Liftoff Characteristics of Turbulent Jet Diffusion Flames

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A theoretical analysis of turbulent jet diffusion flames is developed in which the flame is regarded as an ensemble of laminar diffusion flamelets that are highly distorted. The flow inhomogeneities are considered to be sufficiently strong to produce local quenching events for flamelets as a consequence of excessive flame stretch. The condition for flamelet extinction is derived in terms of the instantaneous scalar dissipation rate, which is ascribed a log-normal distribution. Percolation theory for a random network of stoichiometric sheets is used to predict quenching thresholds that define liftoff heights. Predictions are shown to be in reasonably satisfactory agreement with experimentally measured liftoff heights of methane jet diffusion flames, within experimental uncertainties.

I. Introduction

FUEL issuing from a tube or duct into an oxidizing atmosphere forms a jet in which combustion may occur. The associated combustion process is the most classical example of a diffusion flame. At sufficiently high velocities of fuel flow (fundamentally, at sufficiently large Reynolds numbers) the entire diffusion flame is turbulent. The turbulent jet diffusion flame begins at the mouth of the duct for a range of values of the exit velocity. When a critical exit velocity is exceeded, the flame abruptly is detached from the duct and acquires a new configuration of stabilization in which combustion begins a number of duct diameters downstream. Flames in this state, stabilized in the mixing region, are termed lifted diffusion flames, and the critical exit velocity at which they appear is called the liftoff velocity.

The liftoff height is the centerline distance from the duct exit to the plane of flame stabilization. A further increase in the exit velocity increases the liftoff height without significantly modifying the turbulent flame height (the centerline distance from the duct exit to the plane at which, on the average, combustion ceases). There is a second critical value of the exit velocity, called the blowoff velocity, beyond which the flame cannot be stabilized in the mixing region. The present study addresses questions of the structure of lifted turbulent diffusion flames at exit velocities between liftoff and blowoff values. Attention is focused especially on the calculation of liftoff heights.

Liftoff characteristics for turbulent jet diffusion flames are of practical importance in connection with flame stabilization. Conditions for liftoff and blowoff must be known in developing rational designs of burners, e.g., in diffusion-flame combustors for power production or in flaring applications for the petroleum industry. They are also of interest in connection with extinguishment of certain fires that may occur in oil or gas rigs. The present work is directed toward developing an improved fundamental understanding of liftoff phenomena that may later prove useful for these applications.

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The early studies of turbulent diffusion flames recognized the existence of liftoff and blowoff[1] but were focused instead on flame heights. Often only blowoff velocities were reported along with flame heights, no mention being made of liftoff.[2] Recently some relatively detailed data on liftoff velocities and liftoff heights have been generated[3] for methane and methane-hydrogen jets in air. These data afford a basis for comparison with predictions and will be employed herein for that purpose.

II. Theoretical Concepts of Lifted Flames

A popular view of liftoff and blowoff phenomena treats diffusion flames as if they were premixed flames.[4,5] A characteristic of premixed flames is the existence of a flame velocity at which the flame propagates with respect to the unburnt mixture. In the diffusion flame mixing is considered to occur prior to onset of combustion, and the base of the diffusion flame is assumed to be stabilized by propagation of a premixed flame into the mixture.

This view is convenient for predicting liftoff heights and blowoff velocities. Under turbulent conditions, turbulent mixing is calculated without combustion to obtain local average values for the mixture ratio, the flow velocity, and the turbulence intensity and scale. Knowledge of the mixture ratio, intensity, and scale provides a turbulent burning velocity from measurements on premixed flames. Near the duct exit the average flow velocity everywhere exceeds the turbulent burning velocity. Farther downstream a position is reached at which these two velocities become equal at an optimum transverse location. This position defines the liftoff height. Increasing the jet exit velocity increases the local average flow velocity and thereby causes the liftoff height to increase. At sufficiently high exit velocities the stabilization plane is so far downstream that the average composition becomes fuel-lean across the entire jet, and the turbulent burning velocity begins to decrease sharply. Blowoff is then encountered as a consequence of either inability to satisfy equality of velocities or attainment of a lean flammability limit.

In relating conditions of lifted diffusion flames to those of premixed flames this view implicitly considers mixing to have occurred to the molecular level, since premixed turbulent flame-speed data are available only for gases of spatially and temporally uniform mixture ratios. It is possible to estimate whether sufficient time is available to a fluid element in lifted diffusion flames for its turbulent mixing to approach local uniformity prior to reaching the flame. Typical liftoff heights range from 3 to 30 cm as the exit velocity varies from 10 to 60 m/s.[3] Residence times for fuel elements prior to encountering flames therefore lie between 1 and 5 ms. Since molecular diffusion coefficients are on the order of 10^{-1} cm^2/s in the cold fuel jet, the characteristic distances over which molecules can diffuse in the time available are in the vicinity of 10^{-2} cm. Since exit diameters are on the order of a few millimeters under these conditions,[3] turbulence Reynolds numbers R (based on fluctuating velocity and integral scale), may be estimated to range up to 10^3, and turbulence scales will vary from a maximum eddy size ℓ on the order of 1 cm to a Kolmogorov scale of $\ell/R^{3/4} \sim 10^{-2}$ cm. Thus, in the time available, negligible molecular diffusion occurs through the largest eddies and perhaps from 50 to 90% through the smallest. It seems unlikely that a sufficient amount of premixing at the molecular scale can occur to justify use of the premixed-flame concept. In our view, the range of validity of the premixed-flame concept remains an open question; it is well known that observed blue flame zones and radical emissions, with negligible yellow radiation, do not constitute evidence for premixed flames, since they occur as well for stretched diffusion flames (see Ref. 19).

