AME 514 Applications of Combustion – Spring 2017 – Homework #4

Due Friday 4/28/17, 4:00 pm, in the drop box in OHE 430N (Xerox room in the OHE 430 suite of offices.) While hard copies are preferred, if you're off campus you can email your assignment to ronney@usc.edu. DEN students should submit through the usual channels. ***Late homework marked down 10 points (out of 100 possible) per day late.***

Part 1: paper review

Since there weren't many references in this set of lectures, there won't be a Part 1. Part 2 will count twice as much as did for the other homework sets. You'll notice Part 2 is somewhat more time-consuming than usual, though probably not twice as long as the others. (All the words in this problem set make it look like a long problem set, but in fact it makes it easier to do, not more difficult, since I've given you step by step instructions. Of course, your mileage may vary).

Part 2. The usual type of homework questions

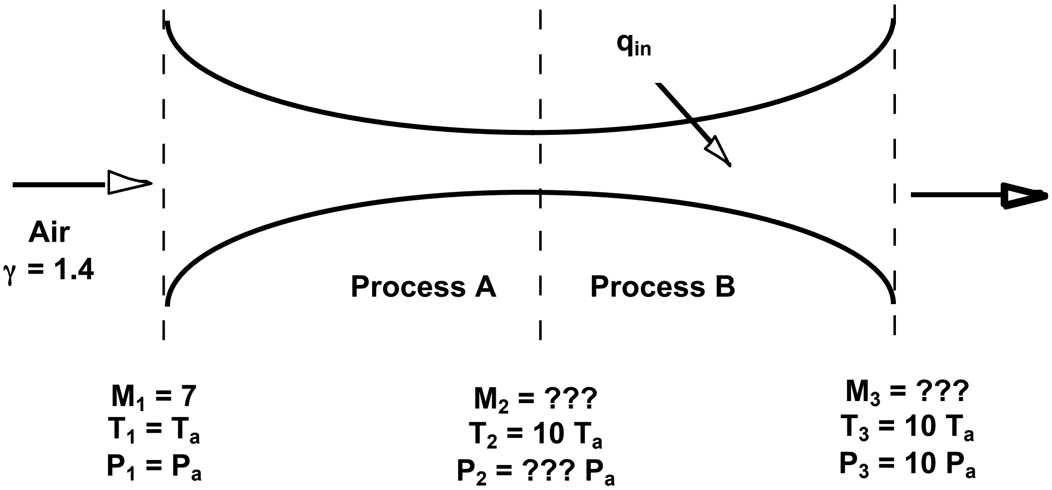
<u>Problem #1 (15 points)</u>

Consider a simple hypersonic propulsion system for an aircraft at an initial Mach number of 15 that consists of two processes:

> **Process A**: Decelerate the incoming flow reversibly and adiabatically until the static temperature is 3000K
>
> **Process B**: Add heat at constant temperature until the static pressure is equal to ambient pressure

a. For a flight an ambient atmosphere at 100,000 feet (227K and 0.0107 atm, with $\gamma = 1.4$), to what Mach number could the air be decelerated? What would the corresponding pressure after deceleration be? What would the area ratio be?
b. After heat addition, what would the exit Mach (M_e) number be? What would the area ratio be?
c. What would the specific thrust be? (Note for this case specific thrust = Thrust/ $\dot{m}_a c_1 = \dot{m}_a(u_e - u_1)/\dot{m}_a c_1 = (M_e c_e - M_1 c_1)/c_1 = M_e(T_e/T_1)^{1/2} - M_1$, which is all stuff you already have)
d. What would the Thrust Specific Fuel Consumption be?
   (Note that TSFC = (Heat input)/Thrust*c_1
   $= [\dot{m}_a(C_P(T_{3t} - T_{2t})c_1]/[\text{Thrust}^*c_1^2]$
   $= [(\dot{m}_a c_1)/\text{Thrust}]\ [(\gamma/(\gamma-1))R(T_{3t}-T_{2t})/(\gamma R T_1)]$
   $= [1/(\text{Specific thrust})]\ [1/(\gamma-1)]\ [(T_{3t}-T_{2t})/T_1]$
   and you have everything needed to calculate T_{3t} and T_{2t})
e. Can any fuel burning in air generate enough heat to accomplish this? Look at stoichiometric hydrogen-air and see if the heat release per unit mass = $f_{stoich}Q_R$ is equal to or greater than the heat input needed = $C_P(T_{3t} - T_{2t})$. (Your answer should be NO, but support with numbers).

