139

Linear Stability Analysis of Nonadiabatic Flames: Diffusional–Thermal Model

G. JOULIN

Laboratoire d'Energétique et de Détonique (LA 193 du CNRS), ENSMA, Rue Guillaume VII, 86034, Poitiers, France

and

P. CLAVIN

Laboratoire de Dynamique et Thermophysique des Fluides (LA 72 du CNRS), Rue H. Poincaré, 13397, Marseilles Cedex 4, France

The method of matched asymptotic expansions, in terms of a suitably reduced activation energy, is applied to investigate the effects of heat losses on linear stability of a planar flame, which is governed by a one-step irreversible Arrhenius reaction.

The density change associated with the heat release is neglected in order to eliminate the Landau hydrodynamic instability. Attention is focused on diffusional–thermal instability mechanisms.

The dispersion relation is obtained in terms of the diffusive properties of the limiting reactant and of the heat-loss intensity. For a given loss intensity—less than the critical value leading to extinction—the two steady planar regimes have different stability properties: (1) the "slow" regimes (which do not reduce to the adiabatic one when the heat-loss intensity goes to zero) are shown to be always unstable; and (2) for the "fast" regimes (which include and generalize the adiabatic one) cellular structures are predicted to occur when the limiting component is sufficiently light. If the limiting component is moderately light, the "fast" regimes are stable and unstructured in nearly adiabatic conditions; however, they are destabilized by an increase of heat losses and must exhibit cells before the extinction limit is reached. Similarly, for mixtures involving a *realistically* heavy limiting component, our analysis predicts the appearance of transverse travelling waves near the extinction limit.

INTRODUCTION

The quenching mechanism of premixed flames by heat loss was first explained by the pioneering approximate analyses of Spalding [1] and Alder [2]. Singular perturbation methods for large reduced activation energy introduced in combustion field [3, 4] allowed investigation of this problem in a more systematic way. Recent analytical investigations [5, 6] confirm the qualitative results quoted in Spalding [1] and Adler [2], namely: (1) for heat-loss intensity higher than a critical quenching value there is no steady flame propagation; and (2) for each value less than this critical value, two steady plane waves with different velocities emerge from the analysis. In the limit of a vanishing heat-loss intensity the faster wave reduces to the adiabatic flame, whereas the slower one goes to a nearly isothermal combustion process with a vanishing velocity and a large amount of unburned reactant.

Problems about the stability of these two branches of solutions have so far not been solved. The present work is devoted to such a study and to the connected problem of the onset of cellular structures on nonadiabatic flames. The first

studies of flame stability, in the adiabatic case, were carried out by Landau [7] and Darrieus [8], by means of a discontinuous flame-front model with a constant normal burning velocity. They treated the flows of fresh and burned mixtures to be incompressible and studied the hydrodynamic linear response of flow variables to the perturbations related to distortions of the flame front. This analysis neglects the perturbations induced by the distortions on the transport processes within the flame. Due to the density jump through the discontinuous front, this approximation leads to the nonrealistic result: flames are unstable for all transverse wavelengths.

To take the inner dissipative effects into account, Markstein [9] and Eckhaus [10] introduced, through phenomenological relations, a normal burning velocity that depends on the flame-front curvature and acceleration. These relations lead to stability for sufficiently short wavelengths of perturbations. The Landau instability, of hydrodynamic nature, remains only for the long waves. An excellent review of such analyses can be found in Markstein [9].

Sivashinsky [11] was recently able to approach these stability problems from a more fundamental point of view. He treated correctly the flame structure in the limit of large activation energies by applying the singular expansions method. His results confirm that (1) for short wavelengths the transport processes always have a stabilizing effect; and (2) for long waves they can introduce an instability mechanism, previously noticed by different authors [12, 13]. This last instability mechanism can be understood in terms of two competitive processes occuring in the perturbed preheat zone; indeed, the transverse heat conduction and the transverse diffusion of the limiting component have opposite influence on the combustion temperature in the reactive zone and thus lead to competitive effects on the local flame velocity [13]. The diffusive instability occurs for a sufficiently low Lewis number, when the molecular diffusion is more efficient than the heat conduction. Furthermore, by comparing the theoretical structures of corrugations of the front with experimental results on cellular flames [9], Sivashinsky gave

strong arguments showing that the appearance of the familiar cells in flames corresponds to the onset of this diffusive instability. The hydrodynamic Landau instability does not play an essential role in the onset of cells; consequently, it can be discarded in a preliminary study by using a thermal–diffusional model [11, 13] in which the density change associated with heat release is neglected.

In the nonadiabatic flame, the hydrodynamic instability must be even less relevant than in the adiabatic case. In fact, due to the heat loss from burned gases, the downstream temperature profile falls down to its upstream value on a characteristic finite length; for transverse waves longer than this characteristic length, the flame appears as a moving sheet without density jump. Therefore, the hydrodynamic instability, which should be more efficient in the limit of long waves than otherwise, is weakened or destroyed by heat losses. Then, the simple thermal–diffusional model appears to be very convenient to study the stability of nonadiabatic flames. We present the results of such an analysis in the following sections of the paper.

FORMULATION

For simplicity, let us consider a flame governed by an overall one-step exothermic reaction and make the usual assumptions of constant molecular weight, specific heat C_p, and thermal conductivity λ of the mixture. It has been shown previously [5] that the quenching mechanism is qualitatively insensitive to the precise temperature dependence of the heat-loss power per unit volume. For simplicity, therefore, we use a linear law, $K(T - T_u)$, where T and T_u are respectively the local and surrounding temperatures—T_u also the temperature of the fresh mixture—and K is a constant coefficient that measures the intensity of heat losses. This linear law is also a good approximation for a flame propagating in a duct when heat losses are due to conduction towards the cold walls [14].

To simplify the molecular diffusion mechanism, the two reactants are assumed to be small traces in an inert gas and are at a stoichiometrically un-

balanced composition.[1] In such a case the only diffusion coefficient appearing in the analysis is the binary diffusion coefficient D of the limiting component A and the inert gas. The other reactive specie concentration is unaffected by the reaction.