An alternative concept, proposed by Peters,[6] focuses on extinction of laminar diffusion flames. The turbulent diffusion flame is viewed as an ensemble of laminar diffusion flamelets, reaction sheets stretched and contorted by the turbulent flow. It is known[7] that if the strain rate imposed on a laminar diffusion flame exceeds a critical value then reaction abruptly ceases in the flame and extinguishment occurs. The peak in the spectrum of strain rates in the turbulent flow shifts to higher values as the exit velocity of the fuel jet increases. For an attached turbulent diffusion flame this increase may lead to a sufficiently large fraction of extinctions of laminar diffusion flamelets at the rim for liftoff to occur. After liftoff, the turbulence of the cold jet at the exit has a higher Reynolds number and correspondingly higher strain rates. However, at a sufficiently large distance downstream from the exit, the peak in the spectrum of strain rates has decreased to a value that allows a reasonable fraction of the laminar diffusion flamelets to remain unextinguished. The diffusion flamelets may therefore remain stabilized in a lifted configuration, with a liftoff height that increases as the exit velocity increases.

In this alternative view, any effects of premixing at the molecular level are neglected, although of course time-average concentrations will exhibit an apparent mixing through turbulence. The laminar diffusion flamelets occur at local, instantaneous stoichiometric surfaces. At the liftoff height most of these surfaces are located away from the jet axis when the exit velocity is just above the critical value for liftoff. Increasing the exit velocity further eventually causes the average location of the stoichiometric surfaces just upstream from the liftoff height to begin to approach the jet axis. At sufficiently high exit velocities there are very few stoichiometric surfaces in regions where strain rates are low enough for unextinguished diffusion flamelets to exist, and stabilization of the turbulent diffusion flame can no longer be achieved (strain rates being low enough only in fuel-lean regions). This defines a sufficient condition for blowoff, although in practice blowoff may occur at lower exit velocities as a consequence of passage of a large, relatively rare coherent structure that carries the flame-stabilization region downstream with it. This second concept of liftoff phenomena is the one explored herein.

III. Critical Value of the Scalar Dissipation Rate for Local Extinction

The criterion for extinction of laminar diffusion flamelets in the present study is taken from the work of Liñán,[7] who analyzed the structure and extinction of counterflow diffusion flames. To avoid excessive complication, we assume here that the overall reaction between fuel F and oxidizer O can be written as $\nu_F F + \nu_O O \rightarrow$ products where ν_F and ν_O are mole-based stoichiometric coefficients; that molecular coefficients of diffusion D for all species and for heat are equal; that radiant energy loss is negligible; that Mach numbers are low; and that fluctuations of temperatures and of concentrations of chemical species in approach streams are sufficiently small to have negligible influences on enthalpies. Then it may be shown[8] that a mixture fraction Z, defined to have a value of zero in the ambient atmosphere and unity at the exit of the fuel duct, obeys a source-free conservation equation,

$$\rho \partial Z/\partial t + \rho v \cdot \nabla Z = \nabla \cdot (\rho D \nabla Z) \tag{1}$$

(where ρ is density and v velocity) and relates the fuel and oxidizer concentrations to the temperature on an instantaneous, local basis. Because of the absence of a chemical source term in Eq. (1), Z has been called a conserved scalar.[8] Bilger has developed the idea of using Eq. (1) in Ref. 8 and in a number of references quoted therein.

With the specific heat at constant pressure c_p assumed constant, in chemically frozen flow the temperature T is $T_{\mathrm{fr}} = T_O + Z(T_F - T_O)$, where T_F and T_O denote the temperatures of the incoming fuel and oxidizer. In terms of Q_F, the heat released per unit mass of fuel consumed, it may be shown[6] that the mass fractions of fuel and oxidizer, respectively, are expressible as

$$Y_F = Y_{FF} Z - (T - T_{\mathrm{fr}}) c_p / Q_F \tag{2}$$

and

$$Y_O = Y_{OO}(1 - Z) - (T - T_{\mathrm{fr}}) c_p / (\nu Q_F) \tag{3}$$

where Y_{FF} is the mass fraction of fuel in the fuel stream, Y_{OO} the mass fraction of oxidizer in the ambient atmosphere, and ν the raio of the mass of fuel to that of oxidizer at stoichiometric conditions $[\nu = (\nu_F W_F)/(\nu_O W_O)$, where W denotes molecular weight]. Multiplication of Eq. (3) by ν, subtraction of the result from Eq. (2), and introduction of the conditions $Y_O = Y_F = 0$ result in an equation that may be solved for Z to show that the stoichiometric value of the mixture fraction, which occurs for example at a thin reaction zone of a diffusion flame, is $Z_{\mathrm{st}} = [1 + Y_{FF}/(\nu Y_{OO})]^{-1}$. The temperature at $Y_O = Y_F = 0$, $Z = Z_{\mathrm{st}}$ is the adiabatic flame temperature, $T_{\mathrm{st}} = T_{\mathrm{fr,st}} + Z_{\mathrm{st}} Y_{FF} Q_F / c_p$, from Eq. (2), where $T_{\mathrm{fr,st}} = T_O + Z_{\mathrm{st}}(T_F - T_O)$, the temperature at $Z = Z_{\mathrm{st}}$, if frozen conditions prevail.

Under the assumptions that have been introduced, the flame structure can be studied in terms of one equation, energy conservation, which may be written as

$$\rho \partial T/\partial t + \rho v \cdot \nabla T = \nabla \cdot (\rho D \nabla T) + w/c_p \tag{4}$$

where w is the energy per unit volume per unit time released by chemical reactions. For purposes of analysis a one-step Arrhenius approximation is adopted for w, with overall reaction orders with respect to both fuel and oxidizer being unity, whence

$$w = (Q_F B_F \rho^2 / W_O) Y_F Y_O e^{-E/RT} \tag{5}$$

where B_F (cm^3/mole s) is the frequency factor for the rate of fuel consumption, E is the overall activation energy, and R the universal gas constant. The quantities, E, B_F, D, and ρ are to be evaluated at T_{st} for the purpose of extinction calculations.

