Influence of Momentum and Heat Losses on the Large-Scale Stability of Quasi-2D Premixed Flames

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ABSTRACT—We consider premixed gaseous flames propagating between parallel plates (Hele-Shaw cell) and qualitatively analyze how the resulting losses of momentum and heat affect the flame stability for wavelengths of wrinkling that noticeably exceed the plate spacing. Modelling the flame as an effective interface and using Euler-Darcy equations for the gas on both sides, we firstly show that friction to adiabatic walls modify the Landau-Darrieus instability in two ways: a damping coefficient is brought in, along with a Saffman-Taylor type of instability. Being due to friction-induced pressure gradients, the latter may even dominate the stabilizing influence of gravity for downwardly propagating fronts in narrow channels. Because they ultimately make the gas density resume its upstream value, heat losses tend to moderate these effects, as well as the Landau-Darrieus instability and the influence of gravity, but the long wavelengths of wrinkling may remain unstable in downward propagations when heat losses are accounted for. This last conclusion is reached upon analysis of the weak-expansion limit.

We also outline a flame-dynamics type of experiment to estimate the heat and momentum exchanges that are involved in our analysis.

1 INTRODUCTION

Among the various instabilities occurring in premixed flames (see Clavin, 1985, or Sivashinsky, 1983, 1990 for reviews) one may single out that discovered by Landau (1944) and Darrieus (1938), which is solely due to the existence of a propagation velocity and of a fresh-to-burnt density drop. Being of hydrodynamical origin, the LD instability mechanism is of ubiquitous importance when the flame wrinkles have long wavelengths (A) compared to the flame thickness. By the same token, it is sensitive to modifications of hydrodynamics acting on the $O(A)$ length-scale.

To date, three broad classes of influences have been shown to modify the “standard” LD picture at the linear level:

— buoyancy or other body forces, which affect the flowfield via a Rayleigh-Taylor type of effect (Markstein, 1964; Searby and Rochwerger, 1991).
— proximity of boundaries where velocity fluctuations are constrained (Joulin, 1987).
— large-scale geometry of the unperturbed front (Istratov and Librovich, 1969; Zel'dovich et al., 1980) or of the incoming flow (Kim and Matalon, 1990; Joulin and Sivashinsky, 1992).
What we envisage here is a modification of hydrodynamics, hence of large-scale stability of the flame, through losses of momentum and heat to boundaries. This could happen to flames propagating in gases that are loaded by filamentous material (e.g. glass wool) or dust. This also applies to the specific situation which we consider in the present article, i.e. flames propagating between two plates that are 2d apart (Figure 1); beside its academic interest this configuration might have implications on safety rules and applications to flame propagations in crevices such as in engine cylinders. Solving exactly this formidable free-boundary problem of compressible, three-dimensional hydrodynamics is out of reach of the available analytical tools (it is not an easy numerical task either!). As is shown here, working out a crude model can provide one with valuable insights into the mechanisms involved, however.

II A MODIFIED LANDAU–DARRIEUS PROBLEM

The Landau–Darrieus model simply consisted of an infinitely-thin (on the wavelength (\(\Lambda\)) scale) flame propagating itself at a constant known normal velocity \(u_L\) relative to fresh gases of density \(\rho_u\) and transforming them into a burned medium of density \(\rho_b\) < \(\rho_u\). Under the assumption that both fluids are governed by piecewise-incompressible Euler equations, a quadratic equation, which we repeat here for future reference, is found to give the growth rate \(\omega\) of transversally-harmonic wrinkles of wavenumber \(|k| (= 2\pi/\Lambda)\):

\[
\omega^2(\rho_u + \rho_b) + 2|k|\rho_u u_L \omega = k^2 \rho_u u_L (u_b - u_L)
\]  

(2.1)

where \(u_b = u_L \rho_u / \rho_b > u_L\); therefore \(\omega/|k| u_L\) is a positive constant.

