AME 514 - Special topics in combustion - Fall 2008
Assignment #4
Due: Monday 12/1/08, 4:30 pm in my office (OHE 430J)

Part 1: paper review

Since they're weren't many references in this set of lectures, I've decided to skip Part I. Part 2 will count twice as much as did for the other homework sets. You'll notice Part 2 is somewhat more time consuming than usual, though probably not twice as long as the others. (I think all the words in this problem set that make it look like a long problem set make it easier to do, not more difficult, since I've give you step by step instructions. Of course, your mileage may vary).

Part 2. The usual type of homework questions

Problem #1

a. Show that for heat addition at **constant temperature** (which really simplifies things) using the first law ($h_1 + u_1^2/2 + q = h_2 + u_2^2/2$), the entropy of an ideal gas (s_2-$s_1 = C_P\ln(T_2/T_1) - R\ln(P_2/P_1)$), the enthalpy of an ideal gas ($h_2 - h_1 = C_P(T_2 - T_1)$), the definition of Mach number ($M = u/c = u/(\gamma RT)^{1/2}$) and the consequence of second law ($ds = dq/T$):

$P_2/P_1 = exp[(\gamma/2)(M_1^2 - M_2^2)]$

(yes you already have this result from the lecture notes but I'd like for you to show it…)

b. By additionally using mass conservation, show that

$A_2/A_1 = (M_1/M_2)exp[(\gamma/2)(M_2^2 - M_1^2)]$

(ditto comment in part a)

c. Now consider a propulsion system based on this. First the air will be decelerated isentropically (not to M = 0) then heat will be added at constant temperature. For a flight Mach number of 15, an ambient atmosphere at 100,000 feet (227K and 0.0107 atm, with $\gamma = 1.4$), to what Mach number could the air be decelerated if the maximum allowable gas temperature is 3000K? What would the corresponding pressure be?

d. From this condition, if heat is added at constant temperature until the ambient pressure was reached (not a good way to operate, but this represents a sort of maximum heat addition), what would the exit Mach (M_e) number be? What would the area ratio be?

e. What would the specific thrust be? (Note for this case specific thrust = Thrust/$\dot{m}_a c_1 = \dot{m}_a(u_e - u_1)/\dot{m}_a a_1 = (M_e c_e - M_1 c_1)/c_1 = M_e(T_e/T_1)^{1/2} - M_1$, which is all stuff you already have)

f. What would the Thrust Specific Fuel Consumption be?
(Note that TSFC = (Heat input)/Thrust*c_1
= [$\dot{m}_a$ ($C_P(T_{3t} - T_{2t})c_1$]/[Thrust*c_1^2]
= [($\dot{m}_a c_1$)/Thrust] [(γ/(γ-1))R(T_{3t}–T_{2t})/(γRT_1)]
= [1/(Specific thrust)] [1/(γ-1)] [(T_{3t}–T_{2t})/T_1]
and you have everything needed to calculate T_{3t} and T_{2t})

g. Can any fuel generate enough heat to accomplish this? Look at stoichiometric hydrogen-air and see if the heat release per unit mass = $f_{stoich}Q_R$ is equal to or greater than the heat input needed = $C_P(T_{3t} - T_{2t})$. (Your answer should be NO, but support with numbers).

Problem #2

Let's (sort of) repeat problem 1 c – f using GASEQ, which you can download from http://www.gaseq.co.uk.

a. Note the enthalpy (h_1) and sound speed (c_1) of air at ambient conditions (227 K, 0.0107 atm), then find the kinetic energy of the ambient air $u_1^2/2= (c_1M_1)^2/2$. Then select process "Adiabatic compression/expansion" (be sure to use air as the reactants, dissociated air as the products, and uncheck the "frozen composition" box). Compress the air to a product temperature T_2 of 3000K by adjusting your guess of P_2 (should be around 300 atm) and hitting the "Calculate" button each time.
b. Now do the combustion. To do this, let's first re-visit the constant-temperature heat addition analysis. The momentum equation is AdP + $\dot{m}$du = 0 or AdP +ρuAdu = 0, and the energy equation is h + $u^2/2$ = constant or dh + udu = 0. Combining these, plus the ideal gas law P = ρRT yields dP/P = dh/RT. T is constant by assumption, but R is not quite constant since R = $\Re$/M and the molecular weight M will changed somewhat during combustion. But if we take a value of M averaged between the reactant and product mixtures, we won't be too far off. So if we assume constant (averaged) M and thus constant R, we obtain

$$\ln(P_3/P_2) = (h_3 - h_2)/RT_2$$

where $T_2 = T_3$ = 3000K. So the process for doing the combustion is:

1. Choose as reactants "hydrogen-air flame" and as products "H2/O2/N2 products." (Note that we've ignored any mixture process and the effect that has on the mass flow, stagnation P and T, etc.) The default mixture strength is stoichiometric, so you shouldn't have to change that. Again be sure "frozen composition" is not checked.
2. Guess P_3
3. For the problem type, choose "Equilibrium at defined T and P", enter the 3000K for T_3 and your guess for P_3, and hit "calculate."
4. Get h_2 and M_2, h_3 and M_3 from GASEQ, calculate the average molecular weight = $M_{avg} = (M_2 + M_3)/2$, and calculate the average R = $\Re/M_{avg}$.