The analysis of diffusion-flame structure and extinction is performed by introducing a local coordinate system that moves with the stoichiometric sheet. A Crocco type of transformation is then employed to replace the coordinate normal to the sheet by Z as an independent variable. In the resulting new coordinate system, let x and y denote orthogonal space coordinates tangential to the sheet, let t denote time, let u and v denote components of velocity in the x and y directions, respectively, let Z_x and Z_y identify derivatives of Z in the transverse directions in the original variables, and employ standard subscript notation for other partial derivatives in the new variables. Then by use of Eq. (1) it may be shown[6] that Eq. (4) becomes

$$\begin{aligned}\rho(T_t+uT_x+vT_y) &= w/c_p+\rho D\,|\nabla Z|^2 T_{ZZ}\\ &+Z_x[2\rho DT_{Zx}+T_x(\rho D)_Z]+Z_y[2\rho DT_{Zy}+T_y(\rho D)_Z]\\ &+\rho D(T_{xx}+T_{yy})+T_x(\rho D)_x+T_y(\rho D)_y \end{aligned}\tag{6}$$

The transformed normal coordinate Z is stretched about Z_{st} by the large factor

$$\beta=E(T_{st}-T_{fr,st})/[2RT_{st}^2 Z_{st}(1-Z_{st})]$$

which is essentially the same as the expansion parameter of Liñán[7] that measures the thickness of the reaction sheet in the Z coordinate. When the length scales of the turbulent eddies are all large compared with the thickness of the reaction zone, this stretching reduces Eq. (6) in the first approximation to

$$T_{ZZ}=-w/(c_p\rho D\,|\nabla Z|^2)\tag{7}$$

It can be shown explicitly[6] that Eq. (7) leads to a problem equivalent to that solved by Liñán.[7] Use of Eqs. (2), (3), and (5) in Eq. (7), introduction of the stretched variables $\eta=\beta(Z-Z_{st})$ and $y=(T_{st}-T)E/(RT_{st}^2)-\gamma\eta$, where

$$\gamma=2Z_{st}-1-2Z_{st}(1-Z_{st})(T_F-T_O)/(T_{st}-T_{fr,st})\tag{8}$$

and expansion to first order in β^{-1} in a proper manner provide the equation

$$d^2y/d\eta^2=\delta(y^2-\eta^2)e^{-(y+\gamma\eta)}\tag{9}$$

where the reduced Damköhler number is

$$\delta=\frac{Z_{st}Y_{FF}\rho B_F e^{-E/(RT_{st})}}{vW_O D\,|\nabla Z|^2_{st}}\left[\frac{RT_{st}^2}{E(T_{st}-T_{fr,st})}\right]^3[2Z_{st}(1-Z_{st})]^2\tag{10}$$

The boundary conditions for Eq. (9) must be obtained from matching to outer solutions that satisfy Eq. (6) with $w=0$. The advantage of the Crocco transformation is that, irrespective of the complexity of the turbulence, these outer equations and their boundary conditions are satisfied by setting T equal to a linear function of Z, independent of x, y, and t; whatever is done to the temperature by the turbulence also is done to the mixture fraction. When the linear functions in the outer solutions are taken to be those of the equilibrium diffusion flame, it is found that the matching conditions for Eq. (9) become $dy/d\eta\to\pm 1$ as $\eta\to\pm\infty$.

Liñán[7] solved Eq. (9) subject to these boundary conditions and showed that solutions exist only for $\delta>\delta_{qu}$, an extinction or quenching value that is given approximately by $\delta_{qu}=e(1-|\gamma|)$. Thus, the laminar diffusion flamelet cannot exist unless the parameter δ, defined in Eq. (10), exceeds a critical value. The turbulence influences δ in Eq. (10) only through the local, instantaneous rate of dissipation of the (scalar) mixture fraction at its stoichiometric value, viz., through $X_{st}=2D\,|\nabla Z|^2_{st}$. Putting $\delta=\delta_{qu}$ in Eq. (10) therefore defines a critical value, X_{qu}, of the instantaneous dissipation rate X_{st}. In most applications, Z_{st} is small enough for γ of Eq. (8) to be negative, and in this case the formula for X_{qu} is seen by substitution to be

$$X_{qu}=\left(\frac{4Y_{FF}\rho B_F}{evW_O}\right)\left[\frac{RT_{st}^2}{E(T_{st}-T_{fr,st})}\right]^3 Z_{st}^2(1-Z_{st})^2 \times\left[1-(1-Z_{st})\left(\frac{T_F-T_O}{T_{st}-T_{fr,st}}\right)\right]^{-1}e^{-E/(RT_{st})}\tag{11}$$

The diffusion flamelets are quenched if $X_{st}>X_{qu}$. It is evident by comparing Eq. (10) with the corresponding Damköhler number of Liñán[7] that the counterpart of his counterflow velocity gradient is basically $D\,|\nabla Z|^2_{st}$, or X_{st}, within a constant factor. Thus, although in turbulence X_{st} is interpreted in terms of dissipation, from the viewpoint of flamelet extinction an interpretation in terms of strain rate may be superior; the flamelet is extinguished if the local, instantaneous strain rate that it experiences becomes too large.

The principal approximations that have been introduced here are the constancy of specific heat, the Lewis number of unity, the one-step, second-order reaction, and the thin reaction zone. It seems clear that the first of these can be removed at the expense of greater complexity in definitions. Estimates suggest that the others often are reasonably good. It is remarkable that, with the quantities in Eq. (11) evaluated at the hot flame sheet, none of the usual assumptions concerning constant properties are needed. Effects of thermal expansion, for example, automatically are included if ρ, X_{st}, etc., are obtained with proper regard paid to these effects.