Introducing a gravity field \(g\) in the direction of propagation \((g > 0\) for upward propagations) modifies (2.1) into the following form (Markstein, 1963):

\[
\omega^2(\rho_u + \rho_b) + 2|k|\rho_u u_L \omega = k^2 \rho_u u_L (u_b - u_L) + (\rho_u - \rho_b)|k|g
\]

(2.2)

Therefore \(\omega \sim |k| u_L > 0\) when \(g \ll |k| u_L^2\), whereas the opposite limit gives \(\omega \sim (\rho_u - \rho_b)g|k|/(\rho_u + \rho_b)\): in the long wavelength limit, the wrinkle dynamics is buoyancy-
controlled, and the real part of $\omega$ is predicted to be negative when $g < 0$ and $k$ is small enough (Figure 2).

The model of loss-affected flame which we consider here attempts to parallel the original LD formulation while accounting for the walls in a simple way. A comparison of the results with (2.1) and (2.2) will reveal the new mechanisms involved. Specifically we assume that a thin flame can again be identified (on the $\Lambda$-scale), on both sides of which the fluids follow non adiabatic Euler–Darcy equations, viz:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (2.3)$$

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \nabla \mathbf{u} \right) = - \nabla p - f \mathbf{u} + g \rho \quad (2.4)$$

$$\rho c \left( \frac{\partial T}{\partial t} + \mathbf{u} \nabla T \right) = - h(T - T_w) \quad (2.5)$$

In these, the two-dimensional velocity field $\mathbf{u} = (u, v)$, the density $\rho$ and the temperature $T$ have to be interpreted as quantities that are already averaged over the channel width. $g$ is a gravity field parallel to the plates, taken to be aligned with the $x$-axis. $f$ and $h$ are exchange coefficients for momentum and energy losses, respectively. Typically one has $f \sim \mu_w/d^2$ and $h \sim \lambda_w/d^2$, where $\mu_w$ and $\lambda_w$ are the dynamic viscosity and the heat conductivity of the gas evaluated at the wall temperature. Because the wall temperatures may be different on both sides of the flame, our model formally allows $f$ and $h$ to take different values on the fresh side ($f_w, h_w$) and on the burnt one ($f_b, h_b$). Note that (2.4) implies a velocity $\mathbf{u}$ which is measured in a frame where the walls are at rest, as is the case when the flame propagates against a flow of fresh mixture in a burner.

The flame is again to be modelled as a reactive discontinuity (lying in the plate plane, Figure 3) across which density and temperature change from their upstream values ($\rho_u, T_u$) to $\rho_b$ and $T_b$, with $\rho_b T_b = \rho_u T_u$ like in any low-Mach-number flame. In reality, the flames are thinner than $2d$ and are therefore curved with a typical radius of

![Figure 2](image-url)  
**FIGURE 2** Gravity-affected Landau–Darrieus growth rate $\omega$ v.s.k (Eq. 2.2).
curvature $\sim \delta$ in a plane normal to the plates (Figure 1); assimilating them to effective lines of discontinuity in the plate plane is therefore valid on the $\Lambda \gg \delta$ scales only. With this proviso, the following jump relationships are to be used "at" the effective front:

$$[\rho \mathbf{n} \cdot (\mathbf{u} - \mathbf{D})] = 0$$  \hspace{1cm} (2.6)

$$[\rho + (\rho \mathbf{n} \cdot (\mathbf{u} - \mathbf{D}))^2] = 0$$  \hspace{1cm} (2.7)

$$[\mathbf{u} \times \mathbf{n}] = 0$$  \hspace{1cm} (2.8)

where $[\mathcal{A}] = \mathcal{A}$ (burnt side)–$\mathcal{A}$ (fresh side) and $\mathbf{D}$ is the front velocity in a fixed frame; $\mathbf{n}$ is the unit normal to the front. Finally, the flame is endowed with a known burning velocity $\mathbf{U}$ relative to the fresh medium, so that:

$$\mathbf{n} \cdot (\mathbf{u} - \mathbf{D})_{\text{fresh}} = \mathbf{U}$$  \hspace{1cm} (2.9)