The energy and specie conservation equations for such a model of isobaric flames are written as

$$\rho C_p \frac{\partial T}{\partial t} + \rho V \cdot \nabla(C_p T)$$

$$= \lambda \Delta T - K(T - T_u) + Q \cdot w(\rho, T, y_A) \quad (1)$$

$$\rho \frac{\partial y_A}{\partial t} + \rho V \cdot \nabla(y_A)$$

$$= \rho D \Delta y_A - w(\rho, T, y_A), \quad (2)$$

where t is the time, y_A is the mass fraction of the limiting component A, V is the local hydrodynamic velocity, ρ is the density of the mixture, Q is a heat of reaction, and $w(\rho, T, y_A)$ is the rate of the chemical reaction, given by an Arrhenius law

$$w(\rho, T, y_A) = \rho^2 \cdot B \cdot y_A \cdot \exp\left(-\frac{E}{RT}\right), \quad (3)$$

where B is a preexponential factor, taken to be constant, R is the universal gas constant, and E is the activation energy. Thereafter E is assumed to be greater than $RT_b^2/(T_b - T_u)$, where $T_b = T_u + (Q/C_p) \cdot y_{A_u}$ is the adiabatic flame temperature. In accordance with the thermal–diffusional model, we take ρ = constant in equations (1)–(3). This approximation, the constancy of pressure, and the ideal gas law, amount to take the thermal expansion parameter $\alpha = [(T_b - T_u)/T_b]$ much smaller than unity ($\alpha \ll 1$). For sufficiently high values of E, this approximation remains consistent with the fundamental assumption of a high value of the reduced activation energy

$$\beta \equiv \frac{E}{RT_b} \cdot \frac{T_b - T_u}{T_b} \quad (\beta \gg 1).$$

[1] The present analysis can be generalized to a nearly stoichiometric fresh mixture [15].

With these notations, the Arrhenius law (3) takes the form:

$$w(\rho, T, y_A) = B \cdot \rho^2 \cdot e^{-E/RT_b} \cdot y_A$$

$$\cdot \exp \frac{\beta(\theta - 1)}{1 + \alpha(\theta - 1)}, \quad (4)$$

where θ is the reduced temperature, $\theta = [(T - T_u)/(T_b - T_u)]$. When β is large it is readily seen in (4) that a small θ variation of β^{-1} order around $\theta = 1$ strongly modifies the production term $w(\rho, T, y_A)$. Thus, even for $\alpha \ll 1$, small temperature changes have to be considered. Since we neglect the change of density across the flame, there is no more coupling mechanism between the hydrodynamic velocity field and the heat release. If the fresh mixture is initially at rest, the velocity of gases remains everywhere equal to zero while the flame propagates. This is the main feature of the thermal–diffusional model.

Let U be the normal propagation velocity of the steady planar deflagration wave, and let

$$X = X_f(Z, t) = -Ut + \epsilon\phi(Z, t) \quad (5)$$

be the equation of the perturbed flame front in the laboratory frame of reference where the gases are at rest. The ϵ is a small dimensionless parameter measuring the amplitude of the front corrugations.

For saving notations, we use only one transverse coordinate Z. When nondimensional coordinates (x, z, τ) are introduced in a moving frame according to

$$(x, z) = (X - X_f, Z) \cdot \frac{\rho U C_p}{\lambda}$$

and

$$\tau = \frac{t \cdot \rho U^2 C_p}{\lambda}, \quad (6)$$

equations (1) and (2) can be written as

$$\frac{\partial\theta}{\partial\tau} + \frac{\partial\theta}{\partial x}\left(1 - \epsilon\frac{\partial\Phi}{\partial\tau}\right) = \Delta\theta + W - \frac{H}{2\beta}\theta \quad (7)$$

$$\frac{\partial y}{\partial \tau} + \frac{\partial y}{\partial x}\left(1 - \epsilon\frac{\partial \Phi}{\partial \tau}\right) = \frac{1}{Le}\Delta y - W, \qquad (8)$$

where y is the reduced mass fraction of A, defined as y_A/y_{A_u}, Φ equals $\phi \cdot [(\rho U C_p)/\lambda]$, and the Laplacian operator Δ is given by

$$\Delta = \left(\frac{\partial}{\partial z} - \epsilon\frac{\partial \Phi}{\partial z}\frac{\partial}{\partial x}\right)^2 + \frac{\partial^2}{\partial x^2}, \qquad (9)$$

and Le denotes a Lewis number, defined as the ratio of the thermal diffusivity of the inert diluent $\lambda/\rho C_p$ to the molecular diffusion coefficient of the limiting component D: $Le = (\lambda/\rho D C_p)$. According to equation (4) and to the small expansion parameter hypothesis ($\alpha \ll 1$), the adimensional source term W is given by

$$W(\theta, y) = L \cdot \beta^2 \cdot y \cdot \exp \beta(\theta - 1), \qquad (10)$$

where L is the usual eigenvalue

$$L \equiv \frac{\lambda \rho_u^2 B}{(\rho U)^2 C_p} \cdot \frac{e^{-E/RT_b}}{\beta^2}. \qquad (11)$$

The reduced heat-loss coefficient H in (7) is given by

$$H \equiv \frac{\lambda K}{(\rho U)^2 C_p^2} \cdot 2\beta. \qquad (12)$$

The scaling factors β^{-2} and 2β are introduced in equations (11) and (12) to obtain quenching values of H and L that remain finite in the limit $\beta \to \infty$. For steady planar conditions, (7) and (8) become

$$\frac{d\bar{\theta}}{dx} = \frac{d^2\bar{\theta}}{dx^2} + W(\bar{y}, \bar{\theta}) - \frac{H \cdot \bar{\theta}}{2\beta} \qquad (13)$$

$$\frac{d\bar{y}}{dx} = \frac{1}{Le}\frac{d^2\bar{y}}{dx^2} - W(\bar{y}, \bar{\theta}). \qquad (14)$$

When (13) and (14) are solved by an asymptotic expansion for $\beta \to \infty$ with the boundary conditions

$$x = -\infty : \bar{\theta} = 0, \bar{y} = 1$$

and

$$x = +\infty : \bar{\theta} = \frac{d\bar{y}}{dx} = 0, \qquad (15)$$

the following relation between L and H is found [5] (cf. Appendix):

$$H = \log(2Le \cdot L) \cdot (1 + 0(\beta^{-1})). \qquad (16)$$

In the adiabatic case, both K and H are zero, and the value of L given by (16) is the adiabatic eigenvalue $L_{ad} \equiv (2\ Le)^{-1}$. Therefore, we can write (16) as

$$H = \log\frac{L}{L_{ad}} + 0(\beta^{-1})$$

$$= -2\log\frac{U}{U_{ad}} + 0(\beta^{-1}), \qquad (17)$$

where U_{ad} is the adiabatic flame velocity. When the definition of H in (12) is applied, (17) can be written in the following equivalent form, correct to the leading order in β^{-1}:

$$\frac{K \cdot \lambda \cdot 2\beta}{(\rho U_{ad}C_p)^2} = -\left(\frac{U}{U_{ad}}\right)^2 \cdot \log\left(\frac{U}{U_{ad}}\right)^2. \qquad (18)$$

