2) \tag{1c}$$

where Q_T is the rate of heat transfer from the product to reactants across the heat exchanger and $Q_{L,i}$ and $Q_{L,e}$ are, respectively, the rates of heat loss the reactant (inlet) and product (exhaust) sides of the heat exchanger to ambient. It is presumed that the volume of the WSR is small compared with the overall size of the heat exchanger and thus heat loss from the WSR is neglected. Initially it is presumed that complete conversion of reactants to products occurs in the WSR, i.e. that the chemical reaction time scale is much shorter than the residence time in the WSR. In this case $Y_3 = 0$ and thus the temperature jump in the WSR (ΔT_R) is given by $T_3 - T_2 = Y_{\infty}Q/C_P = \Delta T_C$, where ΔT_C is the adiabatic temperature rise for complete combustion and $Y_{\infty} =$

 $Y_1 = Y_2$ is the fuel mass fraction in the incoming fresh mixture. Finite-rate chemical reaction will be discussed in a subsequent section.

Equations (1a) and (1b) are the same as employed by Jones *et al.* [8]; these authors estimated the heat recirculation Q_T via the "mixing cup temperature" assumption, that is, the average temperatures for the purposes of heat transfer on the inlet and outlet arms are $(T_1 + T_2)/2$ and $(T_3 + T_4)/2$, respectively. Using this assumption here also and furthermore using mixing cup temperatures to estimate the terms for heat loss to ambient one obtains:

$$Q_T = U_T A_T \left(\frac{T_3 + T_4}{2} - \frac{T_1 + T_2}{2} \right)$$
(2a)

$$Q_{L,i} = U_L A_L \left(\frac{T_1 + T_2}{2} - T_1 \right)$$
(2b)

$$Q_{L,o} = U_L A_L \left(\frac{T_3 + T_4}{2} - T_1 \right)$$
(2c)

where U_T is the overall heat transfer coefficient between the product and reactant sides of the heat exchanger and U_L is the coefficient of heat loss to ambient. If the dividing wall between the inlet and outlet arms of the heat exchanger has negligible thermal resistance compared to the thermal resistance from the hot combustion products to this wall and from this wall to the cold reactants, N can be estimated by noting that the overall heat transfer coefficient $U_T = h_T/2$, where h_T is the usual convective heat transfer coefficient for channel flow, assumed to be the same for the inlet and outlet arms of the exchanger. For a linear exchanger the heat exchange area (A_T) and the area exposed to heat loss (A_I) will be nearly the same, though of course there are two surfaces exposed to heat loss, one on the reactant side and one on the product side, whereas there is only one surface for heat exchange between the product and reactant sides.

Equations (1) and (2) represent 6 equations for the unknown temperatures T_2 , T_3 and T_4 and heat fluxes Q_{T_1} , Q_{L_1} and Q_{L_2} (T_1 and temperature rise due to combustion $\Delta T_R = \Delta T_C = T_3$ - T_2 are specified values.) These equations can be solved to obtain the performance of the exchanger in terms of the excess enthalpy (E) (the temperature rise in the reactants due to heat recirculation non-dimensionalized by the temperature rise due to combustion) as a function of the Number of Transfer Units (N) and a dimensionless heat loss coefficient (α):

$$E = \frac{4N}{4 + \alpha N \left[4 + N \left(2 + \alpha\right)\right]}; E = \frac{T_2 - T_1}{\Delta T_C}; N = \frac{U_T A_T}{\dot{m} C_P}; \alpha = \frac{U_L A_L}{U_T A_T}$$
(3)

Figure 2a shows the effect of α on E predicted by Eq. (3). Without heat loss ($\alpha = 0$), the excess enthalpy is related to the Number of Transfer Units by the simple relation E = N. As α increases, E decreases, especially at higher N (*i.e.*, lower flow rate or more rapid heat transfer from products and reactants). According to this model, for any $\alpha > 0$, when N is sufficiently large, increasing N further actually yields lower E due to recycled thermal enthalpy being lost to ambient rather than increasing the enthalpy of the reactants. This partially explains why a lowvelocity (large N) extinction limit always exists in heat-recirculating combustors despite the fact that N can be extremely large in some experiments [9] – as much as 10 for gas-phase combustion and 330 for catalytic combustion. Such large values of N would lead to extremely large values of E and thus very broad extinction limits in a linear device if it were truly adiabatic. On the other hand heat losses alone do not explain the low-velocity limit as will be discussed in section 2.4.

Since Eq. (3) uses N to represent different operating conditions (e.g. different flow rates and sizes), it requires knowledge of U_T and thus h_T , which in turn must be estimated from the Nusselt number Nu = $h_T d/k_g$ which is not known *a priori*. Moreover, the scaling of h_T with flow rate changes between laminar and turbulent flows. For these reasons it is preferable to characterize the flow in terms of the Reynolds number Re = $\rho v d/\mu$ since all of these properties are known *a priori* from the experimental conditions. Assuming fully-developed channel flow, Nu can usually be approximated by an expression of the form Nu ~ Re^aPr^b, where Pr is the Prandtl number; since Pr is close to unity for most gases and usually b < 1, the effects of Pr will be neglected here. Hence, for geometrically similar devices ($A_T \sim d^2$, $A_X \sim d^2$):

$$N = \frac{U_T A_T}{\dot{m}C_P} \sim \frac{\frac{1}{2} \frac{Nu \, k_g}{d} A_T}{\rho v A_X C_P} \sim Nu \frac{\kappa}{v d} \sim Nu \frac{\rho \kappa}{\mu} \frac{\mu}{\rho v d} \sim Re^a \frac{1}{Pr} \frac{1}{Re} \sim Re^{a-1}$$
(4)

For laminar flow a = 0 whereas for turbulent flow in straight channels, $a \approx 0.8$ for straight channels. For the spiral heat exchangers discussed in Section 2.2, the curvature of the channels may affect the value of a due to the formation of Dean vortices. These effects were discussed in the context of heat-recirculating combustors in [10] and it was found that when the combined effects of turbulence and curvature are considered, for Re > 500 Nu is very nearly proportional to Re¹, thus the scaling relation Nu ~ Re^a at high Re is considered valid. Whether a = 0.8 or 1.0 does not affect the following discussion; it is only important that Nu scales with Re only for geometrically similar heat exchangers without separate influence of other parameters. Consequently, for both laminar and turbulent flows Re may be substituted for N as a scaling parameter. Also, since a is rarely if ever larger than unity, Eq. (4) shows that N will not increase with increasing Re even for turbulent flows because the increase in U_T is offset by the increase in \dot{m} .

For geometrically similar devices α scales according to

$$\alpha \sim \frac{U_L}{\frac{1}{2} \frac{Nu k_g}{d}} \sim \frac{1}{Re^a} \frac{U_L d}{k_g}$$
(5)

Typically the external heat loss is due to buoyant convection or radiative transfer, for both of which the loss per unit area is nearly independent of scale. If this is the case then for fixed Re, $\alpha \sim d^1$, i.e. α increases linearly with increasing scale. For extremely small combustors heat loss to the surroundings may be dominated by conduction instead of convection or radiation, in which case $U_L \sim d^{-1}$ and thus α is independent of scale.

2.2 Spiral exchanger

Linear exchangers suffer from a large ratio of heat loss area (A_{I}) to heat exchange area (A_{T}), generally about 2 as discussed earlier. One way of minimizing the impact of heat losses is to roll up the linear device into an n-turn spiral "Swiss roll" combustor [11, 12]. In this way for a heat exchanger of overall length L, the length exposed to heat loss to ambient decreases from 2L to L/n and thus A_{L} decreases by a factor of about 1/2n. Additionally, the remaining portion of what were the outer walls of the linear heat exchanger exposed to heat loss become heat transfer area and thus A_{T} increases by a factor of (2L - L/n)/L = 2 - 1/n. Hence, rolling a linear exchanger into a spiral device increases N by a factor of about 2 - 1/n and decreases α by a factor of 2(2n-1). Consequently, the Swiss roll is one of the most thermally efficient types of heat exchangers, though it has limitations at high N as will be discussed in Section 2.2.

Analysis for spiral exchangers is more elaborate since the energy balances on individual turns are coupled and their contributions to A_T vary according to distance from the center of the spiral. For the current purposes the device of Fig. 1 was cut into thirds and stacked to simulate (in the thermal sense) a spiral having 3 equal-length "turns," with the outlet of each turn being the inlet to the next (Figure 3). The WSR sits between the end of the last inlet arm of the exchanger and the beginning of the first outlet arm. For the inlet turns the energy conservation equations are (refer to Eqs. (1) and (2)):

$$\dot{m}C_{P}\left(T_{2}-T_{1}\right) = U_{T}A_{T}\left(\frac{T_{7}+T_{8}}{2}-\frac{T_{1}+T_{2}}{2}\right) - U_{L}A_{L}\left(\frac{T_{1}+T_{2}}{2}-T_{1}\right)$$

$$\dot{m}C_{P}\left(T_{3}-T_{2}\right) = U_{T}A_{T}\left(\frac{T_{6}+T_{7}}{2}-\frac{T_{2}+T_{3}}{2}\right) + U_{T}A_{T}\left(\frac{T_{7}+T_{8}}{2}-\frac{T_{2}+T_{3}}{2}\right) - U_{L}A_{L}\left(\frac{T_{2}+T_{3}}{2}-T_{1}\right)$$

$$\dot{m}C_{P}\left(T_{4}-T_{3}\right) = U_{T}A_{T}\left(\frac{T_{5}+T_{6}}{2}-\frac{T_{3}+T_{4}}{2}\right) + U_{T}A_{T}\left(\frac{T_{6}+T_{7}}{2}-\frac{T_{3}+T_{4}}{2}\right) - U_{L}A_{L}\left(\frac{T_{3}+T_{4}}{2}-T_{1}\right)$$
(6a)