IV. Statistical Aspects of Flamelet Extinction in Turbulence

The quantity X_{qu} in Eq. (11) is a chemical quantity dependent upon thermodynamic and chemical-kinetic parameters. As such it can be evaluated without consideration of turbulence. The problem of calculating X_{qu} is addressed in a later section. Here attention is focused on evaluation of X_{st}, for use in the liftoff criterion.

Application of the criterion $X_{st}>X_{qu}$ to liftoff of laminar diffusion flames is relatively straightforward in principle because X_{st} can be calculated explicitly by numerical solution of the partial differential equations that describe the flame structure in the flame-sheet approximation. The statistical character of X_{st} in turbulent flows introduces fundamental complexities. A probabilistic approach of some type seems desirable since liftoff is expected to be a random event in the stationary stochastic process of jet turbulence. Although the dynamics of flamelets may influence this event, at present insufficient knowledge is available to include flamelet dynamics in liftoff descriptions. Therefore a static, probabilistic approach is adopted here.

The distorted stoichiometric sheet will have a distribution of X_{st} along it. Let $P(X|Z=Z_{st})$ denote the local probability density function for this distribution. At positions or times on the sheet for which $X_{st}>X_{qu}$, the flamelet cannot exist and holes will develop along the sheet, within which the reaction rate is negligibly small. The presence of a hole may alter the local value of X_{st} rapidly, possibly lowering it to a point at which $X_{st}<X_{qu}$. This does not mean that the flamelet will be re-established instantaneously, because ignition and extinction conditions typically differ greatly. The dynamics of the edge of a hole need much more study. To obtain a static, probabilistic criterion, we ignore these dynamics and forbid X_{st} from changing as a consequence of the extinctions at holes, that is, $P(X|Z=Z_{st})$ pertains to the hypothetical situation in which the flame sheet is continuous and has not developed holes, irrespective of the magnitude of X_{st}. This idealized probability density function is needed for a static analysis.

The use of $P(X|Z=Z_{st})$ seems clearest in connection with the prediction of liftoff of attached flames. At a small local average value, $\bar{X}_{st}$, the distorted flame sheet is continuous. As $\bar{X}_{st}$ increases, holes develop in the sheet, and when $\bar{X}_{st}$ becomes sufficiently large there are so many holes that continuity of the sheet is disrupted; there no longer exists a continuous path from the downstream flame sheet to the burner rim. Under this condition liftoff must occur because the cold flowing gases will carry the flame sheet downstream. The probabilistic quantity that must be known for calculating liftoff according to this view is the probable fraction of the sheet not occupied by holes. In view of the definitions of X_{qu} and $P(X|Z=Z_{st})$, it seems logical to assume that this probable fraction is

$$p=\int_0^{X_{qu}} P(X|Z=Z_{st})\,\mathrm{d}X \tag{12}$$

In recent years, the theory of continuum percolation in two dimensions has received extensive study.[9] This theory addresses precisely questions of probabilistic distributions of holes in sheets. Through both theory and experiment (i.e., measurement of electrical conductivities of sheets of conductors with holes randomly punched in them[10]) it has been found that as p is decreased the conductivity of the sheet vanishes, i.e., connectivity is disrupted, at a critical threshold value $p=p_{cr}$, where p_{cr} is in the range of 0.6 to 0.7. Use of Eq. (12) with $p=p_{cr}$ therefore may provide a criterion for liftoff.

Equation (12) may also be used to discuss liftoff heights. Since $\bar{X}_{st}$ decreases with increasing distance from the duct exit, it may be expected that p will increase with increasing distance. Therefore if $p<p_{cr}$ near the duct exit, there will be height above which $p>p_{cr}$, and a continuous distorted flame sheet can exist. It seems logical to identify the liftoff height as the height at which $p=p_{cr}$.

Use of Eq. (12) in liftoff calculations requires knowledge of the conditioned probability density function $P(X|Z=Z_{st})$. There appear to be no measurements whatever of such a conditioned function and no measurements of the unconditioned function in flames. Measurements of the unconditioned function in a nonreacting jet[11] are consistent with the log-normal hypothesis of Oboukhov[12] and Kolmogorov[13] and give a variance of $\sigma \simeq 0.5$. Thus, in the absence of better information, we adopt a log-normal form,

$$P(X|Z=Z_{st})=\frac{1}{X\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2\sigma^2}(\ln X-\mu)^2\right] \tag{13}$$

for which the mean and mean-square fluctuation are

$$\bar{X}_{st}=e^{\mu+\sigma^2/2} \qquad \overline{X_{st}'^2}=\bar{X}_{st}^2(e^{\sigma^2}-1) \tag{14}$$

and we employ $\sigma=0.5$. From Eq. (12), the critical condition for liftoff is then found to be

$$p_{cr}=\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left[\frac{\ln(X_{qu}/\bar{X}_{st})+\sigma^2/2}{2\sqrt{\sigma}}\right] \tag{15}$$

in which use of the estimate $p_{cr}=0.63$ gives $\bar{X}_{st}=0.96X_{qu}$ at liftoff. It is certainly within the accuracy of this calculation to assume that the turbulent diffusion flame can exist only if $\bar{X}_{st}<X_{qu}$ (i.e., $0.96\approx 1$).

The result of this discussion of statistics therefore is that for practical calculations of liftoff heights the statistics can be ignored and replaced by the same sorts of liftoff statements that would be applied in laminar flows, employing the average value $\bar{X}_{st}$. This conclusion is not obvious prior to investigation; depending on the values of σ and p_{cr}, the critical value of the ratio $\bar{X}_{st}/X_{qu}$ varies roughly from 0.5 to 10. If improved values of the parameters become available, then reason might be found for modification of $\bar{X}_{st}=X_{qu}$ as the liftoff condition.