Problem #2 (15 points) (from a previous year's final exam)



Consider a simple hypersonic propulsion system for an aircraft at an initial Mach number of 7 that consists of two processes:

> **Process A**: Decelerate the incoming flow reversibly and adiabatically until the static temperature is 10 times the ambient temperature T_a
> **Process B**: Add heat at constant temperature until the pressure is equal to 10 times the ambient pressure

Assume that the gas is ideal with constant C_P, $\gamma = 1.4$ and that the fuel-to-air ratio (FAR) << 1.

a) Compute the Mach number after deceleration (station 2)
b) Compute the static (not stagnation) pressure relative to P_a after deceleration (station 2)
c) Compute the Mach number at the exit (station 3)
d) Compute the non-dimensional specific thrust
e) Compute the overall efficiency
f) Are the area changes between stations 1 and 2 and between stations 2 and 3 shown in the figure qualitatively correct? Why or why not?

Problem #3 (20 points)

Let's (sort of) repeat problem 1 using GASEQ (http://www.gaseq.co.uk)

a. Note the enthalpy (h_1) and sound speed (c_1) of air at ambient conditions (227 K, 0.0107 atm), then find the kinetic energy of the ambient air $u_1^2/2 = (c_1M_1)^2/2$. Then select process "Adiabatic compression/expansion" (be sure to use air as the reactants, dissociated air as the products, and uncheck the "frozen composition" box). Compress the air to a product temperature T_2 of 3000K by adjusting your guess of P_2 (should be around 300 atm) and hitting the "Calculate" button each time.

b. Now do the combustion. To do this, let's first re-visit the constant-temperature heat addition analysis. The momentum equation is $AdP + \dot{m}du = 0$ or $AdP + \rho uAdu = 0$, and the energy equation is $h + u^2/2$ = constant or $dh + udu = 0$. (Note that the heat transfer q does not appear since the enthalpy h includes both chemical and thermal enthalpy in GASEQ; thus the energy equation says that the sum of kinetic energy, thermal enthalpy and chemical enthalpy is constant.) Combining these, plus the ideal gas law $P = \rho RT$ yields $dP/P = dh/RT$. T is constant by assumption, but R is not quite constant since $R = \Re/M$ and the molecular weight M will changed somewhat during combustion. But if we take a value of M averaged between the reactant and product mixtures, we won't be too far off. So if we assume constant (averaged) M and thus constant R, we obtain

$$\ln(P_3/P_2) = (h_3 - h_2)/RT_2$$

where $T_2 = T_3 = 3000K$. So the process for doing the combustion is:

1. Choose as reactants "hydrogen-air flame" and as products "H2/O2/N2 products." (Note that we've ignored any mixture process and the effect that has on the mass flow, stagnation P and T, etc.) The default mixture strength is stoichiometric, so you shouldn't have to change that. Again be sure "frozen composition" is not checked.
2. Guess P_3
3. For the problem type, choose "Equilibrium at defined T and P", enter the 3000K for T_3 and your guess for P_3, and hit "calculate."
4. Get h_2 and M_2, h_3 and M_3 from GASEQ, calculate the average molecular weight = M_{avg} = $(M_2 + M_3)/2$, and calculate the average $R = \Re/M_{avg}$.
5. Is the above equation $\ln(P_3/P_2) = (h_3 - h_2)/RT_2$, satisfied? If not, adjust your guess for P_3 and go back to step 3.