For the outlet turns energy conservation requires:

$$\dot{m}C_{P}\left(T_{8}-T_{7}\right) = U_{T}A_{T}\left(\frac{T_{1}+T_{2}}{2}-\frac{T_{7}+T_{8}}{2}\right) + U_{T}A_{T}\left(\frac{T_{2}+T_{3}}{2}-\frac{T_{7}+T_{8}}{2}\right) - U_{L}A_{L}\left(\frac{T_{7}+T_{8}}{2}-T_{1}\right)$$
$$\dot{m}C_{P}\left(T_{7}-T_{6}\right) = U_{T}A_{T}\left(\frac{T_{2}+T_{3}}{2}-\frac{T_{6}+T_{7}}{2}\right) + U_{T}A_{T}\left(\frac{T_{3}+T_{4}}{2}-\frac{T_{6}+T_{7}}{2}\right) - U_{L}A_{L}\left(\frac{T_{6}+T_{7}}{2}-T_{1}\right)$$
(6b)
$$\dot{m}C_{P}\left(T_{6}-T_{5}\right) = U_{T}A_{T}\left(\frac{T_{3}+T_{4}}{2}-\frac{T_{5}+T_{6}}{2}\right) - U_{L}A_{L}\left(\frac{T_{5}+T_{6}}{2}-T_{1}\right)$$

Simplifying these equations and non-dimensionalizing temperatures with respect to T1 (denoted

by \tilde{T}) yields

$$(2+N+\alpha N)\tilde{T}_{2} - N\tilde{T}_{7} - N\tilde{T}_{8} = 2 - N + \alpha N$$

$$(-2+2N+\alpha N)\tilde{T}_{2} + (2+2N+\alpha N)\tilde{T}_{3} - N\tilde{T}_{6} - 2N\tilde{T}_{7} - N\tilde{T}_{8} = 2\alpha N$$

$$(-2+2N+\alpha N)\tilde{T}_{3} + (2+N+\alpha N)\tilde{T}_{4} - 2N\tilde{T}_{6} - N\tilde{T}_{7} = 2\alpha N + N\Delta\tilde{T}_{C}$$

$$-2N\tilde{T}_{2} - N\tilde{T}_{3} + (-2+2N+\alpha N)\tilde{T}_{7} + (2+2N+\alpha N)\tilde{T}_{8} = N + 2\alpha N$$

$$(7)$$

$$-N\tilde{T}_{2} - 2N\tilde{T}_{3} - N\tilde{T}_{4} + (-2+2N+\alpha N)\tilde{T}_{6} + (2+2N+\alpha N)\tilde{T}_{7} = 2\alpha N$$

$$-N\tilde{T}_{3} + (-2+\alpha N)\tilde{T}_{4} + (2+N+\alpha N)\tilde{T}_{6} = 2\alpha N + (2-N-\alpha N)\Delta\tilde{T}_{C}$$

This is a set of 6 linear equations for the unknowns \tilde{T}_2 , \tilde{T}_3 , \tilde{T}_4 , \tilde{T}_6 , \tilde{T}_7 and \tilde{T}_8 in terms of specified thermal parameters N, α and $\Delta \tilde{T}_R = \Delta \tilde{T}_C = \tilde{T}_5 - \tilde{T}_4$. Results are expressed in Fig. 2b in terms of the excess enthalpy $E = (\tilde{T}_4 - 1) / \Delta \tilde{T}_C = (T_4 - T_1) / \Delta T_C$ as a function of N for varying α .

The analysis results in the same dimensionless parameters as with the linear device but to compare the results of the linear and simulated spiral devices, three observations should be noted. First, in the above equations N is based on the area (A_T) of one heat exchange surface. Since there are 2n - 1 = 5 heat transfer surfaces between products and reactants, A_T for the purposes of defining N must be based on the total area available for heat recirculation, meaning $N_{spiral} = N_{linear}/5$. Second, heat loss occurs only from the first inlet turn and thus to simulate a spiral exchanger α is set to zero for all but the first equation, but for comparison with the linear exchanger the same definition of α is retained, *i.e.*, A_L is the area of one exterior side of the linear exchanger; if instead α is defined based on the area exposed to heat loss to ambient, the values

of α for the spiral exchanger reported here should be decreased by a factor of 3. Third, in this analysis heat loss in the third dimension (out of the plane of the spiral) is neglected; this is justified for a spiral heat exchanger that is sufficiently tall compared to its diameter or one that is wrapped in the third dimension to create a toroidal device. This assumption will be justified in Section 4.2, however, it will also be shown that for a short device this mode of heat loss cannot be neglected.

The results of the analysis of the simulated spiral device are shown in Fig. 2b, along with results of a much more detailed analysis of an adiabatic 3-turn spiral exchanger by Targett et al. [13]. The close agreement with Targett *et al.*'s results indicates that the highly simplified analysis given here is satisfactory for the current purposes. Figure 2b reveals that at small N, E = N as with the linear exchanger, whereas at larger N, even for the adiabatic case E reaches a maximum value then decreases. As discussed by Churchill and collaborators [13, 14], this occurs because if N is too large, heat transfer from one outlet channel to the adjoining inlet channel will be too rapid and the temperature of this inlet channel will become hotter than the next-cooler (farther toward the outside of the device) outlet channel and some heat transfer from this inlet channel to the cooler adjacent outlet channel will result, rather than the inlet channel receiving thermal enthalpy from both adjacent outlet channels. This cannot occur with linear device since heat transfer only occurs from one side of the outlet channel to the adjacent side of the inlet channel. As a result, for a truly adiabatic system the linear device provides larger excess enthalpy for a given N and thus will exhibit broader extinction limits, however, in the presence of heat losses, spiral exchangers provide substantially larger excess enthalpy. Comparing Figs. 2a and 2b it can be seen that for a given N (thus Re), the spiral exchanger can provide the same value of excess enthalpy E at much larger value of the heat loss α . Even in the limit $\alpha \rightarrow \infty$, only the outermost inlet turn becomes ineffective in terms of heat recirculation; this turn insulates the inner turns from heat losses. Of course, as N increases the rate of heat transfer from the inner turns to the

outermost one increases and more enthalpy is lost to ambient, resulting in significant decreases in E at large N even for the spiral device.

Figure 2c shows a comparison of the performance of simulated 3, 4 and 5-turn spiral exchangers, the latter two using sets of equations identical to Eq. (7) extended to include the effects of the additional turns. Results are shown in the two extreme cases of zero and infinite heat loss coefficient α . It can be seen that, as expected, a larger number of turns results in better performance, even for the same N (i.e., the same mass flow rate, heat transfer coefficient and overall exchanger length). Moreover, the performance of an adiabatic n-turn device is practically the same as an n+1 turn device with infinite heat loss coefficient, because in this limit the outermost inlet turn (the one exposed to heat loss) remains at ambient temperature but all other turns are nearly unaffected by this loss.

2.3 Finite-rate chemistry

While the above equations identify the dimensionless groups N and α needed to describe the heat exchanger performance, additionally the finite rate of thermal enthalpy release due to chemical reaction must be considered in order to determine extinction limits. The coupling between heat exchange and chemical reaction exists because any factor that decreases the reactor temperature ($T_R = T_3$ for the linear device, T_5 for the simulated 3-turn device) will decrease the thermal enthalpy release rate; if this rate drops sufficiently some reactants will not be converted to products within the available residence time in the WSR and thus the temperature rise due to combustion ($\Delta T_R = T_3 - T_2$ for the linear device, $T_5 - T_4$ for the simulated 3-turn device) will decrease below the value for complete reaction (ΔT_C), leading to less heat recirculation from products to reactants, leading to a further decrease in reactor temperature and eventually to extinction. Probably the simplest approach to modeling this thermal enthalpy release is a classical WSR with single-step chemical reaction. The motivation for using a WSR model is that experimental [9] and numerical [10, 15] results show that, at least for sufficiently high Re, near extinction limits the reaction zone structures in Swiss roll combustors are more similar to WSRs than propagating premixed flames; compared to propagating flames the reaction zones in Swiss roll combustors have much smaller temperature gradients, lower peak temperatures and longer residence times at high temperature. Combining a first-order reaction rate expression (mass per unit volume per unit time) of the form $Z\rho Y_R \exp(-E_a/\Re T_R)$ with the enthalpy balance across the reactor $Q(Y_{\infty} - Y_R) = C_P \Delta T_R$, a form of the usual Well-Stirred Reactor expression is obtained:

$$\Delta T_R = \frac{\Delta T_C}{1 + 1/[DaN \exp(-E_a/\Re T_R)]}; Da = \frac{\rho C_P Z V}{U_T A_T}$$
(8)

where the DaN = $\rho ZV/\dot{m}$ has been written to show the dependence on N explicitly. Written this way, the Damköhler number Da is a constant (independent of \dot{m}), except for turbulent flows in which case U_T will become approximately linearly proportional to \dot{m} . Note that in the limit of sufficiently high Da or N, reaction is nearly complete and ΔT_R approaches ΔT_C . The scaling of Da is given by

$$Da = \frac{\rho C_P Z V}{U_T A_T} \sim \frac{\rho C_P Z d^3}{\frac{1}{2} \frac{N u k_s}{d} d^2} \sim \frac{Z \rho d^2 \operatorname{Pr}}{N u \mu} \sim \frac{Z \rho d^2}{R e^a \mu}$$
(9a)

and that of DaN by

$$DaN \sim \frac{Z\rho d^2}{Re^a \mu} Re^{a-1} \sim \frac{Z\rho d^2}{Re \ \mu}$$
(9a)

Since μ , ρ and Z are molecular properties independent of scale, for fixed Re, both Da ~ d² and DaN ~ d².

By replacing ΔT_c (for complete combustion) with ΔT_R (for finite-rate reaction) as given by Eq. (3) (for the linear exchanger) or Eq. (7) (for the simulated 3-turn exchanger) and combining these expressions with Eq. (8), the effects of finite-rate chemistry on combustor performance can be assessed. For the linear exchanger a single relation for the reactor temperature $T_R = T_3$ is obtained, which in dimensionless form is

$$\frac{\tilde{T}_3 - 1}{F + 1} = \frac{\Delta \tilde{T}_C}{1 + 1/\left[DaN \exp\left(-\beta/\tilde{T}_3\right)\right]}; F = \frac{4N}{4 + \alpha N\left[4 + N\left(2 + \alpha\right)\right]}$$
(10).