$\mathbf{U}$ is assumed given and, in principle, it could differ from $u_L$ due to curvature on the $\delta$-scale, wall losses and arclength larger than $2\delta$ and may therefore be $\delta$-dependent, especially in near-quenching conditions where $\mathbf{U} \approx u_L/\sqrt{e}$ (Joulin and Clavin (1976)). When $f = h = 0$, (2.3)-(2.9) formally coincide with the classical Landau–Darrieus model. For the above formulation to make some sense, the spacing between the plates of course has to be large enough to allow for flame propagation, i.e., $d > 10 \, D_h/u_L$ where $D_h$ is the upstream heat diffusivity (about half the radius of a cylindrical tube in quenching conditions; Spalding, 1957); $d$ should also be small enough as to prevent instability-induced wrinkling from developing in the direction normal to the plates, i.e. $d < O(10^3 D_h/u_L)$ when the effect of disturbance stretching by curvature on the $d$-scale is accounted for (Zel'dovich et al., 1980).

* For channels, the wall/gas exchange perimeter per unit cross-sectional area is twice as small as for cylindrical tubes, and the heat loss intensity is therefore halved.
III ADIABATIC FLAMES

In a first step we assume $h = 0$, so that $T$ and $\rho$ are piecewise uniform.

1 Steady planar front

For steady "planar" flames normal to the $x$-axis, one has $u = (\bar{u}, 0)$ and $\rho \bar{u} = \rho_a U$, whence $\bar{u} = U \rho_a/\rho_b \equiv U_b$ on the burned size. From (2.4) it is deduced that:

\[
\frac{d\bar{p}}{dx} = \rho_a g - f_a U \quad \text{for} \quad x < 0
\]
\[
= \rho_b g - f_b U_b \quad \text{for} \quad x > 0
\]

Equations (3.1) reveal the important fact that friction induces a longitudinal pressure gradient which may compete with the hydrostatic one.

2 Linear stability

If the front is slightly distorted and represented by

\[
x = F e^{\omega t + ik y}, \quad F \ll \Lambda = 2\pi/|k|
\]

the solutions to (2.3)-(2.9) are modified into:

\[
(u, v, p) = (\bar{u}, 0, \bar{p}) + (u', v', p') e^{\omega t + ik y} + \ldots
\]

where to growth rate $\omega$ has to be found and the primed quantities are linear in $F$.

The linearized continuity and transverse-momentum equations successively give

\[
v' = ik^{-1} du'/dx
\]
\[
k^2 p' = -(\rho_u + f + \rho_u U d/dx) du'/dx
\]

where $f$ (resp. $\rho$) is $f_a$ (resp. $\rho_a$) or $f_b$ (resp. $\rho_b$). The longitudinal momentum equation then implies.

\[
\left( \rho_u U \frac{d}{dx} + f + \rho_u \omega \right) \left( k^2 - \frac{d^2}{dx^2} \right) u' = 0
\]

Accordingly

\[
u' = A e^{ik|x|}, \quad x < 0
\]
\[
u' = B e^{-ik|x|} + C e^{-x f_b + \rho_b \omega \rho_b U}, \quad x > 0
\]

Whereas the potential parts are not modified by friction, the rotational contributions to $u'$ and $v'$ in the burned gases are. On each side, $v'$ then $p'$ can be expressed in terms of the three integration constants $A, B, C$ featuring in $u'$, owing to (3.4) (3.5). Once the linearized forms of (2.6)-(2.9) are employed, the condition for
the resulting linear system in $A, B, C, F$ to give a nontrivial solution is the sought-after dispersion relation:

$$
\omega^2 (\rho_u + \rho_b) + \omega (2|k| \rho_u U + f_b + f_u) \\
= k^2 \rho_u U (U_b - U) + |k| (f_b U_b - f_u U + (\rho_u - \rho_b) g) ~ (3.8)
$$