V. Calculation of the Average Rate of Scalar Dissipation

Application of the liftoff criterion necessitates calculation of $\bar{X}_{st}$. Unfortunately, knowledge of $\bar{X}_{st}$ for turbulent jets rests on relatively poor ground. In the absence of information on the conditioned mean, it is usual to hypothesize statistical independence, employing

$$\bar{X}_{st}\equiv\int_0^\infty XP(X|Z=Z_{st})\,\mathrm{d}X=\int_0^\infty XP(X)\,\mathrm{d}X=\bar{X}_{tb} \tag{16}$$

where $P(X)$ is the unconditioned probability density function for the rate of scalar dissipation in the turbulent fluid. Arguments have been given to the effect that $\bar{X}_{st}=\bar{X}_{tb}$ may be reasonable.[8] It should be remarked that $\bar{X}_{tb}$ is still conditioned on turbulent fluid being present and therefore differs from the fully unconditioned local average rate of scalar dissipation

$$\bar{X}=\int_0^\infty X\bar{I}P(X)\,\mathrm{d}X$$

where $\bar{I}$ is the local average intermittency.[14]

A few theoretical concepts are available for the average rate of dissipation of turbulent kinetic energy in nonreacting jets (e.g., Ref. 15), and existing models for conserved scalars in jet flows (e.g., Ref. 16) enable some deductions concerning $\bar{X}$ to be drawn from these concepts. However, the validity of the deductions is uncertain, and very little data are available for testing validity. Only a few measurements exist of dissipation rates of turbulent kinetic energy in incompressible jets (see Ref. 15 for citations). Apparently, attempts to measure $\bar{X}$ in shear flows have been made in only one experiment,[17] which concerned a jet of preheated air. There is no experimental information at all under conditions involving combustion, and the associated experimental difficulties are substantial.

To illustrate what might be done most simply, neglect combustion and restrict attention to the so-called self-preserving portion of the turbulent jet, $h/d>10$ (where h is the axial distance downstream from the duct exit and d the exit diameter), in which the radius increases in proportion to h and the average velocity decreases inversely with h. Various lines of reasoning[15] suggest that the nondimensional dissipation rate $\bar{X}_{ke}^*=\epsilon_{cl}d/U^3$ (where ϵ_{cl} is the average rate of dissipation of turbulent kinetic energy on the centerline and U is the exit velocity), varies in proportion to $(h/d)^{-4}$. The simplest argument is to assume that ϵ_{cl} is proportional to the cube of a characteristic velocity, divided by a characteristic length, and employ the self-preserving scaling of velocity and length. The available experiments[15] confirm the predicted dependence, giving $\bar{X}_{ke}^*=48(h/d)^{-4}$.

It might be expected that a similar behavior would occur for the nondimensional scalar dissipation rate, $\bar{X}_{cl}^*=\bar{X}_{cl}d/U$, when $\bar{X}_{cl}$ is the average rate of scalar dissipation on the centerline. This behavior can, in fact, be predicted. For example, a balance between production and dissipation of scalar fluctuations suggests that $\bar{X}_{cl}$ will be proportional to an average value of $(\mathrm{d}\bar{Z}/\mathrm{d}r)^2$ where r is the radial coordinate. The usual models[16] show that in the self-preserving region the average magnitude of $\bar{Z}$ varies inversely with h, and the average radius is proportional to h, so $\bar{X}_{cl}^*\sim(h/d)^{-4}$.

The available data[17] do not seem to support this simple power-law dependence. Instead, $\bar{X}_{cl}^*$ appears to decrease at first with increasing h, then remain nearly constant in the self-preserving region. Also, the dissipation rate appears to achieve a minimum on the centerline and a maximum in the region of maximum shear.[17] These observations cast doubt on the use of models to calculate $\bar{X}$ and imply that appreciable uncertainty must exist in estimates of liftoff heights.

Additional complicating factors include the intermittency (mentioned earlier) and the question of the radial position at which $\bar{X}$ should be evaluated for use in the liftoff criterion. Intermittency will introduce an appreciable difference between the $\bar{X}_{tb}$ needed in a liftoff criterion and the $\bar{X}$ measured in experiments that employ the usual unconditioned sampling techniques. From the definitions of $\bar{X}_{tb}$ [see Eq. (12)] and $\bar{X}$ it is evident that $\bar{X}_{tb}=\bar{X}/\bar{I}$, and an appropriate value for $\bar{I}$ is needed. For this purpose, we use the data of Wygnanski and

Fiedler[18] for the self-preserving region to write

$$\bar{I}=\exp[-200(\xi-0.1)^2] \qquad \xi>0.1$$
$$=1 \qquad \xi<0.1 \tag{17}$$

where $\xi=r/h$. The constant cross-sectional integral

$$\int_0^\infty \bar{I}\xi \mathrm{d}\xi = 1.38\times10^{-2}$$

is employed as the factor relating $\bar{X}_{\text{tb}}$ to $\bar{X}$. This factor is much too small if the stoichiometric surface is near the centerline, but may be appropriate for most fuel-air systems, which typically have very small values of Z_{st} (i.e., the flame sheet is near the edge of the turbulent flow).