Figure 4 shows the response of dimensionless reactor temperature \overline{T}_R to N for adiabatic ($\alpha = 0$) and non-adiabatic ($\alpha = 0.1$) conditions for the linear and simulated 3-turn exchangers. As discussed in Section 2.4, the values of Da and β are chosen to match those employed in a prior investigation [16] using a more detailed analysis of linear exchangers, the results of which are also shown in Fig. 4. First note that for both the linear and simulated spiral exchangers, without heat loss there is a low-N (high flow rate) extinction limit but no high-N (low flow rate) limit. The value of N at the low-N limit is nearly the same for the linear and simulated spiral exchangers, which is reasonable because for adiabatic conditions at low values of N, the excess enthalpy (E) is nearly the same for the two types of exchangers (Figure 2b). For large N, the temperature increases much more rapidly for the linear exchanger and unlike the 3-turn device does not reach a maximum value. This is consistent with the response of excess enthalpy (E) to N for the linear device as compared to the simulated 3-turn devices (Fig. 2b.) With heat loss, the low-N limits do not change significantly for either type of device but as N increases, the 3-turn device exhibits considerably better performance. This is also consistent with the results shown in Fig. 2b.

Figure 4 shows a high-N (low flow rate) extinction limit with heat loss as a consequence of the isola response of \tilde{T}_R to N. While dual (low and high-flow rate) limits are common in combustion systems, for heat recirculating combustors this low flow rate limit shown in Fig. 4 is actually an artifact of the use of the mixing-cup approximation which uses average temperature differences to compute heat recirculation and heat loss. The mixing-cup approximation is reasonable if the temperature profiles along the inlet and outlet arms of the heat exchanger are nearly linear, which is valid for moderate heat loss. If there are substantial heat losses then the entire inlet end of the exchanger is at near-ambient temperature with only a small region near the WSR end having temperatures above ambient, in which case the mixing-cup method overestimates the impact of heat loss. A more detailed model that does not suffer from this limitation is presented in the next section.

2.4 Detailed analytical model

In this section the model developed in section 2.2 is replaced by one using energy balances applied to infinitesimal elements of the heat exchanger rather than one applied globally for each arm of the exchanger. The model in this section also includes the effect of heat conduction along the wall dividing the inlet and outlet arms of the exchanger, which is critical to understanding the performance of heat-recirculating combustors [16]. Referring to Fig. 5, energy balances on the wall, inlet arm and outlet arm of the exchanger readily yield

$$k_{w}\tau \frac{d^{2}T_{w}}{dx^{2}} + 2U_{T}(T_{e} - T_{w,e}) - 2U_{T}(T_{i} - T_{w,i}) = 0$$
(11a)

$$\frac{\dot{m}C_{P}}{A_{T}/L}\frac{dT_{i}}{dx} - 2U_{T}(T_{w,i} - T_{i}) + U_{L}(T_{i} - T_{\infty}) = 0$$
(11b)

$$\frac{\dot{m}C_{P}}{A_{T}/L}\frac{dT_{e}}{dx} - 2U_{T}(T_{e} - T_{w,e}) - U_{L}(T_{e} - T_{\infty}) = 0$$
(11c)

In the above relations it has been assumed, as discussed in section 2.1, that the heat transfer coefficients are constant and equal on the inlet and outlet arms of the exchanger with negligible thermal resistance across the dividing wall, in which case $U_T = h_T/2$. Using the mean wall temperature $T_w \approx (T_{w,e} + T_{w,i})/2$, invoking the thermally-thin assumption $(T_{w,e} - T_{i,w} << T_e - T_i)$ and combining Eqs. (11a – c) yields a fourth-order differential equation for wall temperature:

$$\frac{1}{N^2 Bi \,\alpha(2+\alpha)} \frac{d^4 \tilde{T}_w}{d\tilde{x}^4} - \left[\frac{1}{N^2 \alpha(2+\alpha)} + \frac{2+\alpha}{Bi \,\alpha}\right] \frac{d^2 \tilde{T}_w}{d\tilde{x}^2} + \tilde{T}_w = 1; \quad \tilde{T} = \frac{T}{T_1}; \quad \tilde{x} = \frac{x}{L}; \quad Bi = \frac{4U_T L^2}{k_w \tau}$$
(12)

where introducing the effects of wall thermal conductivity generates a new parameter, namely a Biot number (Bi). Note that for geometrically similar devices the scaling of Bi is given by

$$Bi = \frac{4U_T L^2}{k_w \tau} \sim \frac{\frac{Nu k_g}{d} d^2}{k_w d} \sim Re^a \frac{k_g}{k_w}$$
(13)

and thus for a given Re, Bi is independent of scale and for laminar flow (a = 0) is simply a constant except to the extent that k_g and k_w vary with temperature.

The boundary conditions for Eq. (12) are (1) the temperature at the inlet of the exchanger is ambient; (2) the temperature rise across the WSR is given by Eq. (8), (3) the inlet end of the wall is adiabatic and (4) the WSR end of the wall is adiabatic. The assumption of adiabatic wall ends does not affect the results substantially; using a convection boundary condition rather than an adiabatic one changes the results only slightly for realistic choices of property values [16]. Since the first two boundary conditions are in terms of \tilde{T}_i and \tilde{T}_e rather than \tilde{T}_w , to solve this system of equations first $\tilde{T}_w(\tilde{x})$ is found using Eq. (12) then this result is applied to Eqs. (11a) - (11c) to find $\tilde{T}_i(\tilde{x})$ and $\tilde{T}_e(\tilde{x})$. The process is straightforward but tedious and not reproduced here; see [16] for a detailed derivation. Results of the analysis are shown in Figs. 4, 6 and 7.

Examples of the heat exchanger temperature profiles $\tilde{T}_i(\tilde{x})$, $\tilde{T}_w(\tilde{x})$ and $\tilde{T}_e(\tilde{x})$ predicted by this detailed analysis are shown in Figure 6. For infinite reaction rate (Da = ∞), adiabatic (α = 0) and wall-conduction-free (Bi = ∞) conditions, Fig. 6 (top) shows that temperature profiles are linear. For this case, for any N the reactor temperature is $\tilde{T}_R = 1 + N\Delta \tilde{T}_C$, corresponding E = N as predicted by Eq. (3), and the exhaust temperature $\tilde{T}_e(0)$ is simply the adiabatic flame temperature 1+ $\Delta \tilde{T}_C$ as energy conservation requires. The response of \tilde{T}_R to N predicted by this analysis is shown in Fig. 4, where it is compared to the results of the simple analysis discussed in section 2.2. It can be seen that the results of the detailed model are identical to those of the simplified model, which is understandable since the temperature profiles are linear and thus mixing-cup model provides an accurate estimate of the heat transfer across the heat exchanger. For $\alpha = 0$ and Bi = ∞ but finite-rate reaction (Da $\neq \infty$), the temperature profiles will still be linear but the temperature jump at the WSR will decrease according to Eq. (8) and thus the reactor temperature \tilde{T}_R will be lower.

Figure 6 (middle) shows heat exchanger temperature profiles predicted by the detailed analytical model for infinite reaction rate ($Da = \infty$) and wall-conduction-free ($Bi = \infty$) conditions with heat loss ($\alpha > 0$). It can be seen that the temperatures are near ambient except near the WSR end of the exchanger ($\tilde{x} = 1$) and thus there is heat loss only from this end. In contrast, the simplified mixing-cup model essentially assumes linear profiles and thus has no means to capture this behavior. It should also be noted that for the case shown in Fig. 6 (middle), all of the thermal enthalpy generated can be lost to ambient rather than exhausted at the exchanger exit without extinction occurring. Thus, simply equating rates of heat generation and loss cannot always yield an extinction criterion.

The response of \tilde{T}_R to N for this non-adiabatic case is also shown in Fig. 4. It can be seen that for low N (large mass flow rates) where heat loss effects are minimal, the simplified and detailed models predict nearly the same extinction limit, whereas for large N the detailed model predicts no extinction limit whatsoever (as $N \rightarrow \infty$, \tilde{T}_R asymptotes to a fixed value) whereas the simplified model predicts isola behaviour indicating an extinction limit. This is because as $N \rightarrow \infty$, the detailed model predicts that heat recirculation is balanced by heat loss; above a certain value of N, increasing N further (e.g. in an experiment by decreasing the mass flow rate or increasing the length of the heat exchanger) has no effect other than to increase the fraction of the length of the exchanger where both the reactant and product streams remain at near-ambient temperatures. It can be shown that the reactor temperature in the limit $N \rightarrow \infty$ is

$$\tilde{T}_{e}(1) = \frac{(1 + \Delta \tilde{T}_{c})G - 1}{G - 1}; G \equiv \sqrt{\frac{1 + \alpha + \sqrt{\alpha(2 + \alpha)}}{1 + \alpha - \sqrt{\alpha(2 + \alpha)}}} \quad (Bi \to \infty, N \to \infty)$$
(14)

This observation is crucial to understanding the high-N (low Re) extinction limits because it indicates that, even in the presence of heat losses, without thermal conduction along the wall there is no means to reduce the reactor temperature as N is increased. This is quite different from combustors without heat recirculation, where sufficient reduction in mass flow rate or Re (thus increase in N) will nearly always lead to extinction due to heat losses.