5. Is the above equation $\ln(P_3/P_2) = (h_3 - h_2)/RT_2$, satisfied? If not, adjust your guess for P_3 and go back to step 3.

c. Now do the expansion. Select problem type "Adiabatic compression/expansion." Hit "R << P" to transfer the products to reactants. Make sure the "frozen composition" box is unchecked. You should be able to choose a product pressure of 0.01 atm but this doesn't converge. Instead choose a product pressure of 0.1 atm, hit "Calculate," then hit "R << P" to transfer the products to reactants, check the "frozen composition" box, choose a product pressure of 0.0107 atm, hit "Calculate" one more time and you're done. Note the final enthalpy h_e.

d. Compute the product velocity from $h_1 + u_1^2/2 = h_e + u_e^2/2$. You have everything except u_e. **Note that GASEQ gives you enthalpies in kJ/kg, not J/kg, so you need to multiply GASEQ's values of h by 1000 to get the units right. You now have fair warning, I will not be very forgiving if you're numbers are off by $(1000)^{1/2}$!!!**

e. Compute the specific thrust = $(u_e - u_1)/c_1$, which should be a lot lower than in problem 1 because your answer to 1g was NO.

f. Compute TSFC = (Heat input)/Thrust*c_1 = $\dot{m}_a c_1 f_{stoich} Q_R / (\dot{m}_a (u_e - u_1)*c_1^2)$ = (1/(Specific thrust)) $f_{stoich} Q_R / c_1^2$. This should be pretty similar to your answer to problem 1. Also calculate the Specific Impulse = $(1/TSFC)(Q_R/c_1 g_{earth})$. I get about 2100 seconds, much better than the best H_2-O_2 rocket engines (about 450 sec) but not that great considering how hard it will be to get anywhere near this ideal performance.

Problem #3

Estimate the zero Mach number thrust of a Pulse Detonation Engine using propane in the following way.

a. Estimate the dimensionless heat addition H for stoichiometric propane-air assuming T_1 = 300K and P_1 = 1 atm.

b. Compute the detonation Mach number M_1 assuming $\gamma = 1.4$, and the incoming reactant velocity $u_1 = M_1 c_1$

c. Compute the post-shock Mach number M_2, temperature T_2 and pressure P_2 using the analytical formulas (the ones with all the M's and γ's flying around) given in Lecture 13.

d. Compute the pressure P_3, temperature T_3, and sound speed c_3 after heat addition to $M_3 = 1$ in a constant-area duct.

e. We've computed the velocity of the products in the frame of reference attached to the moving detonation front. We need the velocity in the frame of reference of the unburned gas, i.e. in the laboratory frame of reference. So compute u_3 (lab frame) = $u_1 - u_3 = u_1 - c_3 M_3 = u_1 - c_3$.

f. The gas behind the detonation products is moving toward the open end of the tube with a velocity $u_{3,lab}$. But the velocity of the gas at the closed end of the tube must be zero. Thus, the detonation products act like a piston and cause an expansion wave in the products. Compute the pressure P_4, temperature T_4 and sound speed c_4 of the gas after this expansion wave according to the isentropic wave relations from 1D gas dynamics:

$$\frac{P_4}{P_3} = \left(1 - \frac{\gamma - 1}{2}\frac{\Delta u}{c_3}\right)^{2\gamma/\gamma-1}; \frac{T_4}{T_3} = \left(1 - \frac{\gamma - 1}{2}\frac{\Delta u}{c_3}\right)^2$$

where $\Delta u = u_{3,lab} - u_{4,lab} = u_{3,lab} - 0 = u_{3,lab}$.

g. Now compute the specific impulse. If we assume, as discussed in class, that the approximate time the thrust surface "feels" the pressure P_4 is $L/u_1 + L/c_4$, where L is the tube length, then the total impulse is $(P_4 - P_1)AL(1/u_1 + 1/c_4)$, where A is the tube (and thrust surface) cross-sectional area. Then the specific impulse = (total impulse)/(fuel weight), where the fuel weight is (total mass)(fuel mass fraction)g = (ρ_1)(volume)fg = $\rho_1 ALfg$ = $(P_1/RT_1)ALfg$. And finally recall that the specific heat addition H from part (a) is given by $H = fQ_R/RT_1$, so the fuel weight is $(P_1/RT_1)AL(HRT_1/Q_R)g = P_1ALHg/Q_R$. Thus the specific impulse is

$$I_{SP} = \frac{(P_4 - P_1)AL(1/u_1 + 1/c_4)}{P_1 ALHg/Q_R} = \left(\frac{P_4}{P_1} - 1\right)\frac{Q_R}{Hgc_1}\left(\frac{c_1}{u_1} + \frac{c_1}{c_4}\right)$$
$$= \left(\frac{P_4}{P_1} - 1\right)\frac{Q_R}{Hgc_1}\left(\frac{1}{M_1} + \sqrt{\frac{T_1}{T_4}}\right)$$

h. Compute the specific thrust and TSFC.