For purposes of comparison, three different methods are employed here for taking into account the radial position at which $\bar{X}$ is to be evaluated. The selection of the radial position is important because it influences the variation of $\bar{X}$ with h/d. As indicated above, if the centerline is selected then the self-preserving approximation gives $\bar{X}\sim(h/d)^{-4}$. It will be seen later that a weaker dependence is needed for agreement with data on liftoff heights. In the absence of a better theory, we assume self-preserving behavior and a local balance between production and dissipation in the equation for $\bar{Z}'^2$. The latter condition is expressed as

$$\bar{X}=\left(\frac{2\nu_t}{Sc_t}\right)\left(\frac{\partial\bar{Z}}{\partial r}\right)^2 \tag{18}$$

where ν_t and Sc_t denote the turbulent kinematic viscosity and the turbulent Schmidt number, respectively. To obtain $\bar{Z}(r)$ we adopt the self-preserving solution[16]

$$\bar{Z}=\bar{Z}_{\text{cl}}[1+(\gamma_t\xi)^2/4]^{-2Sc_t} \tag{19}$$

For ν_t, the incompressible formula[16] $\nu_t=Ud/70$ is employed. Through algebraic manipulations, these formulas lead to

$$\bar{X}^*=\left(\frac{4Sc_t\gamma_t^2\bar{Z}^2}{35}\right)\left(\frac{d}{h}\right)^2\left(\frac{\bar{Z}}{\bar{Z}_{\text{cl}}}\right)^{1/Sc_t}\left[\left(\frac{\bar{Z}_{\text{cl}}}{\bar{Z}}\right)^{1/(2Sc_t)}-1\right] \tag{20}$$

for the nondimensional average dissipation rate. The three methods differ in the manner in which Eq. (20) is employed in the liftoff criterion.

In the first method, Eq. (20) is used with the approximation $Sc_t=1$ and with $\bar{Z}=Z_{\text{st}}$. The latter selection is consitent with the idea that on the average the dissipation relevant to liftoff occurs at a radial position where the average mixture fraction is stoichiometric, since flamelets are more concentrated in that region than elsewhere. With $Sc_t=1$, the value $\gamma_t=10$ is chosen to fit the data[17] on the spread rate of the jet at $\bar{Z}=\bar{Z}_{\text{cl}}/2$, by use of Eq. (19). The formula[16]

$$\bar{Z}_{\text{cl}}=[(1+2Sc_t)/32]70(d/h) \tag{21}$$

is approximated here as $\bar{Z}_{\text{cl}}=6(d/h)$, and substitutions yield

$$\bar{X}^*_{\text{tb}1}=0.24(d/h)^{1.5}(1-0.096\sqrt{h/d}) \tag{22}$$

where the subscript *1* identifies the result of the first method. The value $Z_{\text{st}}=0.055$, appropriate for methane flames in air, has been employed here and elsewhere. According to Eq. (16), an approximation to $\bar{X}^*_{\text{st}}$ is provided by Eq. (22). It may be noted that, primarily because $\bar{Z}=Z_{\text{st}}\neq\bar{Z}_{\text{cl}}$, the functional dependence of $\bar{X}^*_{\text{tb}1}$ on h/d differs from the centerline power law.

In the second method, a cross-section integral, analogous to that for estimating $\bar{I}$, also is used for estimating $\bar{X}$. Specifically, the formula

$$\bar{X}_{\text{tb}2}=\int_0^\infty \bar{X}\xi\mathrm{d}\xi\Big/\int_0^\infty \bar{I}\xi\mathrm{d}\xi \tag{23}$$

is employed with $\bar{X}$ given by Eq. (18). The integral in the numerator becomes

$$-\left(\frac{2\nu_t}{Sc_t}\right)\left[\int_0^1 \xi\left(\frac{\mathrm{d}\bar{Z}}{\mathrm{d}\xi}\right)\mathrm{d}\bar{Z}\right]\Big/h^2$$

and the integral appearing here is approximated as $[\xi(\mathrm{d}\bar{Z}/\mathrm{d}\xi)]$ evaluated at $\bar{Z}=Z_{\text{st}}$, again with the idea that conditions near the mean stoichiometric surface are most relevant. The better value $Sc_t=0.7$ is now used, and through substitutions and manipulations it is found that

$$\bar{X}^*_{\text{tb}2}=0.46(d/h)^2[1-0.039(h/d)^{1/1.4}] \tag{24}$$

The third method is the same as the first, except that the quantity in the square brackets in Eq. (20) is assigned the constant value 0.78, purely for the purpose of producing better agreement of theoretical predictions with the data on liftoff heights. The resulting formula is

$$\bar{X}^*_{\text{tb}3}=0.018(d/h) \tag{25}$$

In view of the many uncertainties that have been indicated in this section, the empiricism in obtaining Eq. (25) might be deemed acceptable.

VI. Calculation of the Critical Rate of Scalar Dissipation for Extinction

The preceding results may be used to test the liftoff criterion, $\bar{X}_{\text{st}}>\bar{X}_{\text{qu}}$, if the nondimensional strain rate for extinction, $X^*_{\text{qu}}=X_{\text{qu}}d/U$ can be evaluated. Calculation of this quantity from Eq. (11) requires overall kinetic parameters that are not generally available. The parameters needed are beginning to be obtained for some fuels.[19] A more direct approach for methane flames in air is to use experimental data[20] on the extinction of laminar counterflow diffusion flames to calculate X_{qu} directly. This entails developing a theoretical analysis of the particular experiment.

In the experiment,[20] the external velocity gradient at extinction, for methane in air, was found to be $a=320\ \text{s}^{-1}$. Mainly because of the small value of Z_{st}, this number does not provide a good estimate of the strain rate at the stoichiometric surface. If z denotes the coordinate normal to the flame sheet, then this strain rate is $X_{\text{st}}=2D(\partial Z/\partial z)^2_{\text{st}}$ in the experiment. From Liñán's formulation[7] of the counterflow problem, $Z=(\frac{1}{2}\text{erfc}(z/\sqrt{2D/a})$, whence $X_{\text{st}}=(a/\pi)\exp[-az^2_{\text{st}}/D]$. For small values of Z, asymptotic expansion of the complementary error function shows that this formula may be approximated roughly as

$$X_{\text{st}}\approx 4aZ^2_{\text{st}}[\text{erfc}^{-1}(2Z_{\text{st}})]^2 \tag{26}$$

where erfc^{-1} denotes the inverse of the complementary error function. With $Z_{\text{st}}=0.055$, use of the experimental value of a at extinction in this formula gives $X_{\text{qu}}\approx5\ \text{s}^{-1}$. Although there is some uncertainty in this value, e.g., as a consequence of the fact that the analysis[7] assumes constant properties, it seems likely that the value of X_{qu} obtained here is of approximately the correct order of magnitude.