3 DISCUSSION

A comparison of (3.8) with (2.2) indicates that friction modifies the Landau–Darrieus dynamics in two ways:

- the "damping coefficient", i.e. the factor of $\omega$ in (3.8), now includes a $k$-independent contribution which, if alone, would give $\omega \sim -(f_b + f_u)/(\rho_u + \rho_b)$ when $k = 0$ and corresponds to the natural slowing down of the gas motion by friction to the walls; this contribution to damping comes from the very solutions to (3.6) and is dominant over the term $2|k| \rho_u U$ when $k$ is small enough.

- the difference in friction-induced pressure gradients competes with what gravity induces, as was patent already in (3.1); this second contribution of friction involves the combination $f_b U_b - f_u U$ which clearly comes from (3.1) and may be written as:

$$
f_b U_b - f_u U \equiv (f_b - f_u) U_b + f_u (U_b - U) ~ (3.9)
$$

The first term in (3.9) is similar to what causes the instability of the interface between a viscous fluid and a less viscous one which pushes it in a Hele–Shaw cell (Saffman and Taylor, 1958); the second one results from the expansion-induced increase in momentum loss and is thus specific to flames.

When $k$ is large enough, $\omega$ also is and (3.8) gives similar results to (2.2), i.e. $\omega \sim |k| U$, whereas the opposite limit yields:

$$
\omega \sim |k| (f_b U_b - f_u U) + (\rho_u - \rho_b) g / (f_b + f_u) ~ (3.10)
$$

along the branch which vanishes when $k = 0$ (Figure 4). In this limit, the sign of $\omega$ is controlled by the grouping

$$
j \equiv (-g)(\rho_u - \rho_b)/(f_b U_b - f_u U) ~ (3.11)
$$

which measures the relative contributions of gravity and friction to the jump in longitudinal pressure gradients. Only when $j > 1$ does gravity stabilize the longest wavelengths of wrinkling in downwardly propagating flames (Figure 4). To estimate $j$ one may assume a Poiseuille flow between the plates to obtain

$$
f \equiv 3 \mu_u / d^2 ~ (3.12)
$$

Then, with $-g = 10 \text{m/s}^2$, $\mu_u / \rho_u = 2 \times 10^{-5} \text{m}^2/\text{s}, 3U = 1 \text{m/s}, d = 2 \times 10^{-3} \text{m}(2d > \text{quenching distance})$ and $\rho_u = 6 \rho_b$, one has $j \approx 0.4$ if the wall temperature, hence $\mu_u$, is kept constant; if $\mu_u$ is allowed to vary like $\sqrt{T}$, $j \approx 0.15$. In both situations $j < 1$ and friction should allow wrinkles of arbitrarily long wavelengths to grow on downwardly propagating flames between plates. As for upward propagations, friction reinforces the
Rayleigh–Taylor instability mechanism. A way to check the above prediction could be to perform experiments with the annular burner sketched in Figure (5). The very existence of wrinkles with a wavelength (≈ mid-cylinder perimeter) such that the right-hand side of (2.2) is negative would convincingly confirm the above predictions.

For future reference we give below the limiting form of (3.6) when the density contrast is small, along with the difference \((f_u - f_b)/(f_u + f_b)\):

\[
\omega = \frac{\gamma U |k|}{2} \left( 1 + \frac{(f_u - f_b)/\gamma + g\rho_u/U}{|k|\rho_u U + f_{aw}} \right) + \ldots
\]

(3.14)

where \(2f_{aw} = (f_u + f_b)\) and the ellipses stand for \(O(\gamma^2)\) terms.

IV HEAT LOSSES

When momentum losses exist heat losses generally also do, because the Prandtl number is \(O(1)\) in gases. As a consequence \(T\) and \(\rho\) resume their upstream values beyond a distance of order \(c\rho_u U/h\) downstream of the front. If the wavelength of wrinkling markedly exceeds this cooling length, the hydrodynamic fields on the A-scale will hardly notice any change in density. The expected influence of heat losses is thus to moderate the Landau–Darrieus mechanism, the action of buoyancy and the repercussion of the second term in (3.9).