While the detailed model predicts no extinction limit at high N / low flow rates, experiments [9, 10, 11, 12] do show that high-N limits do indeed exist, indicating that an additional mechanism is required to predict such extinction limits. One readily identifiable mechanism is that of heat losses in the out-of-plane dimension, which will be discussed in Section 4.2. Even this mechanism can be essentially eliminated by extending the height of the exchanger in the third dimension or wrapping the exchanger in the third dimension to create a toroidal device. However, even if heat losses in the third dimension are completely eliminated, another mechanism, namely thermal conduction along the wall dividing the inlet and outlet arms of the exchanger, can lead to high-N extinction limits without an additional loss mechanism. Figure 6 (bottom), which shows temperature profiles in the case of an adiabatic exchanger with wall heat conduction effects (Bi $\neq \infty$, $\alpha = 0$), provides insight into this mechanism. The temperature profiles clearly show that even though there is no heat loss in the system whatsoever, wall heat conduction removes thermal enthalpy from the high-temperature gas near $\tilde{x} = 1$ and returns this enthalpy to the gas at lower temperatures (smaller \tilde{x}), resulting in a lower reactor temperature than the adiabatic exchanger without wall heat conduction (Bi = ∞). (Nevertheless, the exit temperature $\tilde{T}_e(0)$ is $1 + \Delta \tilde{T}_c$ for both infinite (Fig. 6, top) and finite Bi (Fig. 6, bottom) because in both cases the system is adiabatic with respect to the surroundings ($\alpha = 0$)).

The significance of streamwise wall heat conduction is further elucidated in Figure 7, which shows the response of WSR temperature $\tilde{T}_R = \tilde{T}_e(1)$ to N for several values of the Biot number (Bi) under adiabatic and non-adiabatic conditions. As already shown in Fig. 4, for infinite reaction rate, adiabatic conditions and no streamwise wall thermal conduction response is monontically increasing corresponding to E = N and with finite-rate reaction a C-shaped extinction curve is found. In contrast, with wall conduction (finite Bi), the C-shaped response of

 \tilde{T}_R to N that occur for both adiabatic and non-adiabatic conditions become isolas with both lower and upper limits on N because conduction of thermal energy away from the WSR vicinity through the wall becomes significant at large N. Once conducted away from the WSR vicinity, some thermal energy is transferred back to the gas via convection and a portion of this energy is then lost to ambient. It is emphasized that this mechanism is important only at large N (low Re or mass flow rates), where wall conduction is competitive with gas-phase convection. Figure 7 also shows that the small-N extinction limit is extended slightly by wall conduction, since heat recirculation (thus WSR temperature) is low at small N (Eq. (3)), thus the increase in heat recirculation provided by wall conduction increases the WSR temperature slightly.

Figure 8 shows the effect of N on the fuel concentration expressed in terms of the minimum adiabatic temperature rise due to combustion $\Delta \tilde{T}_c$ required to sustain combustion (corresponding to the minimum fuel concentration, thus extinction limit). Without streamwise wall conduction (Bi = ∞) no small-*M* extinction limit exists for the reasons given in the previous paragraph. For finite Bi, both small-N (high mass flow rate) and large-N (low mass flow rate) limits exist due to finite residence time and heat losses, respectively. Consistent with Fig. 7, the large-N limit is slightly extended by decreasing Bi (thereby slightly increasing heat recirculation) whereas the small-N limit is drastically narrowed by decreasing Bi due to the mechanism described in the previous paragraph, i.e. conduction removing thermal enthalpy from the WSR region and thus reducing the reactor temperature and reaction rate.

It should be stressed that the value of Bi needed to affect extinction is much smaller than that which might be expected based on simplistic estimates. The overall ratio of streamwise convection to wall conduction is of the order $\dot{m}C_{\rm P}/(k_{\rm w}\tau A_{\rm T}/L^2) = {\rm Bi}/4{\rm N}$. Even for the Bi = 10,000 case shown in Fig. 8, where the effects of wall conduction might be thought to be negligible, the extinction limits are affected for all N > 50, thus Bi/4N > 50. Based on simplistic estimates, no effect wall conduction effects would be expected unless Bi/4N > 1. The powerful wall conduction effects result from the fact that the wall temperature gradients near the WSR are much larger when heat losses are present (Fig. 6, middle), i.e. much larger than the mean gradient under these conditions.

3. Scaling

3.1 Objectives

The analyses of Section 2 suggest that if the dimensionless groups N (or Re), α , Da and Bi are constant, the extinction limits should be the same for a given value of the adiabatic temperature rise due to combustion $\Delta \tilde{T}_c$ regardless of the physical size of the combustor. In particular the following questions arise:

- 1. Can a highly simplified analysis (i.e. one-dimensional heat transfer, constant property values, simplified heat recirculation and heat loss models, well-stirred reactor) be used to identify the dimensionless parameters describing the performance of heat recirculating combustors?
- 2. Are Re, α , Da and Bi a complete set of parameters?
- 3. Are these parameters applicable to both laminar and turbulent flows?

4. Are these parameters applicable to both linear and spiral (Swiss roll) combustors?

One way of addressing these questions is via experimentation, *i.e.*, construct geometrically similar devices of different sizes, test them at the same values of Re, α , Da and Bi, and determine if the operating temperatures for a given $\Delta \tilde{T}_c$ (or equivalence ratio or fuel mass fraction) and the values of $\Delta \tilde{T}_c$ at extinction are the same. In experiments it is very difficult to keep all dimensionless groups constant for combustors of different scales because, as will be shown in the next subsection, this requires adjustment of the heat loss coefficients, surface emissivities and reaction rate parameters. Instead, these questions will be addressed via numerical simulation since the material properties and operating conditions can readily be adjusted in the numerical

model.

The results of this section are presented in terms of the Reynolds number (based on the channel width (d), inlet flow velocity and the viscosity of the incoming fuel-air mixture at ambient conditions) rather than the number of transfer units N because N is built with the overall heat transfer coefficient U_T which is a calculated quantity rather than an input parameter *per se* and because U_T will vary within the exchanger due to changes in mixture temperature and composition. Of course Re also varies within the exchanger due to increases in temperature (which affects both the local flow velocity and mixture viscosity) but if the scaling analysis presented here is valid, the same temperature profiles in the exchanger and combustor will occur regardless of scale and thus temperature-dependent effects will be the same for all scales.

3.2 Computational model

A fully three-dimensional computational model using FLUENT 12.1 was used for the simulations. Details of the model and validation with experiments have been reported previously [10, 17]. For this study three 3.5 turn Swiss roll combustors were modeled. The nominal-scale combustor (Figure 9) was 5 cm tall with a channel width d = 3.5 mm and a wall thickness $\tau = 0.5$ mm; the other two combustors were geometrically identical but half and double the size of the nominal-scale device. The computational domain included the gaseous reactants and products, solid combustor walls and insulation on the top and bottom surfaces. As discussed in Section 4.2, computations with devices of different heights showed that the nominal 5 cm height chosen was sufficient to minimize the effects of heat loss in the direction out of the plane of the spiral. Both convective (nominally $h_L = 10 \text{ W/m}^2\text{K}$, typical of buoyant convection in ambient air) and radiative (nominally $\epsilon_L = 0.8$ for exterior walls and $\epsilon_L = 1$ for insulation) boundary conditions were used to simulate heat loss from the combustors. Symmetry was assumed at the midplane of the device (i.e. the bottom surface in Fig. 3) and thus only half the device was modeled, which would be inaccurate if buoyancy effects were important, however, calculations made without the

assumption of symmetry showed that buoyancy effects were negligible [17]. The Reynolds Stress Model (RSM) was employed to simulate the effects of turbulence on heat transfer. Propane-air combustion was simulated using single-step finite rate gas-phase chemistry with an activation energy $E_a = 40$ kcal/mole and the pre-exponential term adjusted to obtain agreement between the model and experiment at Re = 1000. No model of turbulence-chemistry interactions was used, which is consistent with the use of a WSR model in the analysis of Section 2.

3.3 Extinction limits without scaling

Figure 10 (upper) shows the predicted extinction limits as a function of Re for the three combustors without any adjustment of property values to obtain constant α or Da. (Note that Bi is independent of scale according to Eq. (13).) All three extinction limit curves exhibit the expected U-shaped behavior but the performance of the three combustors is clearly not identical even at the same Re (thus same N). In particular, at lower Re, smaller-scale combustors show better performance (lower lean extinction limits), whereas at higher Re, larger-scale combustors show better performance. These results can be explained as follows. As previously discussed, at low Re, extinction behavior is dominated by heat losses and according to Eq. (5) the heat loss parameter $\alpha \sim d^1$, thus smaller combustors are subject to less impact of heat loss (specifically α_{double} : $\alpha_{half} = 4 : 2 : 1$) and consequently will have wider extinction limits. On the other hand at high Re, extinction limits are caused by insufficient residence time relative to the chemical reaction time, *i.e.*, the WSR blows out. The ratio of residence time to chemical reaction time is of course characterized by Da. According to Eq. (9), Da $\sim d^2$, thus larger combustors have more residence time relative to the chemical reaction time (specifically Da_{double}: Da_{full} : Da

3.4 Extinction limits with scaling

To determine whether Re, α , Da and Bi are sufficient to characterize the performance of Swiss roll combustors, the simulations were repeated with property values adjusted so that these dimensionless parameters are the same for all three combustors. Specifically, the convective heat loss coefficient h_L and the emissivities for external radiative loss $\boldsymbol{\epsilon}_L$ were artificially adjusted in proportion to d-1 and the pre-exponential term in the reaction rate expression was artificially adjusted in proportion to d-2 (see Table 1). The computed results (not shown) were practically identical at high Re but at low Re the larger combustors perform more poorly, implying that other scale-dependent process(es) are significant at low Re. The dominant process was identified [17] as wall-to-wall radiative transfer, which previous studies [15] had shown significantly affects the extinction limits at low Re. In particular, radiation between internal heat exchange surfaces is a means of transferring thermal enthalpy away from the high-temperature reaction zone without recycling the enthalpy to the incoming reactants in a manner very similar to that of streamwise wall heat conduction, and thus is detrimental to combustor performance. (This mode of heat transfer does not exist in linear exchangers and thus could not be identified by the analysis in Section 2.) The radiative heat transfer (Q_R) between neighboring walls j and k of a spiral counter-current heat exchanger can be estimated assuming infinite parallel isothermal walls, i.e.

$$Q_{R} = \frac{\varepsilon_{i}}{2 - \varepsilon_{i}} \sigma A_{T} \left(T_{j}^{4} - T_{k}^{4} \right), \text{ and thus a radiative heat transfer coefficient } U_{R} = Q_{R} / A_{T} (T_{j} - T_{k}) \text{ can}$$

be estimated as

$$U_{R} = \frac{\varepsilon_{i}}{2 - \varepsilon_{i}} \sigma \left(T_{j}^{2} + T_{k}^{2}\right) \left(T_{j} + T_{k}\right)$$
(15).