VII. Comparisons of Theoretical and Experimental Liftoff Heights

The results just obtained for X^*_{qu} may be employed in conjunction with the data[3] on liftoff heights to plot X^*_{qu} as a function of h/d, where h now denotes the liftoff height. The graph is prepared from the experimental data by selecting a

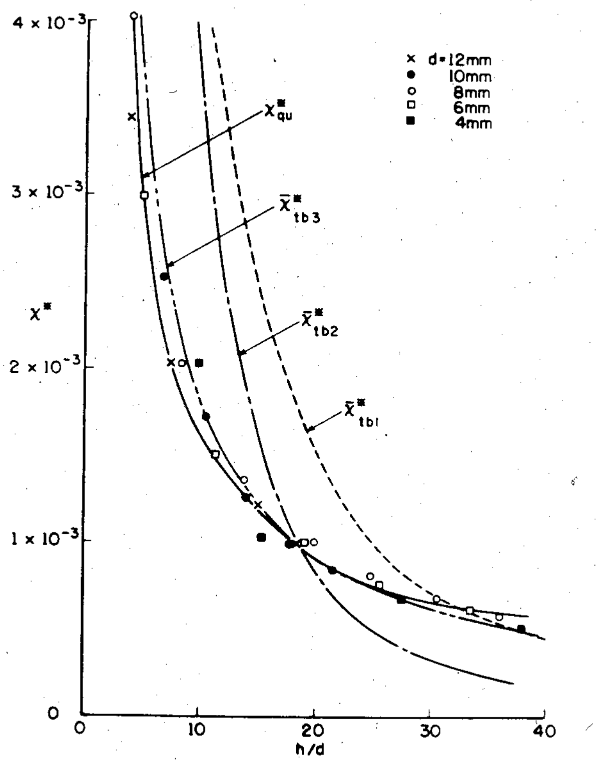

Fig. 1 Nondimensional rate of scalar dissipation as a function of the ratio of the liftoff height to the jet diameter.

value of d, then calculating $5d/U$ and h/d at each value of U for which h was measured. The results are shown by the points in Fig. 1. The solid line labeled X_{qu}^* represents a best fit through these points.

The three theoretical formulas that have been derived for $\bar{X}_{tb}^*$ [Eqs. (22), (24), and (25)] also are shown in Fig. 1. If the previously derived liftoff condition ($\bar{X}_{st} = X_{qu}$) is correct, then these curves should agree with the points. It is seen that the theoretical predictions of liftoff heights are of the right order of magnitude. However, the curves $\bar{X}_{tb1}^*$ and $\bar{X}_{tb2}^*$, which rely less on empiricism, tend to exhibit a stronger dependence on h/d than does X_{qu}^*. The empirical result $\bar{X}_{tb3}^*$, given by Eq. (25), agrees remarkably well with the liftoff data. We may conclude that, within the accuracy with which comparisons can be made, the liftoff criterion $\bar{X}_{st} = X_{qu}$ is consistent with the data on liftoff heights of methane flames.

VIII. Discussion and Conclusions

The many uncertainties in the calculations presented here deserve emphasis. The curve of X_{qu}^* may be in error because of the manner in which X_{qu} was deduced from the laminar extinction experiment, with neglect of variable properties. Better kinetic data, for methane and other fuels, are needed for use in Eq. (11). In addition, it would be worthwhile to generate additional data on liftoff heights for other fuels. Accuracies better than 10% in liftoff data are unlikely because of the turbulent character of the process.

Currently the greatest uncertainty lies in the calculation of the average rate of scalar dissipation. The models that have been employed require many unjustified assumptions. Some phenomena of likely importance have simply been excluded here because of the absence of methods for handling them. For example, the density and temperature changes in the turbulent flames may be expected to have substantial influences on $\bar{X}_{st}$, but there appears to be no suitable way to estimate these effects. The fact that experiment[17] seems to contradict the self-preserving hypothesis for $\bar{X}_{cl}$ that underlies the calculations raises further doubts concerning Eqs. (22), (24), and (25); the relatively flat experimental variation in the downstream region suggests that the theoretical curves in Fig. 1 may be too steep. It appears that abandonment of the self-preserving hypothesis and use of data[17] for $\bar{X}$ may tend to improve agreement (in functional dependence) between theory and experiment in Fig. 1, but the data are so sparse and uncertain that this approach does not yet seem justified. Furthermore, functional dependences of $\bar{X}_{st}^*$ on h/d might well be modified by the heating and expansion in the flame. It seems quite likely that uncertainties in the theoretical curves in Fig. 1 exceed an order of magnitude. More and better data on rates of scalar dissipation in turbulent jets are needed before accuracies of the predictions can be improved. Modeling, no matter how complex, does not yet constitute a reliable approach to obtaining $\bar{X}_{st}$.

Notwithstanding these doubts, there is one aspect of Fig. 1 that favors the idea that liftoff is controlled by the quenching of laminar diffusion flamelets. The data on liftoff heights for jets of various diameters appear to fall along a single curve in Fig. 1. This behavior is consistent with the scalar dissipation rate (a strain rate) being the phenomenon that determines liftoff. Moreover, the behavior is uninfluenced by the manner in which the data were treated (for example, a change in X_{qu} would merely shift the level of the curve). It would be of interest to perform further experiments on liftoff heights with the objective of testing more carefully correlations based on plotting h/d as a function of U/d. If the viewpoint proposed herein is proper, then universal curves, independent of d, should be obtained separately for each fuel. Relative heights of the correlation curves should provide an inverse measure of the reactivity of the fuel.