Equation (18) clearly shows that K must be less than or equal to the critical quenching value:

$$K^* \equiv \frac{(\rho U_{ad} \cdot C_p)^2}{\lambda} \cdot \frac{e^{-1}}{2\beta}.$$

For $K > K^*$ there is no steady flame propagation, and for $0 < K < K^*$ two different possible values of U are obtained from (18). These two values of nonadiabatic flame velocity belong to two different ranges: the first value (larger one) is included in the domain $]U^*, U_{ad}]$, whereas the other one is smaller than U^*. The "fast" regimes, according to (17), are associated with $0 < H < 1$; they are true generalizations of the adiabatic one since $U \to U_{ad}$ when $K \to 0$. The "slow" regimes, corresponding to $H > 1$, are new regimes since $U \to 0$ when $K \to 0$. The extinction $H^* = 1$ separates these two ranges and occurs with a nonzero velocity, $U =$

$U^* \equiv e^{-0.5}U_{\mathrm{ad}}$. The present work is mainly concerned with the linear stability of the aforementioned two regimes that one obtains from (18) when K is fixed to be smaller than K^*.

Time-dependent solutions of (7) and (8) are investigated by the method of small harmonic perturbations. One seeks θ, y, and Φ of the form

$$\Phi(z, \tau) = \exp(\omega\tau + ikz)$$

$$\theta(x, z, \tau) = \bar{\theta}(x) + \epsilon \cdot \Phi \cdot \hat{\theta}(x)$$

$$y(x, z, \tau) = \bar{y}(x) + \epsilon \cdot \Phi \cdot \hat{y}(x), \qquad (19)$$

in which ϵ is assumed to be small ($\epsilon \to 0$).

The linearized equations for the perturbations $\hat{\theta}$ and $\hat{y}$ are easily obtained from (7), (8), (13), and (14):

$$(\omega + k^2)\hat{\theta} + \frac{d\hat{\theta}}{dx} - \frac{d^2\hat{\theta}}{dx^2} - \hat{w} + \frac{H \cdot \hat{\theta}}{2\beta}$$

$$= (\omega + k^2)\frac{d\bar{\theta}}{dx} \qquad (20)$$

$$\left(\omega + \frac{k^2}{Le}\right)\hat{y} + \frac{d\hat{y}}{dx} - \frac{1}{Le}\frac{d^2\hat{y}}{dx^2} + \hat{w}$$

$$= \left(\omega + \frac{k^2}{Le}\right)\frac{d\bar{y}}{dx}, \qquad (21)$$

where

$$\hat{w}(x) = \hat{\theta} \cdot \frac{\partial W}{\partial\bar{\theta}}(\bar{\theta}, \bar{y}) + \hat{y} \cdot \frac{\partial W}{\partial\bar{y}}(\bar{\theta}, \bar{y}) \qquad (22)$$

Since we are concerned only with the intrinsic stability of flames, no upstream forcing terms[2] are introduced, and the following boundary conditions are used:

$$x = -\infty : \hat{\theta} = \hat{y} = 0 \qquad (23)$$

Furthermore, we require that the perturbations $\hat{y}$ and $\hat{\theta}$ remain everywhere bounded. Once (20) and (21) are solved with the aforementioned condi-

[2] Such forcing terms appear in the turbulent-flame problem [16].

tions, one obtains the dispersion relation, which expresses the amplification rate ω as a function of the transverse wave number k, with β, Le, and H as parameters. Rate ω may be a complex number, but the transverse wave number has to be real since we are interested with the stability of transversely unbounded flames. If the real part of ω is positive, the corresponding wave number is unstable.

THE DISPERSION RELATION

Difficulties in solving the problem are due to the production term $\hat{w}$, which introduces through (22) nonconstant coefficients in the differential equations (20) and (21). However, according to the assumption of large values of β, $\hat{w}$ is everywhere negligible, except in a thin reactive zone, the width of which is $O(\beta^{-1})$ in $(\lambda/\rho UC_p)$ units. As the upstream diffusive thickness $(\lambda/\rho UC_p)$ is of order unity—by definition—and the downstream cooling length is of order β, the reactive zone can be seen in a first approximation as a boundary sheet between two larger nonreactive zones.

In fact, (5) is the equation of the perturbed reactive surface. Boundary conditions on both sides of the reactive sheet are obtained by solving the inner equations inside the reactive zone, and then matching the inner solutions with the outer ones. When time dependence is sufficiently slow and transverse wavelengths are sufficiently long, the structure of this inner zone is qualitatively unaffected by the flame distortions. These boundary conditions are then obtained in a similar way to the stationary planar case (cf. Appendix).

In the outer nonreactive zone, with $\hat{w}$ negligible, the governing equations (20) and (21) become linear differential equations with constant coefficients. Thus the corresponding bounded solutions are easily obtained. When expressed in terms of boundary values on the reactive sheet, they take the following form:

$$\hat{\theta}^\pm(x) = \frac{d\bar{\theta}^\pm}{dx}(x) + \left(\hat{\theta}^\pm(0) - \frac{d\bar{\theta}^\pm}{dx}(0)\right)e^{x \cdot x^\pm} \qquad (24)$$

$$\hat{y}^\pm(x) = \frac{d\bar{y}^\pm}{dx}(x) + \left(\hat{y}^\pm(0) - \frac{d\bar{y}^\pm}{dx}(0)\right)e^{x \cdot \nu^\pm} \qquad (25)$$

with

$$\chi^\pm \equiv \frac{1}{2}\left[1 \mp \sqrt{1 + 4\left(\omega + k^2 + \frac{H}{2\beta}\right)}\right]$$

$$\nu^\pm \equiv \frac{Le}{2}\left[1 \mp \sqrt{1 + \frac{4}{Le}\left(\omega + \frac{k^2}{Le}\right)}\right], \qquad (26)$$

where the symbols + and − denote the downstream cooling region and the upstream preheat zone, respectively, and $\bar{\theta}^\pm$ and $\bar{y}^\pm$ are the reduced temperature and concentration outer profiles of the unperturbed steady problem (13)-(15), specifically:

$$\bar{\theta}^+(x) \equiv \bar{\theta}^+(0)\exp\left[\frac{x}{2}\left(1 - \sqrt{1 + \frac{2H}{\beta}}\right)\right];$$

$$\bar{y}^+(x) \equiv 0 \qquad (27)$$

$$\bar{\theta}^-(x) \equiv \bar{\theta}^-(0)\exp\left[\frac{x}{2}\left(1 + \sqrt{1 + \frac{2H}{\beta}}\right)\right];$$

$$\bar{y}^-(x) \equiv 1 - e^{Le \cdot x} \qquad (28)$$

According to (6), $x = 0$ corresponds to the reactive sheet, and the x axis is oriented toward the downstream burned gases.