c. Now do the expansion. Select problem type "Adiabatic compression/expansion." Hit "R << P" to transfer the products to reactants. Make sure the "frozen composition" box is unchecked. You should be able to choose a product pressure of 0.01 atm but this doesn't converge. Instead choose a product pressure of 0.1 atm, hit "Calculate," then hit "R << P" to transfer the products to reactants, check the "frozen composition" box, choose a product pressure of 0.0107 atm, hit "Calculate" one more time and you're done. Note the final enthalpy h_e.

d. Compute the product velocity from $h_1 + u_1^2/2 = h_e + u_e^2/2$. You have everything except u_e. **Note that GASEQ gives you enthalpies in kJ/kg, not J/kg, so you need to multiply GASEQ's values of h by 1000 to get the units right. You now have fair warning, I will not be very forgiving if you're numbers are off by $(1000)^{1/2}$!**

e. Compute the specific thrust = $(u_e - u_1)/c_1$, which should be a lot lower than in problem 1 because your answer to 1g was NO.

f. Compute TSFC = (Heat input)/Thrust*c_1 = $\dot{m}_a c_1 f_{stoich} Q_R/(\dot{m}_a (u_e - u_1)^* c_1^2)$ = (1/(Specific thrust)) $f_{stoich} Q_R/c_1^2$. This should be pretty similar to your answer to problem 1. Also calculate the Specific Impulse = $(1/TSFC)(Q_R/c_1 g_{earth})$. I get about 2100 seconds, much better than the best H_2-O_2 rocket engines (about 450 sec) but not that great considering how hard it will be to get anywhere near this ideal performance.

Problem #4 (15 points)

Estimate the zero Mach number thrust of a Pulse Detonation Engine using propane in the following way.

a. Estimate the dimensionless heat addition H for stoichiometric propane-air assuming T_1 = 300K and P_1 = 1 atm.
b. Compute the detonation Mach number M_1 and the incoming reactant velocity $u_1 = M_1c_1$
c. Compute the post-shock Mach number M_2, temperature T_2 and pressure P_2 using the analytical formulas (the ones with all the M's and γ's flying around) given in Lecture 11.
d. Compute the pressure P_3, temperature T_3, and sound speed c_3 after heat addition to $M_3 = 1$ in a constant-area duct.
e. We've computed the velocity of the products in the frame of reference attached to the moving detonation front. We need the velocity in the frame of reference of the unburned gas, i.e. in the laboratory frame of reference. So compute u_3 (lab frame) = $u_1 - u_3 = u_1 - c_3M_3 = u_1 - c_3$.
f. The gas behind the detonation products is moving toward the open end of the tube with a velocity $u_{3,lab}$. But the velocity of the gas at the closed end of the tube must be zero. Thus, the detonation products act like a piston and cause an expansion wave in the products. Compute the pressure P_4, temperature T_4 and sound speed c_4 of the gas after this expansion wave according to the isentropic wave relations from 1D gas dynamics:
g. Now compute the specific impulse. If we assume, as discussed in class, that the approximate time the thrust surface "feels" the pressure P_4 is $L/u_1 + L/c_4$, where L is the tube length, then the total impulse is $(P_4 - P_1)AL(1/u_1 + 1/c_4)$, where A is the tube (and thrust surface) cross-sectional area. Then the specific impulse = (total impulse)/(fuel weight), where the fuel weight is (total mass)(fuel mass fraction)g = (ρ_1)(volume)fg = $\rho_1 ALfg = (P_1/RT_1)ALfg$. And finally recall that the specific heat addition H from part (a) is given by $H = fQ_R/RT_1$, so the fuel weight is $(P_1/RT_1)AL(HRT_1/Q_R)g = P_1ALHg/Q_R$. Thus the specific impulse is
h. Compute the specific thrust and TSFC.

