Part 1: paper review

Since there weren’t many references in this set of lectures, there won’t be a Part I. Part 2 will count twice as much as did for the other homework sets. You’ll notice Part 2 is somewhat more time consuming than usual, though probably not twice as long as the others. (All the words in this problem set make it look like a long problem set, but in fact it makes it easier to do, not more difficult, since I’ve given you step by step instructions. Of course, your mileage may vary).

Part 2. The usual type of homework questions

Problem #1 (15 points)

a. Show that for heat addition at constant temperature (which really simplifies things) using the first law \((h_1 + u_1^2/2 + q = h_2 + u_2^2/2)\), the entropy of an ideal gas \(s_2 - s_1 = C_P \ln(T_2/T_1) - R \ln(p_2/p_1)\), the enthalpy of an ideal gas \(h_2 - h_1 = C_P (T_2 - T_1)\), the definition of Mach number \(M = u/c = u/((\gamma RT)^{1/2})\) and the consequence of second law \((ds = dq/T)\):

\[P_2/P_1 = \exp[(\gamma/2)(M_1^{5/2} - M_2^{5/2})]\]

(escape you already have this result from the lecture notes but I’d like for you to show it…)

b. By additionally using mass conservation, show that

\[A_2/A_1 = (M_1/M_2) \exp[(\gamma/2)(M_2^{5/2} - M_1^{5/2})]\]

(ditto comment in part a)

c. Now consider a propulsion system based on this. First the air will be decelerated isentropically (not to \(M = 0\)) then heat will be added at constant temperature. For a flight Mach number of 15, an ambient atmosphere at 100,000 feet (227K and 0.0107 atm, with \(\gamma = 1.4\)), to what Mach number could the air be decelerated if the maximum allowable gas temperature is 3000K? What would the corresponding pressure be?

d. From this condition, if heat is added at constant temperature until the ambient pressure was reached (not a good way to operate, but this represents a sort of maximum heat addition), what would the exit Mach \((M_e)\) number be? What would the area ratio be?
e. What would the specific thrust be? (Note for this case specific thrust = Thrust/\dot{m}_ac_1 = \dot{m}_a(u_e - u_i)/\dot{m}_ac_1 = (M_1c_e - M_i c_i)/c_1 = M_1(T_c/T_1)^{1/2} - M_i, which is all stuff you already have)

f. What would the Thrust Specific Fuel Consumption be?
(Note that TSFC = (Heat input)/Thrust*c_1
= [\dot{m}_ac_1]/Thrust] \gamma/(\gamma-1)R(T_3 - T_2)/\gamma RT_i]
= [1/(Specific thrust)] [1/(\gamma-1)] [(T_3 - T_2)/T_i]
and you have everything needed to calculate T_3 and T_2)

g. Can any fuel burning in air generate enough heat to accomplish this? Look at stoichiometric hydrogen-air and see if the heat release per unit mass = f_{stoi} Q_{R} is equal to or greater than the heat input needed = C_p(T_3 - T_2). (Your answer should be NO, but support with numbers).

Problem #2 (20 points)

Let’s (sort of) repeat problem 1 e – f using GASEQ, which you can download from http://www.gaseq.co.uk.

a. Note the enthalpy (h_i) and sound speed (c_i) of air at ambient conditions (227 K, 0.0107 atm), then find the kinetic energy of the ambient air u_i^2/2 = (c_i M_1)^2/2. Then select process “Adiabatic compression/expansion” (be sure to use air as the reactants, dissociated air as the products, and uncheck the “frozen composition” box). Compress the air to a product temperature T_2 of 3000K by adjusting your guess of P_2 (should be around 300 atm) and hitting the “Calculate” button each time.

b. Now do the combustion. To do this, let’s first re-visit the constant-temperature heat addition analysis. The momentum equation is AdP + \dot{m}du = 0 or AdP + \rho u \Delta u = 0, and the energy equation is h + u^2/2 = constant or dh + u du = 0. (Note that the heat transfer q does not appear since the enthalpy h includes both chemical and thermal enthalpy in GASEQ; thus the energy equation says that the sum of kinetic energy, thermal enthalpy and chemical enthalpy is constant.) Combining these, plus the ideal gas law P = \rho RT yields dP/P = dh/RT. T is constant by assumption, but R is not quite constant since R = \mathcal{R}/M and the molecular weight M will changed somewhat during combustion. But if we take a value of M averaged between the reactant and product mixtures, we won’t be too far off. So if we assume constant (averaged) M and thus constant R, we obtain