The longitudinal gradients of $\hat{\theta}$ and $\hat{y}$ on both sides of the reactive sheet can be expressed, using (24) and (25), in terms of the corresponding boundary values:

$$\frac{d\hat{\theta}^\pm}{dx}(0) = \frac{d^2\bar{\theta}^\pm}{dx^2}(0) + \chi^\pm$$
$$\cdot \left(\hat{\theta}^\pm(0) - \frac{d\bar{\theta}^\pm}{dx}(0)\right) \qquad (29)$$

$$\frac{d\hat{y}^\pm}{dx}(0) = \frac{d^2\bar{y}^\pm}{dx^2}(0) + \nu^\pm$$
$$\cdot \left(\hat{y}^\pm(0) - \frac{d\bar{y}^\pm}{dx}(0)\right) \qquad (30)$$

Since the production term $\hat{w}$ is transcendentally

small in the outer nonreactive zones, relations (24)-(28) are accurate up to all algebraic orders in β^{-1}.

The key of the method is that the inner reactive zone can be solved by an asymptotic expansion for large values of β without any assumption on the amplitude of front corrugations. Matching of these perturbed inner solutions with the outer ones in (24) and (25) provides us with different relations between the lowest-order terms of the β expansions of boundary values $\hat{\theta}^\pm(0)$, $\hat{y}^\pm(0)$, and space derivatives $(d\hat{\theta}^\pm/dx)_{x=0}$, and $(d\hat{y}^\pm/dx)_{x=0}$. These relations, associated with those obtained from (29) and (30), are in sufficient number to obtain all the terms involved in (29) and (30). They also give a compatibility relation between the wave number k and amplification coefficient ω (i.e., the dispersion relation).

When the following expansions of the outer solutions

$$(\hat{\theta}^\pm(x), \hat{y}^\pm(x)) = (\hat{\theta}_0^\pm(x), \hat{y}_0^\pm(x)) + \beta^{-1}(\hat{\theta}_1^\pm(x),$$

$$\hat{y}_1^\pm(x)) + 0(\beta^{-2}) \qquad (31)$$

$$(\chi^\pm, \nu^\pm) = (\chi_0^\pm, \nu_0^\pm) + \beta^{-1}(\chi_1^\pm, \nu_1^\pm)$$

$$+ 0(\beta^{-2}) \qquad (32)$$

are introduced in (29) and (30), together with the analogous expansions of $\bar{y}^\pm$ and $\bar{\theta}^\pm$ previously obtained [5] (cf. Appendix), a first set of relations emerges:

$$\frac{d\hat{\theta}_0^-}{dx}(0) = 1 - \chi_0^-, \quad \frac{d\hat{\theta}_1^-}{dx}(0)$$

$$= \chi_0^-\left(\hat{\theta}_1(0) + \frac{H_0}{2}\right) - \chi_1^-,$$

$$\frac{d\hat{\theta}_0^+}{dx}(0) = 0, \quad \frac{d\hat{\theta}_1^+}{dx}(0)$$

$$= \chi_0^+ \cdot \left(\hat{\theta}_1(0) + \frac{H_0}{2}\right) \qquad (33)$$

$$\frac{d\hat{y}_0{}^-}{dx}(0) = -l_0(l_0 - \nu_0{}^-), \frac{d\hat{y}_1{}^-}{dx}(0)$$

$$= -l_1(l_0 - \nu_0{}^-) - l_0(l_1 - \nu_1{}^-)$$

$$\frac{d\hat{y}_0{}^+}{dx}(0) = 0, \frac{d\hat{y}_1{}^+}{dx}(0) = 0, \tag{34}$$

where the following boundary conditions obtained from the inner zone solutions are used (cf. Appendix):

$$\hat{\theta}_0{}^+(0) = \hat{\theta}_0{}^-(0) = 0, \ \hat{y}_0{}^+(0) = \hat{y}_0{}^-(0) = 0 \tag{35}$$

$$\hat{\theta}_1{}^+(0) = \hat{\theta}_1{}^-(0) = \hat{\theta}_1(0),$$

$$\hat{y}_1{}^+(0) = \hat{y}_1{}^+(0) = 0 \tag{36}$$

and where l_0 and l_1 are defined by

$$Le = l_0 + \beta^{-1}l_1 + 0(\beta^{-2}). \tag{37}$$

Similar expansions for H, ω, and k^2 are introduced in (26) to obtain the values of all the terms involved in (31) and (32):

$$H = H_0 + 0(\beta^{-1}), \text{ with } H_0 \equiv -2\log\frac{U}{U_{\text{ad}}} \tag{38}$$

and

$$k^2 = k_0{}^2 + \beta^{-1}(k^2)_1 + 0(\beta^{-2})$$

$$\omega = \omega_0 + \beta^{-1}\omega_1 + 0(\beta^{-2}). \tag{39}$$

A second kind of relation between the boundary values and derivatives is obtained by solving the inner equations and matching them with the outer ones (cf. Appendix):

$$\frac{d\hat{\theta}_0{}^-}{dx}(0) = \frac{-1}{l_0}\frac{d\hat{y}_0{}^-}{dx}(0) = \frac{1}{2}\hat{\theta}_1(0) \tag{40}$$

$$\left[\frac{d\hat{\theta}_1}{dx}\right]_-^+ + \frac{-1}{l_0}\left[\frac{d\hat{y}_1}{dx}\right]_-^+ - \frac{l_1}{l_0{}^2}\left[\frac{d\hat{y}_0}{dx}\right]_-^+$$

$$= 0, \tag{41}$$

where the brackets [ ]$^\pm$ denote jumps between $x = 0^\pm$. When compared to (33), the first part of (40) gives $l_0(1 - \chi_0{}^-) = l_0(l_0 - \nu_0{}^-)$. By introducing (37)–(39), this last result leads to

$$l_0\left(1 - \sqrt{1 + \frac{4}{l_0}\left(\omega_0 + \frac{k_0{}^2}{l_0}\right)}\right)$$

$$= 1 - \sqrt{1 + 4(\omega_0 + k_0{}^2)}. \tag{42}$$

When we look at the response of the system to a perturbation with a wavelength of the same order of magnitude as the thermal length ($\lambda/\rho U C_p$), k_0 is different from zero.