Problem #5 (15 points)

Now use GASEQ again, which conveniently offers a CJ detonation solver.

a. Choose reactants "propane-air flame" and products "HC/O2/N2 products." (Again the default mixture strength is stoichiometric, so you shouldn't have to change that.) Use 300K and 1 atm as the initial conditions. Choose Problem type "C-J-Detonation" and hit "Calculate." Note the incoming reactant velocity $u_1 = c_1M_1$ and the sound speed (c_3) and specific heat ratio (γ_3) of the products. Note that $M_3 = 1$ as required for a CJ detonation. Compute $u_{3,lab} = u_1 - u_3 = u_1 - c_3M_3 = u_1 - c_3$.
b. Estimate the final pressure P_4 after the expansion wave from the relation

$$\frac{P_4}{P_3} = \left(1 - \frac{\gamma - 1}{2}\frac{\Delta u}{c_3}\right)^{2\gamma/\gamma - 1}$$

which is not strictly valid since γ is not constant between states 3 and 4 when we consider gases with non-constant specific heats and dissociation, but γ changes so little during this process we'll neglect that.

c. Now hit "R << P" to transfer the products to reactants, select process "adiabatic compression/expansion," select product pressure P_4, and hit "Calculate." Note the sound speed (c_4) of the expanded products.
d. Compute the specific thrust, TSFC and specific impulse in the usual way. I get I_{SP} between 1200 and 1400 seconds – not exactly spectacular.

Problem #6 (10 points). NO formation

a) For a stoichiometric premixed laminar methane-air flame, plot the log of NO concentration as a function of distance from the flame front. (Make it a big plot since you will be drawing several other curves on the same plot.) Explain the shape of this plot briefly.
b) On the same plot, draw the log of NO concentration as a function of distance from the flame front for a very lean premixed methane-air flame. Explain the shape of this plot briefly.
c) Repeat b) for a stoichiometric premixed methane-air flame with just enough exhaust gas recirculation to have the same adiabatic flame temperature as the flame of part b)
d) Repeat b) for a rich premixed methane-air flame with the same adiabatic flame temperature as the flame of part b
e) Repeat b) for a rich premixed CO-air flame with the same adiabatic flame temperature as the flame of part b)

(Consider both thermal and prompt NO formation, and the relative magnitudes of both types of NO for each of these flames).

f) In terms of **maximum NO concentration**, how would a nonpremixed methane-air flame with no fuel or air dilution compare to the premixed flames in a) – e), i.e. which premixed flames would have a higher maximum NO, and which would have a lower maximum NO?

Problem #7 (10 points). Soot formation

Rank each member of the following groups (rank the members of each group separately, don't try to rank across groups) in terms of their propensity to form soot, and explain why. For example group 1: d, a, c, b; group 2: a, d, c, b.

Group 1:

a. Stoichiometric premixed methane-air flame
b. Non-premixed methane-air flame with no fuel or air dilution
c. Non-premixed methane-air flame with synthetic air having 30% O_2 rather than the usual 21%
d. Non-premixed ethane-air flame with no fuel or air dilution

Group 2 (all with N_2 dilution adjusted to obtain same peak temperature):

a. Slightly rich premixed propane-air flame (propane: $H_3C\text{-}CH_2\text{-}CH_3$)

b. Slightly rich premixed butane-air flame (butane: $H_3C\text{-}CH_2\text{-}CH_2\text{-}CH_3$)
c. Slightly rich premixed propylene-air flame (propylene: $H_3C{=}CH\text{-}CH_3$)
d. Non-premixed propane-air flame

Group 3

a. Slightly rich premixed propane-air flame with no heat losses
b. Same flame as 3a but with substantial heat losses in burned gas
c. Non-premixed propane-air flame with same peak temperature as in 3a and no heat losses
d. Same flame as in 3c but with substantial heat losses