<u>Problem #4</u>

Now use GASEQ again, which conveniently offers a CJ detonation solver.

a. Choose reactants "propane-air flame" and products "HC/O2/N2 products." (Again the default mixture strength is stoichiometric, so you shouldn't have to change that.) Use 300K and 1 atm as the initial conditions. Choose Problem type "C-J-Detonation" and hit "Calculate." Note the incoming reactant velocity $u_1 = c_1M_1$ and the sound speed (c_3) and specific heat ratio (γ_3) of the products. Note that $M_3 = 1$ as required for a CJ detonation. Compute $u_{3,lab} = u_1 - u_3 = u_1 - c_3M_3 = u_1 - c_3$.
b. Estimate the final pressure P_4 after the expansion wave from the relation

$$\frac{P_4}{P_3} = \left(1 - \frac{\gamma - 1}{2}\frac{\Delta u}{c_3}\right)^{2\gamma/\gamma-1}$$

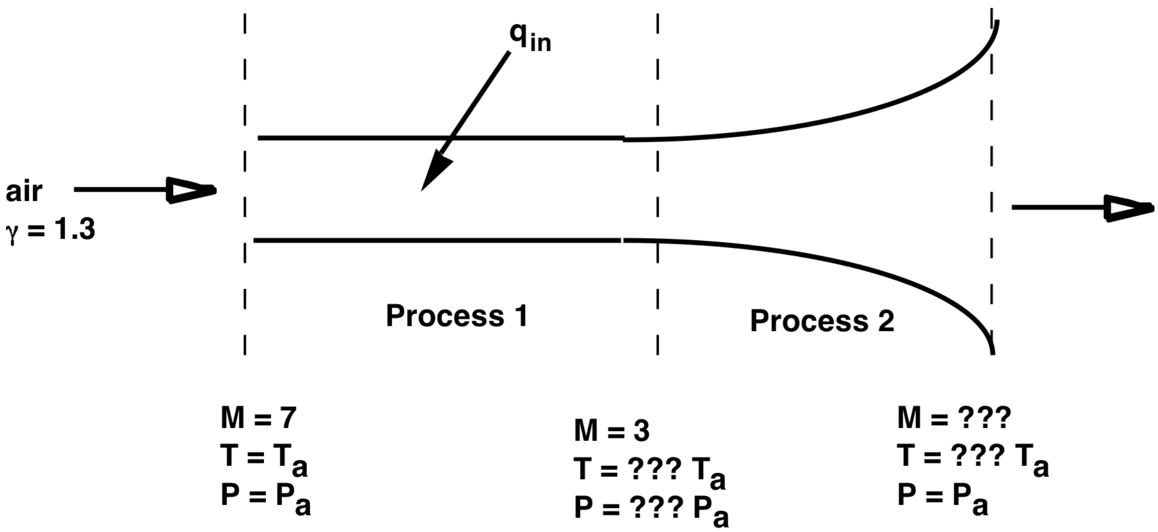
which is not strictly valid since γ is not constant between states 3 and 4 when we consider gases with non-constant specific heats and dissociation, γ changes so little during this process we'll neglect that.

c. Now hit “R << P” to transfer the products to reactants, select process “adiabatic compression/expansion,” select product pressure P_4, and hit “Calculate.” Note the sound speed (c_4) of the expanded products.

$$I_{SP} = \left(\frac{P_4}{P_1} - 1\right)\frac{Q_R}{Hgc_1}\left(\frac{1}{M_1} + \frac{c_1}{c_4}\right)$$

d. Compute the specific thrust, TSFC and specific impulse in the usual way. I get I_{SP} between 1200 and 1400 seconds – not exactly spectacular.

Problem #5



Consider a simple hypersonic propulsion system at an initial Mach number of 7 that consists of:

Process 1: Heat addition at constant area from M = 7 to M = 3
Process 2: Isentropic expansion back to $P_e = P_a$

a. Compute the static (not stagnation) temperature relative to ambient temperature (T_a) after process 1
b. Compute the static (not stagnation) pressure relative to ambient pressure (P_a) after process 1
c. Compute the Mach number after process 2
d. Compute the static (not stagnation) temperature relative to T_a after process 2, and compute the specific thrust
e. Would an isentropic inlet to decrease the Mach number from 7 to 4, followed by heat addition to M = 3, change the Specific Thrust? (You don’t have to show numbers, just state whether ST increases, decreases or stays the same and explain why.)

f. Would an isentropic inlet to decrease the Mach number from 7 to 4, followed by heat addition to $M = 3$, change the Thrust Specific Fuel Consumption? (You don't have to show numbers, just state whether TSFC increases, decreases or stays the same and explain why.)