Then the only way to satisfy (42) is to set

$$l_0 = 1. \tag{43}$$

This result clearly shows that the present analysis is restricted to Lewis numbers close to unity. In gases, the Lewis number often is close to unity and, in fact, assumption (43) is not restrictive for the study of combustion in gas mixtures.

With (33) and (34), the second part of (40) furnishes the value of the departure of the maximum temperature from its adiabatic value:

$$\hat{\theta}_1(0) = 2(1 - \chi_0{}^-) \tag{44}$$

When equations (32) and (44) are combined with (41), one gets a supplementary relation between $\chi_0{}^-$, $\chi_1{}^-$, $\nu_0{}^-$, and $\nu_1{}^-$:

$$\left(2(1 - \chi_0{}^-) + \frac{H_0}{2}\right)(\chi_0{}^+ - \chi_0{}^-) + \chi_1{}^-$$

$$= \nu_1{}^- - l_1. \tag{45}$$

When expressions (26) and expansions (37) and (39) are introduced in (45), it is readily found that the terms ω_1 and $(k^2)_1$ cancel (fortunately). Thus we are left with the following relation between ω_0 and $k_0{}^2$, where H_0 and l_1, which characterize the loss intensity and the Lewis number, respec-

tively, appear as parameters:

$$(1 - \Gamma)\left(\frac{H_0}{2}(\Gamma + 1) - \Gamma^2\right)$$

$$= \frac{l_1}{2}(1 - \Gamma + 2\omega_0) \tag{46}$$

with

$$\Gamma = \sqrt{1 + 4(\omega_0 + k_0{}^2)}, \ \text{Re}\,(\Gamma) > 1. \tag{47}$$

Equation (46) is the dispersion relation, which describes the stability properties of nonadiabatic flames for perturbation wavelengths of the same order of magnitude as flame thickness.

DISCUSSION OF RESULTS

With $H_0 = 0$, that is, for a vanishing heat-loss intensity, (46) reduces to the dispersion relation recently obtained by Sivashinsky [11] for the diffusional–thermal model of adiabatic flames. Before pointing out the typical effects of heat losses, let us briefly recall the main results obtained for the adiabatic case.

In the case of Lewis numbers $Le = 1 + (l_1/\beta) + \cdots$ such that $-2 < l_1 < 10.5 \cdots$, all the wave numbers k are shown to be stable. For usual values of activation energies and of Lewis numbers occurring in gas mixtures, l_1 always is less than the above upper bound. Then, for the diffusional stability of adiabatic flames, the only useful critical value of l_1 is (-2). When $l_1 < -2$, a critical wavelength $\Lambda_c = (2\pi/k_c)$ appears, which corresponds to neutral stability. All wavelengths Λ greater than Λ_c are unstable and those less than Λ_c are stable. Furthermore, since planar perturbations ($\Lambda = \infty$) are stable, there is a finite wavelength $\Lambda_m (\Lambda_m > \Lambda_c; \Lambda_m = 2\pi/k_m)$ for which the amplification rate has a maximum, ω_m. Critical wavelength Λ_m can be reasonably used to characterize the dimensions of cells experimentally observed in unstable situations (see Fig. 3a). When l_1 approaches (-2) from below, Λ_c and Λ_m go to infinity and the maximum growth rate ω_m goes to zero. Thus $Le = 1 + [(-2)/\beta]$ appears to be a bifurcation point. Since $l_1 < 0$ corresponds to $Le <$

1, this bifurcation occurs for sufficiently light limiting reactant. The instability leading to bifurcation appears when the transverse molecular diffusion induced by wrinkles becomes sufficiently efficient (compared to heat diffusion) to decrease the temperature of reacting zones pointed toward burned gases. For the same reasons, the temperature increases locally when the reactive sheet is pointed toward the fresh mixture. In this case, corrugations are amplified because the local velocity of the reactive sheet increases with temperature. As the heat losses affect only the heat transfer, they modify the stability limits.

Let us begin to investigate the behavior of perturbations of very large wavelengths. For $k_0 = 0$, (46) reduces to

$$\Gamma^2 - H_0 \cdot \frac{\Gamma + 1}{2} = \frac{l_1}{2} \cdot \frac{\Gamma - 1}{2}. \tag{48}$$

From (48) it is readily seen that the real part of ω_0 is always positive when $H_0 > 1$ (i.e, $U < U^*$). Thus the "slow" regimes are always unstable for planar perturbations and cannot be observed. The result is quite different for $0 < H_0 < 1$, that is, for the "fast" regimes ($U^* < U < U_{ad}$). The real part of ω_0 is plotted on Fig. 1 for different values l_1 and H_0 and for $k_0 = 0$.

In the range $6 < l_1 < 4(1 + \sqrt{3}) = 10.93 \cdots$, two particular values of H_0 appear:

$$H_0 = H^{(3)}(l_1) \equiv 2 \cdot \sqrt{l_1 + 3} - \left(\frac{l_1}{2} + 2\right)$$

$$H_0 = H^{(4)}(l_1) \equiv \sqrt{8(l_1 + 2)} - \left(\frac{l_1}{2} + 4\right)$$

with

$$H^{(3)}(6) = H^{(4)}(6) = 1;$$

$$H^{(3)}(4(1 + \sqrt{3})) = H^{(4)}(16) = 0.$$

As seen in Fig. 1, infinite wavelengths are stable $[Re(\omega_0) < 0]$ only for $0 < H < H^{(3)}(l_1)$; they exhibit instabilities of oscillatory type $[Re(\omega_0) > 0, Im(\omega_0) \neq 0]$ for $H^{(3)}(l_1) < H_0 < H^{(4)}(l_1)$, or of monotonic type $[Re(\omega_0) > 0,$