Of course U_R is not a true coefficient since it depends on the temperature and thus will vary within the combustor, but if the scaling parameter for internal radiation is valid the temperature at a given location with the combustor and thus U_R will be the same regardless of scale.

Consequently another dimensionless group, namely the internal radiation coefficient R, can be identified:

$$R = \frac{U_R}{U_T} = \frac{\sigma}{U_T} \frac{\varepsilon_i}{2 - \varepsilon_i} \left(T_i^2 + T_j^2\right) \left(T_i + T_j\right)$$
(16).

The scaling of R is given by

$$R = \frac{\sigma}{U_T} \frac{\varepsilon_i}{2 - \varepsilon_i} \left(T_i^2 + T_j^2\right) \left(T_i + T_j\right) \sim \frac{\sigma d}{N u \, k_g} \frac{\varepsilon_i}{2 - \varepsilon_i} \sim \frac{\sigma d}{\operatorname{Re}^a k_g} \frac{\varepsilon_i}{2 - \varepsilon_i}$$
(17).

Since $R \sim d^1$, the three combustors will have different R according to R_{double} : R_{full} : $R_{half} = 4:2:1$. The larger impact of internal radiative transfer at larger scales explains why the larger combustors perform more poorly at small Re even when α and Da are the same for all combustors. To keep R constant for combustors of varying scale, ε_i was adjusted as shown in Table 1. (Note that $\epsilon_i/(2-\epsilon_i)$ rather than ϵ_i itself must be adjusted in proportion to d⁻¹.) The extinction limits computed with the adjusted values of ε_i are shown in Figure 10 (lower). It can be see that the limits are nearly the same for all combustors at all Re, encompassing both laminar and turbulent flow regimes. It should be emphasized that the parameter values listed in Table 1 were adjusted based entirely on scaling considerations and not readjusted in an empirical fashion to obtain the favorable results shown in Fig. 10 (lower). Additionally Figure 11 shows that the maximum temperatures at the extinction limits for the three combustors are nearly identical, which justifies the use of R as a scaling parameter even though U_R is not temperature-independent. Moreover, this similarity of temperatures validates the definition of Da employed here which does not include the temperature-dependent Arrhenius term ($\exp(-E_a/\Re T_3)$). These results demonstrate that the dimensionless parameters Re, α , Da, Bi and R are sufficient to characterize the performance of heat-recirculating combustors with similar geometry but widely varying scale.

3.5 Linear vs. Swiss roll heat exchangers

While the simple analyses of the linear and simulated Swiss roll devices in Section 2

identified the governing dimensionless parameters, it is still instructive to compare combustor performance in the two different geometries using detailed numerical simulations. With this motivation an "unrolled" linear combustor model having the same channel width d, total heat exchanger length L and same central volume and shape as the nominal-scale Swiss roll combustor was created. A problem arose in the simulations in that the linear combustor could not sustain combustion at low Re (high N) even with a stoichiometric mixture. Such difficulties could be expected based on the poorer performance of non-adiabatic linear exchangers vs. spiral ones as shown in Fig. 2a vs. Fig. 2b. To rectify this situation, for the linear device the external heat loss coefficients h_L and ε_L were artificially reduced to 25% of their values for the Swiss roll device. (Even with this scheme, the effective α is larger in the linear device than the Swiss roll since rolling a linear exchanger into a 3.5 turn spiral reduces A_L by a factor of about 7.)

Figure 12 shows the predicted extinction limits for the Swiss roll combustor and the linear combustor (the latter with modified h_L and ε_L), along with the corresponding results for adiabatic conditions and no internal radiation ($h_L = \varepsilon_L = \varepsilon_i = 0$). First note that even at high Re, where a comparison of the adiabatic and non-adiabatic results show that heat losses are insignificant, the Swiss roll combustor still shows broader extinction limits. This is probably because the inlet arm of the Swiss roll exchanger receives thermal enthalpy from both walls of the channel whereas the linear exchanger receives enthalpy from only one wall, so the Swiss roll device has about twice the effective heat transfer area (A_T) and thus twice as large a value of N at the same Re. Consistent with this explanation, the equivalence ratios at the extinction limits for the Swiss roll combustor are approximately half that of the linear combustor; to obtain the same T_3 with half the N and thus half the E, twice as much temperature rise due to combustion ($T_3 - T_2$) is required and thus twice as high a fuel concentration in the fresh mixture ($Y_{i,\infty}$ or equivalence ratio) is required.

Figure 12 also shows that in the (unrealistic) adiabatic case, as Re decreases (and thus N

increases) the benefit of the Swiss roll combustor over the linear device in terms of extinction limits decreases and at very low Re / high N, the extinction limits of linear device are leaner than those of the Swiss roll despite the linear device having half the N at the same Re. This result can be understood considering the properties of spiral heat exchangers discussed in Section 2. Since $N_{linear} \approx N_{spiral}/2$ and for adiabatic conditions $N_{linear} = E$, the cross-over to leaner extinction limits for the linear device should occur when $N_{spiral} \approx E/2$. For the 3-turn device modeled here, the condition for which $N_{spiral} = E/2$ occurs is at N ≈ 6.1 for $\alpha = 0$, corresponding to Re $\approx 400/N$ = 66. This is comparable to the cross-over value Re ≈ 90 seen in Fig. 12, which suggests that the aforementioned behavior of spiral heat exchangers is responsible for the cross-over in extinction limits seen at low Re for adiabatic combustors.

4. Practical perspectives

4.1 Number of turns

The scaling analyses presented in Sections 3.1 - 3.3 presume that there is no limitation on the ability to manufacture arbitrarily large or small devices while enforcing geometrical similarity. A more practical situation would be a constraint on the volume available for the device, with a fixed wall thickness limited by manufacturing capabilities. Given these constraints, for an n-turn device with average channel length per turn of L^{*} (which would be nearly constant for a device of fixed overall size), a scaling analysis similar to that leading to Eq. (4) yields

$$N = \frac{U_T A_T}{\dot{m} C_P} \sim \frac{\frac{1}{2} \frac{N u k_g}{d} A_T}{\rho v A_X C_P} \sim R e^{a-1} \frac{L^*}{d} n$$
(18)

For fixed overall size, the channel width (d) is inversely proportional to the number of turns and thus for fixed Re, $N \sim n^2$. This would seem to indicate a significant advantage to increasing the number of turns, thus increasing N. However, for spiral exchangers increasing N is beneficial only up to a certain limit as Figures 2(b) and 2(c) show. Moreover, for the stated constraints, more turns also implies more wall material and thus more heat transfer along the walls in the direction perpendicular to the plane of the spiral, which results in additional heat loss to ambient. Clearly if the number of turns is sufficient, the structure will be mostly wall material with massive heat losses in the third dimension (out of the plane of the spiral) and little space for reactant and product flow. (While the impact of heat loss in the third dimension was stated in Section 3.2 to be insignificant for the 3.5 turn exchanger, this is not necessarily true for an increased n and thus increased wall material.) Consequently, an optimum value of n must exist.

With this motivation, a set of two-dimensional calculations were performed using a model for heat loss in the third dimension [15] that has been shown [17] to closely match results of fully 3D calculations. Combustors of n = 3.5 to 12 having the same overall dimensions and wall thickness (Fig. 13) were simulated at varying Re and the extinction limits determined. Again a one-step Arrhenius reaction rate model for propane was employed. Results are shown in Fig. 14. It can be see that an optimum n does indeed exist though it is rather flat for the conditions examined. The optimum is somewhat sharper and shifted to smaller n for smaller Re. For the dimensions of the combustors shown in Fig. 12, N $\approx (24n^2/\text{Re})/(1 - 0.012n)$ which varies from about 6 to 81 for Re = 50 and proportionally less for the higher values of Re. Since the optimal N for spiral exchangers varies roughly in proportion to N (Fig. 2c), due to the n² effect the larger values of n (more turns) correspond to values of N beyond the optimum. This, coupled to the additional heat loss at large n just mentioned, results in an optimum not only the number of transfer units (N) but also in the number of turns (n). At higher Re, the effect is not as pronounced since N is smaller (not far beyond the optimal value if at all) and the impact of heat loss is less, thus the adverse effect of too many turns is not as pronounced.

4.2 Effect of heat exchanger height and out-of-plane heat losses

The scaling analysis of Section 3 was performed without any consideration of heat loss in the third dimension, i.e. perpendicular to the plane of the spiral. In contrast, the computations described in Section 3.2 were fully three-dimensional and did include these losses. Figure 15 shows a comparison of the extinction limits in the nominal-size (5 cm total height) combustor with identical devices extruded to 10 cm or cut to 2.85 cm height. There is very little difference in the extinction limits between the 5 and 10 cm devices, justifying the neglect of heat losses in the third dimension for the computations in Section 3, however, the shorter device does show significantly decreased performance due to an increased out-of-plane heat loss rate relative to the heat release rate, indicating that out-of-plane losses must be considered in such cases. Moreover, the limits are practically independent of height at high Re where extinction results from residence time limitations (low Da) and heat losses are unimportant whereas the limits are very dependent on height at low Re where heat losses are responsible for the extinction limits. These results are consistent with experiments employing Swiss roll propane-air combustors of varying heights but otherwise identical geometries [18, 19] where it is found that the taller device has wider extinction limits (e.g. device "D" vs. "S" in [19]). On the other hand due to their small height and thick dividing walls, these burners suffer substantial heat losses in the third dimension and thus are not technically heat recirculating combustors; comparing a 2-turn device "RD" with a 4turn device "D" of equal channel width, height and wall thickness it is found that the extinction limits are nearly identical. This is consistent with Figure 6 (middle), where the analysis of section 2.4 shows that with substantial heat losses the part of the exchanger farthest away from the reaction region does not provide any preheating of the reactants because exhaust enthalpy is lost to ambient instead of being transferred to the reactants, and thus a longer device (more turns) would have the same performance as one with fewer turns. Additionally, because of these losses, the extinction limits in these devices are within the conventional flammability limits whereas with taller devices and thinner dividing walls, the extinction limits may be substantially outside the conventional flammability limits (e.g. Fig. 17).