\* After unpublished preliminary experiments of S. H. Sohrab, Northwestern Univ., IL (USA).
1 Steady planar front

Upstream of the front, the previous solutions prevail. For \( x > 0 \), \( \rho_\mu = \rho_u U \) still holds, whereas the temperature profile now reads:

\[
\frac{T}{T_u} = 1 + \gamma \exp\left(-hx/c_\rho U\right)/(1 - \gamma)
\]

(4.1)

Density follows from \( \rho T = \rho_u T_u \), i.e.:

\[
\frac{\rho}{\rho_u} = \frac{(1 - \gamma)/(1 - \gamma + \gamma e^{-hx/c_\rho U})}{\rho_u}
\]

(4.2)

Continuity \( \rho \mu = \rho_u U \) gives \( \mu \), then (2.4) yields the pressure gradient. In particular the unburnt to burnt pressure-gradient now reads as:

\[
\left[ \frac{d\bar{p}}{dx} \right]^- = -\rho_u U - (f_b U_b - f_u U) + \frac{hU}{c} \frac{\gamma}{1 - \gamma}
\]

and explicitly account for a heat-loss induced contribution of \( du/dx \) in the burnt gas.

2 Stability analysis

Unfortunately, we could not exactly solve the linearized equations when \( \gamma = O(1) \), and had recourse to the small-\( \gamma \) limit to check the influence of heat losses; then, the linearized equations to be solved on both sides of the flame have constant coefficients to leading order and can be solved analytically. Straightforward (albeit rather lengthy) algebra ultimately yields:

\[
\omega = \frac{\gamma U|k|}{2} \left[ \frac{\rho_u U|k|}{\rho_u U|k| + h/c} + \frac{(f_b - f_u)/\gamma}{(\rho_u U|k| + f_{av})(\rho_u U|k| + h/c)} + \frac{g_{\rho_u} U}{(\rho_u U|k| + f_{av})} \right] + \cdots
\]

(4.3)
which is the leading-order dispersion relation. For \( h = 0 \), it reduces to (3.14), as it should do.

3 Discussion

One may rewrite the above \( \omega \) in the following form, which is equivalent to (4.3) to leading order in the small-\( \gamma \) limit:

\[
\omega(2|k|\rho_u U + f_b + f_a) = (k^2 \rho_u U(U_b - U) + |k|(\rho_u - \rho_b)g + |k|f_a(U_b - U))N\left(\frac{\rho_u \omega U |k| c}{h}\right)
\]

\[+ |k|(f_b - f_a)U_b + \cdots \quad \text{(4.4)}\]

All the sources of instability that were due to gas expansion in (3.8) are now weighted by the common factor

\[N \equiv \frac{\rho_u U |k|}{(\rho_u U |k| h/c)} \quad \text{(4.5)}\]

which vanishes when \( k \to 0 \), if \( h \neq 0 \). It obviously accounts for the now finite length \( \sim c \rho_u h \) over which \( \rho \) significantly departs from its upstream value; only the first contribution of (3.9) and of the Saffman–Taylor-like instability is unaffected, as expected.

In the preceding paragraph, we considered a case where \( f_b > f_u \) and adiabatic walls. One may now inquire about the other extreme, corresponding to cold walls, for which \( f_b = f_u \) but \( h \neq 0 \). The sign of \( \omega \) in the small-k limit is controlled by

\[(-g)\frac{(\rho_u - \rho_b)}{f_a(U_b - U)} \quad \text{(4.6)}\]

Accordingly, long-wave instability is again predicted if \( j \) is markedly less than unity. The main difference from the adiabatic case is that, now, \( \omega \sim k^2 \) in the small-k limit. Comparing (4.4) with (3.8) immediately indicates which interpolation formula may be constructed to cover all cases when \( \gamma = O(1) \), upon weighting the density changes and \( U_b - U \) featuring in (3.8) by the factor \( N \) defined in (4.5); the experiments will reveal whether it is necessary to go beyond, e.g., upon numerical integration of the linearized equations.