\ln(P_3/P_2) = (h_1 - h_2)/RT_2

where T_2 = T_3 = 3000K. So the process for doing the combustion is:

1. Choose as reactants “hydrogen-air flame” and as products “H2/O2/N2 products.” (Note that we’ve ignored any mixture process and the effect that has on the mass flow, stagnation P and T, etc.) The default mixture strength is
stoichiometric, so you shouldn’t have to change that. Again be sure “frozen composition” is not checked.

2. Guess $P_3$

3. For the problem type, choose “Equilibrium at defined $T$ and $P$”, enter the 3000K for $T_3$ and your guess for $P_3$, and hit “calculate.”

4. Get $h_2$ and $M_3$, $h_3$ and $M_3$ from GASEQ, calculate the average molecular weight $= M_{avg} = (M_2 + M_3)/2$, and calculate the average $R = \mathcal{R}/M_{avg}$.

5. Is the above equation $\ln(P_3/P_2) = (h_3 - h_2)/RT_2$, satisfied? If not, adjust your guess for $P_3$ and go back to step 3.

c. Now do the expansion. Select problem type “Adiabatic compression/expansion.” Hit “$R \ll P$” to transfer the products to reactants. Make sure the “frozen composition” box is unchecked. You should be able to choose a product pressure of 0.01 atm but this doesn’t converge. Instead choose a product pressure of 0.1 atm, hit “Calculate,” then hit “$R \ll P$” to transfer the products to reactants, check the “frozen composition” box, choose a product pressure of 0.0107 atm, hit “Calculate” one more time and you’re done. Note the final enthalpy $h_e$.

d. Compute the product velocity from $h_1 + u_1^2/2 = h_e + u_e^2/2$. You have everything except $u_e$. Note that GASEQ gives you enthalpies in kJ/kg, not J/kg, so you need to multiply GASEQ’s values of $h$ by 1000 to get the units right. You now have fair warning, I will not be very forgiving if you’re numbers are off by $(1000)^{1/2}$!

e. Compute the specific thrust $= (u_e - u_1)/c_1$, which should be a lot lower than in problem 1 because your answer to 1g was NO.

f. Compute $TSFC = (\text{Heat input})/\text{Thrust}*c_1 = \dot{m}_a c_1 f_{stoch} Q_R/(\dot{m}_a (u_e - u_1)*c_1^3) = (1/(\text{Specific thrust})) f_{stoch} Q_R/c_1^2$. This should be pretty similar to your answer to problem 1. Also calculate the Specific Impulse $= (1/TSFC)(Q_R/c_1 g_{earth})$. I get about 2100 seconds, much better than the best H_2-O_2 rocket engines (about 450 sec) but not that great considering how hard it will be to get anywhere near this ideal performance.

Problem #3 (15 points)

Estimate the zero Mach number thrust of a Pulse Detonation Engine using propane in the following way.

a. Estimate the dimensionless heat addition $H$ for stoichiometric propane-air assuming $T_1 = 300K$ and $P_1 = 1$ atm.

b. Compute the detonation Mach number $M_1$ and the incoming reactant velocity $u_1 = M_1 c_1$

c. Compute the post-shock Mach number $M_2$, temperature $T_2$ and pressure $P_2$ using the analytical formulas (the ones with all the M’s and $\gamma$’s flying around) given in Lecture 11.

d. Compute the pressure $P_3$, temperature $T_3$, and sound speed $c_3$ after heat addition to $M_3 = 1$ in a constant-area duct.
e. We’ve computed the velocity of the products in the frame of reference attached to the moving detonation front. We need the velocity in the frame of reference of the unburned gas, i.e. in the laboratory frame of reference. So compute \( u_3 \) (lab frame) = \( u_1 - u_3 = u_1 - c_3 M_1 \).