4.3 Optimal thermal conductivity

The analytical presented in section 2.4 show that streamwise wall heat conduction is detrimental to performance, however, for zero wall thermal conductivity (k_w), no heat recirculation is possible, thus, an optimum k_w causing the widest possible extinction limits must exist. (This behavior was not predicted by the analysis in section 2.4 because thermally-thin heat exchanger walls were assumed *a priori*, thus neglecting the thermal resistance across the wall.) Figure 16 shows the computed effect of k_w on extinction limits for varying Re for a 3.5-turn combustor with one-step propane-air chemistry. It can be seen that the optimum k_w is extremely small - in fact, smaller than the conductivity of air (0.026 W/m°C). The following estimate of this optimum is proposed. There is no disadvantage to lower k_w until the wall thermal resistance $\sim \tau/k_w$ is comparable to the thermal resistance between gas and wall $\sim 1/U_T$. At higher k_w streamwise wall conduction reduces performance, whereas at lower kw heat recirculation via conduction across the wall is diminished. Thus at the optimum condition $k_w \approx U_T \tau \approx$ $((10.6/2)W/m^2K)(0.0005 \text{ mm}) = 0.0028 W/mK$ (here U_T is the computed value averaged over all interior walls), which is comparable to the calculated optimal value seen in Fig. 15. The optimum is more pronounced at lower Re (higher N) where the impact of both streamwise wall heat conduction and heat losses are greater. At high Re (lower N), where the impact of these effects is greatly reduced, there is no effect of the wall thermal conductivity except at very low values of k_w. These results demonstrate that while a theoretical optimal thermal conductivity exists, for any solid material from which heat exchanger walls might be constructed, lower thermal conductivity is always advantageous. (While one could reach such low values of the effective thermal conductivity using a Dewar-type wall construction, this would only suppress heat transfer across the wall, i.e. in the direction for which heat transfer is desired, and would not decrease heat transfer in the streamwise direction along the wall, for which heat transfer is detrimental. Consequently Dewar-type walls would be of no value for this application.)

4.4 Turbulence effects

While invocation of the word "microcombustion" suggests low Reynolds numbers for which the effects of turbulent flow would be absent, the possibility of transitional values of Re cannot be ruled out. The primary effect of turbulence has on heat recirculating combustors is of course the increase in U_T and thus increased N when Re is sufficiently high. This would increase the excess enthalpy E (except for large N for spiral reactors, as discussed in section 2.2) and thus reduce the fuel concentration and equivalence ratio at the extinction limit compared to that which would occur without turbulence. Of course this supposition cannot be tested experimentally because one cannot arbitrarily suppress turbulence at will, but this can be tested computationally. Figure 17 (upper) shows a comparison of the experimentally-measured extinction limits of lean propane-air mixtures to those predicted with a 3D Reynolds Stress Model of turbulent flow and heat transfer enabled and disabled. What is remarkable about this figure is that the predictions are practically the same with and without the turbulence model enabled. The reason for this is that without turbulence, Dean vortices form in the curved channels that enhance heat transport and thus heat recirculation by nearly the same amount as turbulence does [10]. Note also that the predictions of both 3D models are in good agreement with the experimental extinction limits. Figure 17 (lower) shows that, in contrast, for the 2D simulations the extinction limits are very different with and without the turbulence model activated because Dean vortices cannot form in 2D simulations. Without the turbulence model, the 2D simulation significantly underpredicts the amount of heat transfer and thus heat recirculation, leading to a requirement for a higher fuel concentration to reach a given flame temperature and thus a higher equivalence ratio at the lean extinction limit.

Figure 18 shows a comparison of temperatures predicted by 2D simulations [15] at the same locations as the thermocouples in the experiments [9]. (These computed temperatures are not the maxima over the entire computational domain but provide the most realistic comparison between model and experiment.) An interesting feature of Figure 18 is that the limit

temperatures with and without turbulent transport are nearly identical even though the limit T_{ad} and mixture strengths (see for example Fig. 17, lower) are very different. This is because at high-Re conditions where turbulent flow is present, extinction is caused by insufficient residence time compared to the reaction time scale rather than heat losses. This residence time is set by the flow velocity (thus Re). Consequently, Re sets the chemical reaction rate required to avoid extinction, which is far more sensitive to temperature than any other property. Thus, to a very close approximation it can be stated that a given Re requires a given reaction temperature to avoid extinction, regardless of the transport environment required to obtain this temperature.

4.5 Catalysis

The analysis and experiments on heat-recirculating combustors reported above correspond to conditions where only gas-phase reactions can occur. In practice, it is often advantageous to employ a catalyst to extend the extinction limits, particularly for conditions where high reaction temperatures are undesirable or unsustainable (e.g., due to heat losses). Catalytic combustion at small scales is discussed in more detail elsewhere in this book, however, some discussion of the effect of catalytic combustion on the performance of heat-recirculating combustors is given in this chapter. Figure 19 shows extinction limits measured in a 3.5-turn Swiss roll combustor, 5 cm in height with 7.5 cm overall width and depth, with and without the use of a catalyst. For experiments with catalyst, strips of bare platinum foil were placed along the walls in the central section of the combustor, with an exposed area of $\approx 30 \text{ cm}^2$. Four are shown in Fig. 19 indicating extinction limits for catalytic combustion using Pt treated via combustion in an ammonia-air mixture, gas-phase combustion (i.e. without catalyst installed) and the boundaries of the "out-of-center reaction zone" regime discussed below. The salient features of Fig. 19 include:

• With the ammonia-treated catalyst, combustion can be sustained over a range of Re from about 1 to 2000 (higher Re are potentially attainable but corresponded to

minimum temperatures required to sustain combustion in excess of limit for the Inconel from which the combustor was built.) This range of Re corresponds to a 2000:1 range of mass flow; this "turn-down ratio" far exceeds that of any other type of combustor known to the author.

- Without catalyst, the minimum Re is much higher (about 40) and thus the attainable turn-down ratio smaller.
- The catalyst improves lean-limit performance only slightly at moderate Re and not at all at the highest Re tested. This is probably because at sufficiently high Re the ratio of the mass flux to the catalyst to the total mass flux becomes very small and only a small fraction of the fuel can be burned on the catalyst, in which case gas-phase combustion is the only means to self-sustain reaction.
- The rich limits are extended drastically with catalyst; for example the equivalence ratio at the rich extinction limit is about 40 for Re = 15. Product analysis [9] shows that these rich mixtures are not "reformed" or partially oxidized; the products do not contain significant amounts CO or H₂ but instead only unburned fuel, CO₂ and H₂O.
- Ammonia treatment of the catalyst extends the extinction limits, but only at low Re corresponding to low temperatures (see Fig. 20).
- For Re < 15, the <u>lean</u> catalytic extinction limit is actually <u>rich</u> of stoichiometric. No similar trend was found without catalyst, for which the limits are nearly symmetric about stoichiometric. The asymmetry with catalytic combustion was attributed to (1) at low Re and thus low temperatures, desorption of O(s) or CO(s) is the rate-limiting process and (2) there is a change in the Pt surface coverage from O(s) to CO(s) for sufficiently rich mixtures, with a corresponding increase in net reaction rate due to the lower activation energy for desorption of CO(s) than O(s) [20].

• For sufficiently high reaction temperatures (i.e., near-stoichiometric mixtures and higher Re), reaction is sufficiently strong to self-sustain without the benefit of heat recirculation and the reaction zone moves out of the center of the spiral towards the inlet. The out-of-center limit is the same for catalytic and gas-phase combustion, which is expected since the catalyst cannot affect reaction when it is completed upstream of the catalyst.

Figure 20 shows the maximum measured temperatures <u>at the extinction limit</u> as a function of Re for both gas-phase and catalytic combustion. Even without catalyst, temperatures required to support combustion (typically 1100K) are lower than for propagating hydrocarbonair flames (1500K), indicating conditions more similar to plug-flow reactors than flames (the difference being that plug-flow reactors have prescribed temperature profiles adjusted using electrical heaters, whereas for the Swiss roll combustor the temperature profile is strongly coupled to heat transfer and heat release). For Re > 750, temperatures gas-phase and catalytic temperatures converge, corresponding to the convergence of lean extinction limits (Fig. 19). Minimum temperatures required to support combustion with catalyst are typically 400K less than for gas-phase reaction, though the difference in limit mixture compositions are small (Fig. 19). For this reason an additional curve is given in Figure 20 that shows T_{max} at the limit with catalytic reaction *for the mixture at the gas-phase lean extinction limit.* For this case it can be see that the temperatures with catalyst are actually slightly lower than with gas-phase reaction only. This shows that the catalyst is actually slightly detrimental to gas-phase reaction; catalytic reaction is possible.

Perhaps the most noteworthy aspect of Fig. 20 is the low temperatures capable of sustaining combustion with catalyst. The minimum temperature observed is only 78°C at Re = 1.2, which to the author's knowledge is the lowest reported self-sustaining hydrocarbon flame temperature (as described in [7], care was taken to ensure that the maximum catalyst temperature was measured.) For both catalytic and gas-phase reaction, the minimum temperature required to

support combustion increases as Re increases. This is expected since higher Re means higher velocities (since neither channel width nor viscosity was changed), thus shorter residence times and consequently faster reaction required to sustain combustion. Faster reaction in turn requires higher temperatures. In fact, Fig. 20 can be redrawn as an Arrhenius plot (ln(Re) vs. 1/T); the resulting plot (not shown) is nearly linear with slopes corresponding to effective activation energies (E) of about 19 and 6.4 kcal/mole for gas-phase and catalytic limits, respectively. On first glance the latter seems rather low, but the dimensionless activation energy (E/RT) for the Re = 1.2 limit case is 9.2, indicating significant sensitivity to temperature.