Although attention here has been focused on liftoff heights, similar reasoning may be applied to the prediction of blowoff conditions. For example, with the approximations leading to Eq. (22) or (23), a sufficient condition for blowoff is $\bar{Z}_{cl} \leq Z_{st}$. Use of this condition in Eq. (21) for methane in air indicates that $h/d = 95$ is sufficient for blowoff. Experimentally, blowoff occurs earlier, at approximately half this value of h/d. Therefore the sufficient condition is not a necessary condition for blowoff. Blowoff is a relatively variable event, exhibiting scatter in data exceeding that for liftoff heights. Blowoff may be produced by large coherent structures.

References

[1] Hottel, H. C. and Hawthorne, W. R., "Diffusion in Laminar Jet Flames," in *Third Symposium on Combustion, Flame and Explosion Phenomena,* Williams and Wilkins Co., Baltimore, Md., 1949, pp. 254-256.

[2] Wohl, K., Gazley, C., and Kapp, N., "Diffusion Flames," in *Third Symposium on Combustion, Flame and Explosion Phenomena,* Williams and Wilkins Co., Baltimore, Md., 1949, pp. 288-300.

[3] Horch, K., "Zur Stabilität von Freistrahl-Diffusionsflammen," Ph.D. Thesis, Universität Karlsruhe, FRG, 1978.

[4] Brzustowski, T. A., "Mixing and Chemical Reactions in Industrial Flares and Their Models," *Physico-Chemical Hydrodynamics,* Vol. 1, 1980, pp. 27-40; Kalghatgi, G. T., "Blow-Out Stability of Gaseous Jet Diffusion Flames. Part I: In Still Air," *Combustion Science and Technology,* Vol. 26, 1981, pp. 233-239.

[5] Günther, R., Horch, K., and Lenze, B., "The Stabilization Mechanism of Free Jet Diffusion Flames," *Colloque International Berthelot-Vieille-Mallard-LeChatelier, First Specialists Meeting (International) of The Combustion Institute,* The Combustion Institute, Pittsburgh, Pa., 1981, pp. 117-122.

[6] Peters, N., "Local Quenching Due to Flame Stretch and Non-Premixed Combustion," Paper 80-4 presented at the 1980 Spring Meeting, Western States Section, The Combustion Institute, University of California, Irvine, Calif., April 1980; to appear in *Combustion Science and Technology.*

[7] Liñán, A., "The Asymptotic Structure of Counterflow Diffusion Flames for Large Activation Energies," *Acta Astronautica,* Vol. 1, 1979, pp. 1007-1039.

[8] Bilger, R. W., "Turbulent Flows with Nonpremixed Reactants," in *Turbulent Reacting Flows,* edited by P. A. Libby and F. A. Williams, Springer-Verlag, Berlin, 1980, pp. 65-113.

[9]Gawlinski, E. T. and Stanley, H. E., "Continuum Percolation in Two Dimensions: Monte Carlo Tests of Scaling and Universality for Non-Interacting Discs," *Journal of Physics,* Vol. A14, 1981, pp. L291-L299.

[10]Last, B. J. and Thouless, D. J., "Percolation Theory and Electrical Conductivity," *Physical Review Letters,* Vol. 27, 1971, pp. 1719-1721.

[11]Masiello, P. J., "Intermittency of the Fine Structure of Turbulent Velocity and Temperature Fields Measured at High Reynolds Number," Ph.D. Thesis, University of California, San Diego, Calif., 1974.

[12]Oboukhov, H. M., "Some Features of Atmospheric Turbulence," *Journal of Fluid Mechanics,* Vol. 13, 1962, pp. 77-81.

[13]Kolmogorov, A. N., "A Refinement of Previous Hypotheses Concerning the Local Structure of Turbulence in a Viscous Incompressible Fluid at High Reynolds Number," *Journal of Fluid Mechanics,* Vol. 13, 1962, pp. 82-85.

[14]Libby, P. A. and Williams, F. A., "Some Implications of Recent Theoretical Studies in Turbulent Combustion," *AIAA Journal,* Vol. 19, March 1980, pp. 261-274.

[15]Antonia, R. A., Satyaprakash, B. R., and Hussain, A. K. M. F., "Measurements of Dissipation Rate and Some Other Characteristics of Turbulent Plane and Circular Jets," *Physics of Fluids,* Vol. 23, April 1980, pp. 695-699.

[16]Peters, N., "An Asymptotic Analysis of Nitric Oxide Formation in Turbulent Diffusion Flames," *Combustion Science and Technology,* Vol. 19, 1978, pp. 39-49.

[17]Lockwood, F. C. and Moneib, H. A., "Fluctuating Temperature Measurements in a Heated Round Free Jet," *Combustion Science and Technology,* Vol. 22, 1980, pp. 63-81.

[18]Wygnanski, I. and Fiedler, H., "Some Measurements in a Self-Preserving Jet," *Journal of Fluid Mechanics,* Vol. 38, 1969, pp. 577-612.

[19]Williams, F. A., "A Review of Flame Extinction," *Fire Safety Journal,* Vol. 3, 1981, pp. 163-175; "Recent Advances in Theoretical Descriptions of Turbulent Diffusion Flames," in *Turbulent Mixing in Nonreactive and Reactive Flows,* edited by S. N. B. Murthy, Plenum Press, New York, 1975, pp. 189-208.

[20]Tsuji, H. and Yamaoka, F., "Structure of Counterflow Diffusion Flames in the Forward Stagnation Region of a Porous Cylinder," *Twelfth Symposium (International) on Combustion,* The Combustion Institute, Pittsburgh, Pa., 1969, pp. 997-1005.