$Re\ \omega_0$

$l_1 > 16$

$l_1 = 16$

$4(1+\sqrt{3}) < l_1 < 16$

$l_1 = 4(1+\sqrt{3})$

$6 < l_1 < 4(1+\sqrt{3})$

$l_1 = 6$

$l_1 < 6$

$l_1 > 16$

$-l_1 > 1$

0 $H^{(3)}(l_1)$ $H^{(4)}(l_1)$ 1 H_0

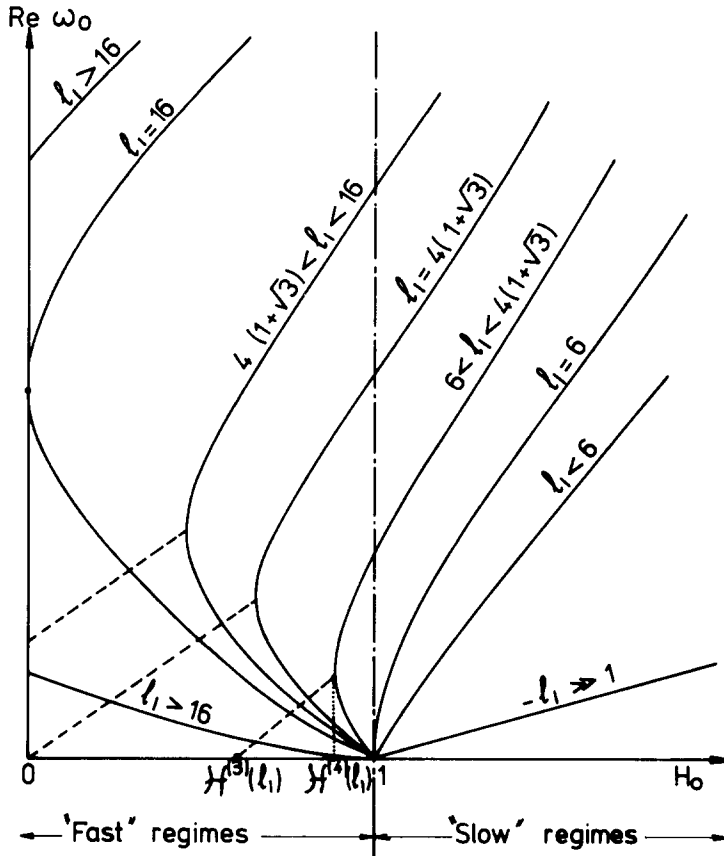—'Fast' regimes ——————|——— 'Slow' regimes ———➤

Fig. 1. Real part of the amplification coefficient ω_0 for planar disturbances: solid lines —— correspond to real ω_0; dotted lines ---- correspond to complex ω_0.

$Im(\omega_0) = 0$] for $H^{(4)}(l_1) < H_0 < 1$. Notice that near the extinction limit $(1 - H_0 \ll 1)$, the critical value of l_1 associated with the planar instability decreases to 6, which is a rather accessible value $(\beta = 16$ and $l_1 = 6$ give $Le = 1.375)$.

Such a result is a first illustration of the following property: near the thermal extinction limit $(H_0 \cong 1)$ one can observe behaviors that were predicted to occur in the adiabatic case for irrelevant values of the Lewis number.

For finite wavelengths $(k_0 \neq 0)$, (46) and (47) involve two other limiting curves, $H_0 = H^{(1)}(l_1) \equiv 1 + (l_1/2)$ and $H_0 \equiv H^{(2)}(l_1)$ (defined later), sketched in Fig. 2.

For a given value of H_0, the first one gives the critical value of Le associated with the instability described at the beginning of this section for

adiabatic flames, and $H_0 = H^{(1)}(l_1)$ is a line of bi furcation points in the (H_0, l_1) plane. Around thi: line, $\delta \equiv H_0 - 1 - (l_1/2)$ is small, and the disper sion relation (46) simplifies to

$$(1 - H_0)\omega_0 = \delta k_0^2 - (4 - H_0)k_0^4,$$

with

$$k_0^2 = 0(\delta), \quad \omega_0 = 0(\delta^2). \tag{49}$$

In (49) the small parameter δ has to be smalle than $(1 - H_0)$. Near the extinction, $\sigma \equiv 1 - H_0$ is also small and, instead of (49), (46) gives an equation with two independent small parameter δ and σ:

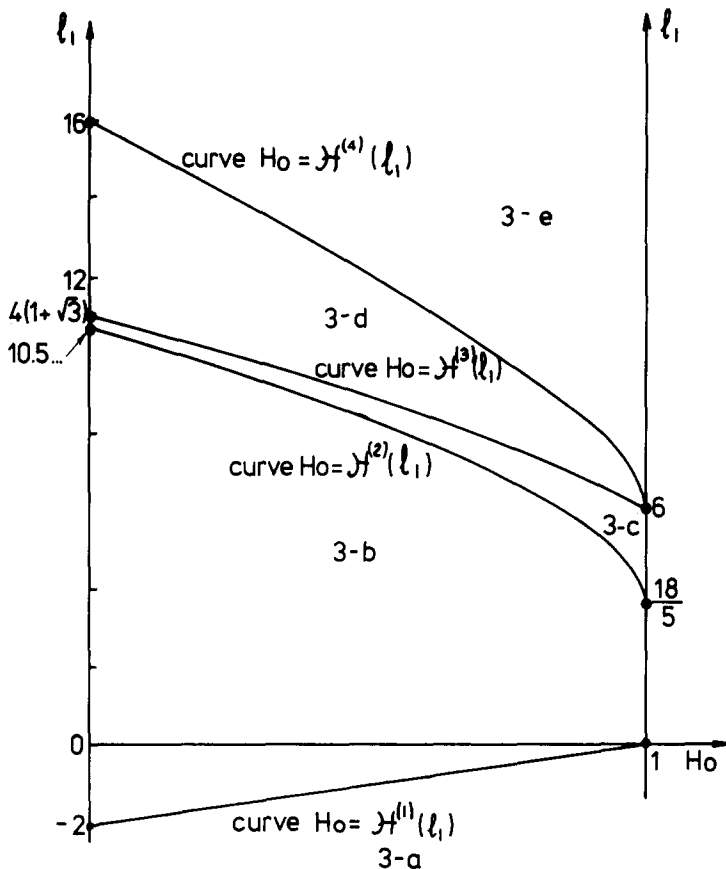$$(\delta + \sigma)\omega_0 = \delta(\omega_0 + k_0^2) - 3(\omega_0 + k_0^2)^2,$$

Fig. 2. Stability domains in the (H_0, l_1) plane.

with

$$k_0{}^2 \sim \omega_0 \sim \delta = 0(\sigma). \qquad (50)$$

Equation (49) is similar to the Landau equation for the second-order phase transition, where the order parameter should be the wave number $k_m = [\delta/2(4 - H_0)]^{1/2}$ associated with the maximum growth rate ω_m. The shape of (49) is given in Fig. 3a.