Having low steady-state flame temperatures is of some merit because it results in smaller heat losses, fewer issues with thermal expansion and allows a wider choice of materials. However, in some applications higher temperatures are desired to supply thermal power to thermoelectric or pyroelectric generators, solid oxide fuel cells, etc. In contrast, having low ignition temperatures is always meritorious because it reduces the energy storage (e.g. from a battery or supplemental fuel) required for a self-contained micropower generation system. Figure 21 shows the ignition temperature (determined by slowly increasing the power delivered to an electrically-heated wire in the center of the spiral and measuring the temperature at which a sudden jump corresponding to ignition occurs) for a Swiss roll combustor using the ammonia-treated Pt catalyst. Also shown in Fig. 21 is the corresponding combustion temperature measured after steady-state conditions are achieved. The lean and rich extinction limits correspond to conditions where the steady-state combustion temperature is nearly as low as the ignition temperature. It can be seen that due to the thermal management provided by the Swiss roll and the low-temperature reaction provided by the catalyst, ignition temperatures as low as 85°C are readily obtained. It is well known that hydrogen will ignite at room temperature on Pt catalyst; hydrocarbons do not, but to the author's knowledge 85°C is the lowest reported ignition temperature for self-sustaining hydrocarbon combustion.

5. Conclusions

Heat-recirculating combustors are an important component of many micropower generation systems because they enable self-sustaining chemical reaction over a wider range of mixtures and flow rates than conventional combustors. However, at small scales heatrecirculating combustors suffer from many of the same issues as other types of combustors, namely heat losses, insufficient residence time and undesirable redistribution of thermal energy via solid-phase heat conduction. This chapter discussed ways of assessing these issues particularly with regards to identification and use of scaling parameters. The key design parameters examined in this chapter are:

- Geometry rolling a linear counter-current heat exchanger into a spiral (Swiss roll) yields two advantages: a significantly smaller heat loss parameter (α) and a doubling of the number of transfer units (N), both of which benefit performance substantially. While at low Re (large N) it is theoretically possible a linear exchanger to exhibit better performance than a spiral one, in practice it is unlikely that heat losses can be minimized to the point where this would be observable
- Number of turns for a given overall device size and wall thickness, there is an optimal choice of the number of turns of the spiral. A device with too few turns has a small value of N and thus small excess enthalpy (E); a device with too many turns also has small E (Figs. 2b, c) and additionally suffers from greater heat losses due to heat conduction through the walls in the direction out of the plane of the spiral (Fig. 14).
- Device height a short device will have substantial out-of-plane heat losses and thus a taller device is preferred, however, if total volume is constrained then an optimal height will exist which minimizes out-of-plane heat loss (Fig. 15) yet provides space for a larger number of spiral turns and/or larger channel width (d).

- Dividing wall thermal conductivity although a perfectly insulating wall between the inlet and outlet arms of the heat exchanger does not allow heat recirculation, for all practical purposes a wall material having lowest possible thermal conductivity provides the widest extinction limits. Among dense (non-porous) solid materials, polymers have the lowest thermal conductivities. Polymers generally cannot tolerate sufficiently high temperatures to sustain gas-phase hydrocarbon combustion, but with catalysis the combustion temperatures are well within the temperature limits of many polymers (particularly polyimides), and have been used to construct Swiss roll combustors [21].
- Wall emissivity while difficult to modify, lower wall emissivity leads less turn-toturn heat transfer and improves performance in a manner similar to lower wall thermal conductivity.
- Catalytic combustion the use of appropriate catalysts results in greatly reduced flame temperatures and thus smaller heat losses, fewer issues with thermal expansion and a wider choice of materials. Additionally, ignition temperatures are substantially lower for catalytic combustion than gas-phase combustion and room-temperature ignition of hydrocarbons may be possible. Considering that the lowest observed ignition temperature is about 85°C (Fig. 21) and the apparent activation energy for low-temperature catalytic combustion is 6.4 kcal/mole, to obtain ignition at 25°C would require a factor of 6 increase in residence time within the combustor (i.e., a 6-fold increase in N) without a corresponding increase in heat losses (meaning a 6-fold decrease in the heat loss α, since at large N, Eq. (3) shows that E ~ 1/αN) a challenging but possibly achievable goal.

Finally, it is noted that the scaling parameters identified in Section 2 and tested in Section 3 can be used not only to predict the behavior of combustors of varying scale, but also to

extrapolate the performance of relatively large, easily tested laboratory-scale devices to predict the performance of small-scale devices. The scaling is sometimes unintuitive, for example, the radiation parameter R (Eq. (17)) scales in proportion to d^1 for laminar flow and thus radiation may not be significant at small scales but if not considered in the extrapolation of large-scale devices to smaller scales, inaccurate predictions may result.

Acknowledgments

The author wishes to thank graduate students Jeongmin Ahn, Chien-Hua Chen and James Kuo for their contributions to the study of heat-recirculating combustors. This research has been supported by the Defense Advanced Research Projects Agency, National Aeronautics and Space Administration and the Defense Threat Reduction Agency. This paper is dedicated to the memory of Prof. Felix J. Weinberg, a valued colleague and scholar.

References

- [1] D. Dunn-Rankin, E. M. Leal and D. C. Walther, *Personal Power Systems*, Progress in Energy and Combustion Science 31 (2005), 422 465.
- [2] D. C. Walther and J. Ahn, *Advances and challenges in the development of power generation systems at small scales*, Progress in Energy and Combustion Science 37 (2011), 583-610.
- [3] K. Maruta, *Micro and mesoscale combustion: technology development and fundamental research*, Proceedings of the Combustion Institute 33 (2011), 125 150.
- [4] S. N. Kaisare and D. Vlachos, *A review on microcombustion: fundamentals, devices and applications*, Progress in Energy and Combustion Science 38 (2012), 321-359.
- [5] Y. Ju and K. Maruta, *Microscale combustion: technology development and fundamental research*, Progress in Energy and Combustion Science 37 (2011), 669 - 715.
- [6] A. M. Karim, J. A. Federici and D. G. Vlachos, *Portable power production from methanol in an integrated thermoeletric/microreactor system*, Journal of Power Sources 179 (2008), 113 120.
- [7] J. Ahn, Z. Shao, P. D. Ronney and S. M. Haile, *A Thermally Self-Sustaining Miniature Solid* Oxide Fuel Cell, Journal of Fuel Cell Science and Technology, Nov. 2009.
- [8] A. R. Jones, S. A. Lloyd and F. J. Weinberg, *Combustion in heat exchangers*, Proceedings of the Royal Society of London Series A: Mathematical and Physical Science 360 (1978), 97-115.
- [9] J. Ahn, C. Eastwood, L. Sitzki and P. D. Ronney, *Gas-phase and catalytic combustion in heatrecirculating burners*, Proceedings of the Combustion Institute 30 (2005), 2463-2472.
- [10] C.-H. Chen and P. D. Ronney, *Three-dimensional effects in counterflow heat-recirculating combustors*, Proceedings of the Combustion Institute 33 (2011), 3285-3291.
- [11] S. A. Lloyd and F. J. Weinberg, *A burner for mixtures of very low heat content*, Nature 251 (1974), 47-49.
- [12] S. A. Lloyd and F. J. Weinberg, *Limits to energy release utilization from chemical fuels*, Nature 257 (1975), 367-370.
- [13] M. Targett, W. Retallick and S. Churchill, *Solutions in closed form for a double-spiral heat exchanger*, Industrial and Engineering Chemical Research 31 (1992), 658-669.
- [14] M. Strenger, S. Churchill and W. Retallick, Operational characteristics of a double-spiral heat exchanger for the catalytic incineration of contaminated air, Industrial and Engineering Chemical Research 29 (1990), 1977-1984.
- [15] C. Kuo and P. D. Ronney, *Numerical modeling of non-adiabatic heat-recirculating combustors*, Proceedings of the Combustion Institute 31 (2007), 3277-3284.
- [16] P. D. Ronney, Analysis of non-adiabatic heat-recirculating combustors, Combustion and Flame 135 (2003) 421-439.

- [17] C.-H. Chen and P. D. Ronney, "Scale and geometry effects on heat-recirculating combustors," Combustion Theory and Modelling, to appear (2013)
- [18] N. I. Kim, S. Kato, T. Kataoka, T. Yokomori, S. Maruyama, T. Fujimori, K. Maruta, *Flame stabilization and emission of small Swiss-roll combustors as heaters*, Combustion and Flame 141 (2005), 229-240.
- [19] N. I. Kim, S. Aizumi, T. Yokomori, S. Kato, T. Fujimori', K. Maruta, *Development and scale effects of small Swiss-roll combustors*, Proceedings of the Combustion Institute 31 (2007), 3243–3250.
- [20] K. Maruta, K., K. Takeda, J. Ahn, K. Borer, L. Sitzki, P. D. Ronney, O. Deutschman, *Extinction Limits of Catalytic Combustion in Microchannels*, Proceedings of the Combustion Institute 29 (2002), 957-963.
- [21] L. L. Sanford, S. Y. J. Huang, C. S. Lin, J. M. Lee, J. M. Ahn, P. D. Ronney, *Plastic mesoscale combustors/heat exchangers*, Proceedings of the ASME International Mechanical Engineering Congress and Exposition 2007, Vol. 6: Energy Systems: Analysis, Thermodynamics and Sustainability (2008), pp. 141 145.