f. The gas behind the detonation products is moving toward the open end of the tube with a velocity \( u_{3, \text{lab}} \). But the velocity of the gas at the closed end of the tube must be zero. Thus, the detonation products act like a piston and cause an expansion wave in the products. Compute the pressure \( P_4 \), temperature \( T_4 \) and sound speed \( c_4 \) of the gas after this expansion wave according to the isentropic wave relations from 1D gas dynamics:

\[
\frac{P_4}{P_3} = \left( 1 - \frac{\gamma - 1}{\gamma} \frac{\Delta u}{c_3} \right)^{\gamma/\gamma-1}
\]

which is not strictly valid since \( \gamma \) is not constant between states 3 and 4 when we consider gases with non-constant specific heats and dissociation, but \( \gamma \) changes so little during this process we’ll neglect that.

g. Now hit “R << P” to transfer the products to reactants, select process “adiabatic compression/expansion,” select product pressure \( P_4 \), and hit “Calculate.” Note the sound speed \( c_4 \) of the expanded products.

d. Compute the specific thrust, TSFC and specific impulse in the usual way. I get I\(_{sp}\) between 1200 and 1400 seconds – not exactly spectacular.

**Problem #4 (20 points)**

Now use GASEQ again, which conveniently offers a CJ detonation solver.

a. Choose reactants “propane-air flame” and products “HC/O2/N2 products.” (Again the default mixture strength is stoichiometric, so you shouldn’t have to change that.) Use 300K and 1 atm as the initial conditions. Choose Problem type “CJ-Detonation” and hit “Calculate.” Note the incoming reactant velocity \( u_1 = c_1 M_1 \) and the sound speed \( c_3 \) and specific heat ratio \( \gamma_3 \) of the products. Note that \( M_3 = 1 \) as required for a CJ detonation. Compute \( u_{3, \text{lab}} = u_1 - u_3 = u_1 - c_3 M_1 = u_1 - c_3 \).

b. Estimate the final pressure \( P_4 \) after the expansion wave from the relation

\[
\frac{P_4}{P_3} = \left( 1 - \frac{\gamma - 1}{\gamma} \frac{\Delta u}{c_3} \right)^{\gamma/\gamma-1}
\]
Problem #5 (15 points) (from a previous year’s final exam)

Consider a very simple propulsion cycle for an aircraft flying in air ($\gamma = 1.4$) at $M_1 = 5$ consisting of

**Process 1 (state 1 to state 2):** Heat addition at constant area until $M_2 = 2$

**Process 2 (state 2 to state 3):** Heat addition at constant $T$ until exit $P$ ($P_3$) = ambient $P$ ($P_1$)

(a) Find the static pressure at state 2 in terms of $P_1$, that is find $C$ in the expression $P_2 = CP_1$
(b) Find the static temperature at state 2 in terms of $T_1$, that is find $C$ in the expression $T_2 = CT_1$
(c) Find the exit Mach number ($M_3$)
(d) Find the stagnation temperature at state 3 in terms of $T_1$, that is find $C$ in the expression $T_{3t} = CT_1$
(e) Find the specific thrust assuming FAR $<< 1$
(f) Find the Thrust Specific Fuel Consumption and Overall Efficiency

Problem #6 (15 points) (from a previous year’s final exam)

Two hypersonic engine designs, A and B, are being considered for a high-speed transport aircraft operating at a flight Mach number of 7.

Engine A produces a flow at the exit with a stagnation pressure 50 times the ambient pressure and a stagnation temperature 15 times the ambient temperature.

Engine B produces a flow at the exit with a stagnation pressure 100 times the ambient pressure and a stagnation temperature 10 times the ambient temperature.

Because these two engines are made by rival companies with trade secrets, little is known about what happens inside the engines. It is not known for either engine it uses a compressor or not, whether combustion occurs at constant $P$, $T$, $A$ or none of the above, if the diffuser and nozzle are reversible or not, nor is $\tau$ known. All that is known is that for both engines (1) the same fuel is used, (2) expansion occurs in the exhaust nozzle to ambient pressure, (3) the gas has constant specific heats with $\gamma = 1.3$, and (4) the fuel to air ratio (FAR) is much less than 1.

a) Which engine, A or B, has the higher exhaust velocity?
b) Which engine, A or B, has the higher mass flow per unit throat area?
c) Which engine, A or B, has the higher specific thrust?
d) Which engine, A or B, has the higher thrust specific fuel consumption?