The critical value of l_1 associated with the diffusive instability, $l_1 = -2(1 - H_0)$, varies from -2 to 0 as the heat-loss intensity increases from $H_0 = 0$ (adiabatic case) to $H_0 = 1$ (extinction limit). Thus the corresponding critical Lewis number goes to unity by lower values when heat-loss power approaches the extinction value. Therefore, initially stable regimes such that $1 - (2/\beta) < Le < 1$ can be

destabilized by an increase of heat losses. In other words, this instability occurs in nonadiabatic cases with limiting reactants that are comparatively heavier than those needed in the adiabatic case. One can thus predict that the cellular structures are more easily obtained near the thermal extinction limit. Indeed, for limiting reactant of Lewis numbers less than unity, which is often the case for usual mixtures, our results show that cells must occur prior to extinction (see Fig. 2). Similar phenomena are known to occur near the composition limits of flammability that are often reached with a cellular structure [9].

Notice that for Lewis numbers very close to (but less than) unity, $0 < -l_1 \ll 1$, the diffusive instability is obtained only in the vicinity of extinction; moreover, it is differently time-scaled and perhaps of different character since (50) involves
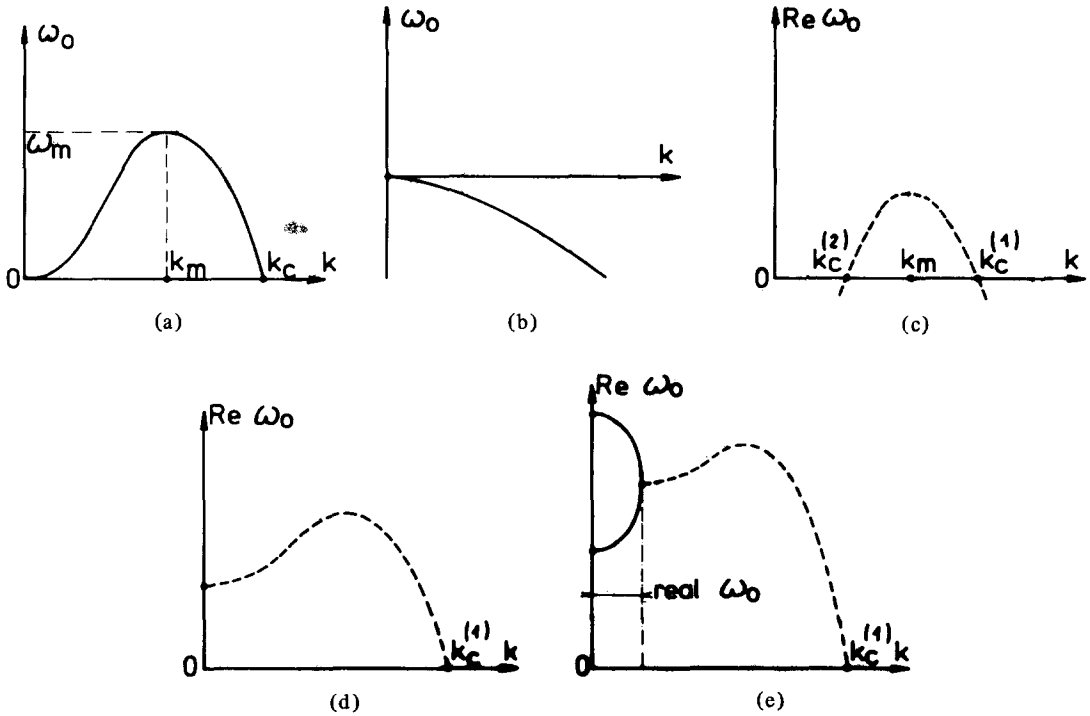
Fig. 3. Dispersion relation shapes. The numbers of the figure correspond to domains of Fig. 2. Solid lines —— correspond to real ω_0; dotted lines ---- correspond to complex ω_0.

ω_0^2 (i.e., second time derivative) in addition to ω_0.

Another important typical effect of heat loss concerns fresh mixtures with Lewis numbers greather than unity (sufficiently heavy limiting component or inerts with a large thermal diffusivity). Each point of the domain limited by the curves $H_0 = H^{(1)}(l_1)$ and $H^{(2)}(l_1)$ represents flames that are stable for all the wavelengths (see Figs. 2 and 3b); $H_0 = H^{(2)}(l_1)$ is also a line of bifurcation points, but concerning unstable wavelengths around a finite value $[2\pi/k_m(H_0)]$ (Fig. 3c) and with a nonzero imaginary rate of growth $[Re(\omega_0) > 0, Im(\omega_0) \neq 0]$. On the curve $H_0 = H^{(2)}(l_1)$, obtained from (46) by numerical means, $Re(\omega_0)$ is zero.

This instability is of oscillatory kind and corresponds to the growth of transverse traveling waves with a dimensionless phase velocity, k_m^{-1} $(H_0) \cdot Im(\omega_0(H_0))$.

When heat loss increases, the critical Lewis

number decreases down to $Le = 1 + (18/5\beta)$, which is an experimentally accessible value $[\beta = 16, l_1 = (18/5)$ give $Le = 1.225]$. Thus, near the extinction limit, our analysis predicts a new kind of instability for lean mixtures of heavy limiting components with low diffusion coefficients and/or diluents with large thermal diffusivities. To our knowledge, self-propagating waves of this type have not yet been observed for this kind of mixtures. To check such behaviors, it would be interesting to perform experiments—with adequate mixtures—on cooled annular burners of width slightly greater than the quenching distance, and with adjustable ambient pressure or mixture strength in order to approach the extinction limit.