1	ľ	ıŁ)1	e

Property	Half	Full	Double
$h_{\rm L}$ (W/m ² K)	10	5	2.5
$\boldsymbol{\epsilon}_{\mathrm{L}}$ (external wall)	0.8	0.4	0.2
$\boldsymbol{\epsilon}_{\mathrm{L}}$ (insulation)	1	0.5	0.25
Z (m-sec-kmole units)	1.44 x 10 ¹¹	$3.6 \ge 10^{10}$	9.0 x 10 ⁹
$\boldsymbol{\epsilon}_{i}$ (internal wall)	0.8	0.5	0.2857

Table 1. Values of the heat loss coefficients, emissivities and pre-exponential term for combustors of varying scale. The values for the "Full" scale device are the nominal properties for the device that was tested experimentally [10] to verify the accuracy of the computational model whereas the values for the "Half" and "Double" scale devices are artificially adjusted to obtain constant values of the dimensionless parameters α , Da and R.

Figures



Figure 1. Schematic diagram of the linear counter-current heat-recirculating combustor.



Figure 2. Dimensionless excess enthalpy (E) vs. Number of Transfer Units (N) for different heat loss coefficients (α): (a) Linear exchanger (Eq. (3)) (data from [17]).



Figure 2. Dimensionless excess enthalpy (E) vs. Number of Transfer Units (N) for different heat loss coefficients (α): (b) simulated 3-turn spiral exchanger (Eq. (7)), with comparison to results by Targett *et al.* [13] (data from [17]).



Figure 2. Dimensionless excess enthalpy (E) vs. Number of Transfer Units (N) for different heat loss coefficients (α): (c) simulated 3, 4 and 5-turn spiral exchangers.



Figure 3. Stacked linear device used to simulate 3-turn spiral Swiss roll combustor.



Figure 4. Reactor temperature (T_R) as a function of the Number of Transfer Units (N) for linear and simulated 3-turn spiral heat exchangers with Damköhler number Da = 2 x 10⁷, nondimensional activation energy β = 70 and non-dimensional heat loss coefficient α = 0.1. Also shown are comparisons between simplified and detailed models for the linear device.

Figure 5. Schematic diagram of enthalpy fluxes assumed in detailed analysis of linear countercurrent heat recirculating combustor.



Figure 6. Temperature profiles in the linear heat exchanger for $Da = \infty$, N = 2.5, $\Delta \tilde{T}_c = 1.5$. Top: $\alpha = 0$, $Bi = \infty$; middle: $\alpha = 2$, $Bi = \infty$; bottom: $\alpha = 0$, Bi = 10. Adapted from [16].



Figure 7. Effect of N on WSR temperature in the counter-current combustor for finite reaction rates ($Da = 2 \ge 10^7$) with $\Delta \tilde{T}_c = 1.5$. For reference, the adiabatic, wall-conduction-free curve from Fig. 4 is also shown (data from [16].)



Figure 8. Effect of N on adiabatic temperature rise $\Delta \tilde{T}_C$ (i.e. fuel concentration) at extinction limit in the non-adiabatic ($\alpha = 0.1$) counter-current combustor for finite reaction rates (Da = 2 x 10⁷) and varying Biot number (Bi) (data from [16].)



Figure 9. Wire-frame model of the spiral combustor used for scaling analyses showing grid resolution.



Figure 10. Computed propane-air equivalence ratios at the extinction limits for geometricallysimilar Swiss roll combustors of three different overall sizes (data from [17]). Top: with baseline property values; bottom: with property values adjusted (Table 1) to obtain constant values of the dimensionless parameters describing heat loss (α), reaction rate (Da) and internal radiative transfer (R).



Figure 11. Computed maximum temperatures at the extinction limits of propane-air mixtures for geometrically-similar Swiss roll combustors of three different overall sizes with property values adjusted to obtain constant α , Da and R (data from [17]).



Figure 12. Comparison of extinction limits of Swiss roll and linear combustors with the same exchanger length, with and without heat losses (data from [17]). For the linear combustor, the heat loss coefficients are set to 25% of their nominal values (see text.)



Figure 13. 3.5-turn and 12-turn spiral combustors used for the computational study of the effects of the number of turns on combustor performance when the overall device size and wall thickness are fixed. Not shown for brevity: 5.5, 7 and 10-turn devices.



Figure 14. Computed propane-air equivalence ratios at the extinction limits for Swiss roll combustors with varying numbers of turns (n) but the same overall size and same wall thickness.



Figure 15. Computed propane-air equivalence ratios at the extinction limits for nominal-scale Swiss roll combustors of different heights (data from [17]).



Figure 16. Computed propane-air equivalence ratios at the extinction limits for 3.5-turn Swiss roll combustors with varying wall thermal conductivity (k_w) . While the values of k_w vary with temperature within the combustor, the values show on the horizontal axis are those corresponding to the reactants at ambient conditions.



Figure 17. Upper: Comparison of measured extinction limits in Swiss-roll combustors to predictions of the 3-dimensional numerical model with and without the RSM turbulence model activated [10]. Lower: Comparison of extinction limits predicted by the 2D simulations with and without the RSM turbulence model activated and the 3D simulations with the RSM activated.



Figure 18. Maximum calculated [15] temperatures at extinction limits obtained using a 2D model with and without turbulent flow and transport suppressed.



Figure 19. Experimental extinction limit map for catalytic and gas-phase combustion in a 3.5-turn Swiss roll using propane-air mixtures (data from [9]).



Figure 20. Maximum burner temperatures at the extinction limits (data from [9]).



Figure 21. Ignition temperatures and steady-state flame temperatures in a Swiss-roll combustor using propane-air mixtures as a function of equivalence ratio for Re = 12.

Figure captions

Figure 1. Schematic diagram of the linear counter-current heat-recirculating combustor.

Figure 2. Dimensionless excess enthalpy (E) vs. Number of Transfer Units (N) for different heat loss coefficients (α):

(a) Linear exchanger (Eq. (3)) (data from [17]).

(b) simulated 3-turn spiral exchanger (Eq. (7)), with comparison to results by Targett *et al.* [13] (Data from [17].)

(c) simulated 3, 4 and 5-turn spiral exchangers.

Figure 3. Stacked linear device used to simulate 3-turn spiral Swiss roll combustor.

Figure 4. Reactor temperature (T_R) as a function of the Number of Transfer Units (N) for linear and simulated 3-turn spiral heat exchangers with Damköhler number Da = 2 x 10⁷, nondimensional activation energy β = 70 and non-dimensional heat loss coefficient α = 0.1. Also shown are comparisons between simplified and detailed models for the linear device.

Figure 5. Schematic diagram of enthalpy fluxes assumed in detailed analysis of linear countercurrent heat recirculating combustor.

Figure 6. Temperature profiles in the linear heat exchanger for $Da = \infty$, N = 2.5, $\Delta \tilde{T}_C = 1.5$. Top: $\alpha = 0$, $Bi = \infty$; middle: $\alpha = 2$, $Bi = \infty$; bottom: $\alpha = 0$, Bi = 10. Adapted from [16].

Figure 7. Effect of N on WSR temperature in the counter-current combustor for finite reaction rates ($Da = 2 \times 10^7$) with $\Delta \tilde{T}_c = 1.5$. For reference, the adiabatic, wall-conduction-free curve from Fig. 4 is also shown. Data from [16].

Figure 8. Effect of N on adiabatic temperature rise $\Delta \tilde{T}_C$ (i.e. fuel concentration) at extinction limit in the non-adiabatic ($\alpha = 0.1$) counter-current combustor for finite reaction rates (Da = 2 x 10⁷) and varying Biot number (Bi) (data from [16].) Figure 9. Wire-frame model of the spiral combustor used for scaling analyses showing grid resolution.

Figure 10. Computed propane-air equivalence ratios at the extinction limits for geometricallysimilar Swiss roll combustors of three different overall sizes (data from [17]). Top: with baseline property values; bottom: with property values adjusted (Table 1) to obtain constant values of the dimensionless parameters describing heat loss (α), reaction rate (Da) and internal radiative transfer (R).

Figure 11. Computed maximum temperatures at the extinction limits of propane-air mixtures for geometrically-similar Swiss roll combustors of three different overall sizes with property values adjusted to obtain constant α , Da and R (data from [17]).

Figure 12. Comparison of extinction limits of Swiss roll and linear combustors with the same exchanger length, with and without heat losses (data from [17]). For the linear combustor, the heat loss coefficients are set to 25% of their nominal values (see text.)

Figure 13. 3.5-turn and 12-turn spiral combustors used for the computational study of the effects of the number of turns on combustor performance when the overall device size and wall thickness are fixed. Not shown for brevity: 5.5, 7 and 10-turn devices.

Figure 14. Computed propane-air equivalence ratios at the extinction limits for Swiss roll combustors with varying numbers of turns (n) but the same overall size and same wall thickness.

Figure 15. Computed propane-air equivalence ratios at the extinction limits for nominal-scale Swiss roll combustors of different heights (data from [17]).

Figure 16. Computed propane-air equivalence ratios at the extinction limits for 3.5-turn Swiss roll combustors with varying wall thermal conductivity (k_w) . While the values of k_w vary with temperature within the combustor, the values show on the horizontal axis are those corresponding to the reactants at ambient conditions.

Figure 17. Upper: Comparison of measured extinction limits in Swiss-roll combustors to predictions of the 3-dimensional numerical model with and without the RSM turbulence model activated [10]. Lower: Comparison of extinction limits predicted by the 2D simulations with and without the RSM turbulence model activated and the 3D simulations with the RSM activated.

Figure 18. Maximum calculated [15] temperatures at extinction limits obtained using a 2D model with and without turbulent flow and transport suppressed.

Figure 19. Experimental extinction limit map for catalytic and gas-phase combustion in a 3.5turn Swiss roll using propane-air mixtures (data from [9].)

Figure 20. Maximum burner temperatures at the extinction limits (data from [9]).

Figure 21. Ignition temperatures and steady-state flame temperatures in a Swiss-roll combustor using propane-air mixtures as a function of equivalence ratio for Re = 12.