V Measuring the Wall Exchanges

In the above model, the momentum and heat losses have been accounted for in a crude way and, even though estimates such as (3.12) can be proposed, it would be valuable to provide the experimentalists with alternative ways of measuring the effective friction and cooling laws. To this end we consider the slightly extended, 1-D versions of (2.3)–(2.5):

\[
\frac{\partial}{\partial t} (\rho u^\alpha) + \frac{\partial}{\partial r}(\rho u u^\alpha) = 0 \quad n = 0, 1 \quad \text{(5.1)}
\]

\[
\rho c \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r}\right) = -H(T - T_w) \quad \text{(5.2)}
\]
which govern effectively onedimensional, nonadiabatic flows with planar \((n = 0)\) or cylindrical \((n = 1)\) symmetry; the function \(H(\cdot)\) is any positive function, e.g. \(H(x) = hx\), which vanishes when its argument does. Note that the momentum equation is decoupled from the continuity and energy balance in such situations. Once supplemented by an equation of state \((\rho T = \rho_u T_u)\) and by the boundary conditions:

\[
\begin{align*}
  u &= 0, \quad \text{at } r = 0 \\
  \rho_u \left( \frac{dR}{dt} - u \right) &= \rho_u U \quad \text{and} \quad T = T_u, \quad \text{at } r = R(t)
\end{align*}
\]

Eqs. (5.1), (5.4) are enough to compute the trajectory \(r = R(t)\) of an "effective flame" which expands horizontally from a center \((r = 0)\) into a mixture that is originally at rest.

Upon use of a mass-weighted coordinate in the above hyperbolic system, one can show (see the Appendix) that \(R(t)\) satisfies the integral equation

\[
R^{n+1}(t) = (n + 1) \int_0^t R^n(\tau) \frac{T_0(t - \tau)}{T_u} U \, d\tau
\]

where \(T_0(t)\) is defined by (5.2) with \(u = 0\), i.e.:

\[
\rho c \frac{dT_0}{dt} = -H(T_0 - T_u), \quad T_0(0) = T_b
\]

Due to heat losses the burned gas shrink and \(R(t) \neq U \cdot t\); more precisely one may show (Figure 6) that:

\[
dR/dt = \begin{cases} 
  U_b & \text{if } \frac{ht}{\rho u c} \ll 1 \\
  U & \text{if } \frac{ht}{\rho u c} \gg 1
\end{cases}
\]

Heat losses moderate the expansion, in much the same way as they did the LD effect. This is patent when \(n = 0\), in which case (5.5) gives \(R(t)\) explicitly. Recording \(R(t)\), when \(n = 1\) as to minimize spurious edge effects and to delay symmetry-breaking instabilities (Istratov and Librovich, 1969), one can in principle invert the integral equation (5.5) to

![FIGURE 6 Heat-loss affected speed of an expanding flame (sketched).](image-url)
obtain $T(t)$; then (5.6) gives access to the cooling law. Next, considering the momentum equation

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} \right) = -\frac{\partial P}{\partial r} - F(u)$$

(5.8)

could give access to the friction law $F(u)$ upon measurements of the (small) pressure variations at fixed locations, because $u(t,r)$ is in principle accessible from a knowledge of $\rho \sim 1/T$ and from the continuity equation.

Working out the converging cases (where $R(t)$ decreases to zero and $u = 0$ in the fresh gases) could give very valuable complementary results; the situation where $\eta = 1$ and the flame is stabilized around a source of fluid is also certainly worth to consider, e.g. to yield measurements of $U$ and possibly of the “Markstein” length $L$ (see below).

VI CONCLUDING REMARKS

The variations on the Landau–Darrieus theme which we presented here show that, at the linear level, momentum and/or heat losses to the walls are likely to drastically affect the large-scale stability of a “quasi-2D” flame. Such losses would also modify the dynamics of finite-amplitude wrinkles. In the small-expansion approximation ($\gamma \ll 1$) it is a simple matter to write down a nonlinear equation for the flame shape (actually its projection onto the plate planes shown in Figure (3)) $x = F(t,y)$, viz:

$$\frac{\partial F}{\partial t} + \frac{U}{2} \left( \frac{\partial F}{\partial y} \right)^2 = I(F,y)$$

(6.1)

where the linear operator $I(\cdot, y)$ is defined by its action on $e^{iky}$ as the multiplication by the right-hand-side of (4.3); the nonlinearity featuring in (6.1) has indeed a purely geometrical origin (Sivashinsky, 1977) and is generic for weakly unstable ($\gamma \ll 1$) flames. As it stands, (6.1) is ill-posed dynamically, however, because (4.3) predicts an unlimited growth rate $\omega$ in the short-wavelength limit. Equation (6.1) thus needs be regularized, e.g. through a curvature term $UL \delta^2 F/\delta y^2$ added to the r.h.s. How to choose $L > 0$ is unclear because many lengths are involved (cooling and/or friction lengths, plate spacing $2d$, flame thickness $D_{th}/u_k$). Presumably $L \sim d$ because this is the effective width of the flame, it is an interesting (but yet unsolved) problem to get reliable estimates of $L$, given that the basic problem corresponding to $F \equiv 0$ ought to be solved numerically in reality to find $U$ and the flowfield at $O(d)$ distances from the actual flame. At any rate this curvature term is not expected to play a great role, apart from a regularization, if the wavelengths of $F(t,y)$ are long enough compared to $L$ (except at the crests of $F(t,y)$).

Alternatively, the truncated equation (6.1) may be used to find steady patterns $(F(t,y) = -Vt + F(y))$, as McConnaughey et al. (1983) did for the lossless flames.

At least when heat losses are absent, the situations we considered may also be viewed as a formal generalization of the problem of Saffman and Taylor (1958). From a mathematical standpoint it would be very interesting to understand how a small mass-flux ($\rho_k U$) through the interface modifies this now well mastered (Tanveer, 1991) classical problem. Last, but not least, careful experiments would be welcomed.
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REFERENCES


APPENDIX

We introduce a mass-weighted coordinate \( \psi = \int_0^r \rho(t,r)dr \) to transform (5.2) into:

\[
\frac{\partial T}{\partial \psi} \bigg|_{\psi = \text{const}} = -H(T - T_a)
\]

(A.1)

whereby \( T = T_a(t - \theta(\psi)) \). The second boundary condition in (5.4) and (5.6) imply that the integration “constant” \( \theta(\psi) \) is the inverse function of \( \Psi(\cdot) \):

\[
\tau = \theta(\phi) \Leftrightarrow \phi = \Psi(t)
\]

(A.2)

where \( \Psi \) is defined by

\[
\Psi(t) = \int_0^{\rho(t)} r^2 \rho(t,r)dr
\]

(A.3)
Differentiating (A.3) and exploiting (5.1) yields:

$$\frac{d\Psi}{dt}(t) = \rho_u U R^\ast(t)$$  \hspace{1cm} (A.4)

once the first condition in (5.4) is employed. Besides, the definitions of $\Psi$ and $\Psi$ combined with the equation of state $\rho T = \rho_u T_u$ imply:

$$R^{n+1}(t) = (n + 1) \int_{0}^{\Psi(t)} \frac{1}{\rho_u T_u} (t - \theta(\phi)) d\phi$$  \hspace{1cm} (A.5)

Changing the variable of integration in (A.5) from $\phi$ to $\tau = \theta(\phi)$ yields (5.5), thanks to (A.2) and (A.4).