APPENDIX: JUMP CONDITIONS ON THE REACTIVE SURFACE

The boundary conditions for the outer solutions on both sides of the reactive surface are obtained

by solving the conservation equations (7) and (8) inside the reactive sheet and then matching with the outer solutions. The inner solutions are obtained in the form of asymptotic expansions for large values of β. As the thickness of the reactive sheet is assumed to be of order β^{-1}, the stretched variable $\eta = \beta x$ is introduced. The inner profiles are sought in the form

$$\theta(x, z, \tau) = 1 + \beta^{-1}\Theta_1(\eta, z, \tau)$$
$$+ \beta^{-2} \cdot \Theta_2(\eta, z, \tau) + O(\beta^{-3})$$

$$y(x, z, \tau) = \beta^{-1} \cdot Y_1(\eta, z, \tau)$$
$$+ \beta^{-2} \cdot Y_2(\eta, z, \tau) + O(\beta^{-3}). \quad \text{(A-1)}$$

For sufficiently slow and large-scaled perturbations, $(\partial/\partial\tau) \sim (\partial/\partial z) = O(1)$, the leading orders of the conservation equations (7) and (8) take the form

$$\frac{\partial^2\Theta_1}{\partial\eta^2} = -\frac{L_0}{1 + \epsilon^2(\partial\Phi/\partial z)^2} \cdot Y_1 \cdot e^{\Theta_1} \quad \text{(A-2)}$$

and

$$\frac{\partial^2 Y_1}{\partial\eta^2} + l_0 \frac{\partial^2\Theta_1}{\partial\eta^2} = 0 \quad \text{(A-3)}$$

with the asymptotic behaviors

$$\eta \to -\infty : Y_1 \to +\infty, \Theta_1 \to -\infty.$$

Furthermore, it is anticipated that in the downstream field, that is, for $\eta \to +\infty$, y_1 goes to zero sufficiently quickly to ensure the completion of the reaction in the inner zone. As it has been shown previously [5], this implies a reaction order less than or equal to unity; for higher reaction orders, unburned amounts of the limiting component appear in the burned gases and the problem is more difficult.

With these asymptotic behaviors it is readily seen from (A-2) that θ_1 and Y_1 have asymptotes for $\eta \to -\infty$ and $\eta \to +\infty$. Matching this asymptotic behavior with β expansions such as (31) for the

outer solutions $\theta^\pm(x, z, \tau)$ and $y^\pm(x, z, \tau)$ implies

$$\eta \to -\infty : \Theta_1 = \frac{\partial\theta_0^-}{\partial x}(0, z, \tau) \cdot \eta$$
$$+ \theta_1^-(0, z, \tau) + \text{(transc.)} \quad \text{(A-4)}$$

$$Y_1 = \frac{\partial y_0^-}{\partial x}(0, z, \tau) \cdot \eta$$
$$+ y_1^-(0, z, \tau) + \text{(transc.)} \quad \text{(A-5)}$$

and

$$\eta \to +\infty : \Theta_1 = \frac{\partial\theta_0^+}{\partial x}(0, z, \tau) \cdot \eta$$
$$+ \theta_1^+(0, z, \tau) + \text{(transc.)} \quad \text{(A-6)}$$

$$Y_1 = \text{(transc.)}, \quad \text{(A-7)}$$

where (transc.) denotes transcendentally small terms. Equations (A-4) and (A-5) imply that $\theta_0^-(0, z, \tau) = 1$ and $y_0^-(0, z, \tau) = 0$; (A-6) and (A-7) imply that $\theta_0^+(0, z, \tau) = 1$ and $y_0^+(0, z, \tau) = y_1^+(0, z, \tau) = 0$. At this stage, it is worthwhile to notice that one can choose the origin $\eta = 0$ for having $y^-(0, z, \tau) = 0$. All these results correspond to (35).

Furthermore, when the outer downstream solution θ_0^+ verifies, as in (27) and (33), $(\partial\theta_0^+/\partial x)_{x=0} = 0$, (A-6) gives

$$\eta \to +\infty : \Theta_1 = \theta_1^+(0, z, \tau) + \text{(transc.)}. \quad \text{(A-8)}$$

The integration of (A-3), with the boundary conditions (A-7) and (A-8), yields

$$(Y_1 + l_0\Theta_1) = l_0\theta_1^+(0, z, \tau). \quad \text{(A-9)}$$

At the upstream boundary, (A-9), (A-4), and (A-5) give

$$\theta_1^-(0, z, \tau) = \theta_1^+(0, z, \tau) \quad \text{(A-10)}$$

$$\frac{\partial y_0^-}{\partial x}(0, z, \tau) = -l_0 \frac{\partial\theta_0^-}{\partial x}(0, z, \tau). \quad \text{(A-11)}$$

Equations (A-10) and (A-11), of course, also hold for the steady solutions; therefore, (36) and the first equality in (40) are demonstrated.

If (A-9) is used and (A-2) is integrated with the boundary condition (A-8), one gets

$$\left(\frac{\partial \Theta_1}{\partial \eta}\right)^2 = \frac{2L_0 l_0 e^{\theta_1^+(0,z,\tau)}}{1 + \epsilon^2 (\partial \Phi/\partial z)^2}$$

$$\cdot \int_0^{\Theta_1 - \theta_1^+(0,z,\tau)} e^u u \, du. \qquad \text{(A-12)}$$

From (A-12) it is readily found that Y_1 goes to zero exponentially with $\eta \to +\infty$, as we supposed at the beginning. At the other side, $\eta \to -\infty$, (A-12) yields

$$\left(\frac{\partial \theta_0^-}{\partial x}(0, z, \tau)\right)^2 = \frac{2L_0 l_0 e^{\theta_1^+(0,z,\tau)}}{1 + \epsilon^2 (\partial \Phi/\partial z)^2}. \qquad \text{(A-13)}$$

When (A-13) is expanded in powers of ϵ, one obtains

$$1 = \left(\frac{d\bar\theta_0^-}{dx}(0)\right)^2 = 2L_0 l_1 e^{\bar\theta_1^+(0)} \qquad \text{(A-14)}$$

for the steady solution $\bar\theta$ and

$$\frac{d\hat\theta_0^-}{dx}(0) = \frac{\hat\theta_1(0)}{2} \qquad \text{(A-15)}$$

for the x-dependent part of the perturbation $\theta - \bar\theta$. Equation (A-15) is the second equality in (40).

When the differential equations for Θ_2 and Y_2 are investigated, the following asymptotic behaviors are obtained:

$$\eta \to \pm\infty : \Theta_2 = \frac{\partial^2 \theta_0^\pm}{\partial x^2}(0, z, \tau) \cdot \frac{\eta^2}{2}$$

$$+ \frac{\partial \theta_1^+}{\partial x}(0, z, \tau) \cdot \eta$$

$$+ \theta_2^\pm(0, z, \tau) + \text{(transc.)} \qquad \text{(A-16)}$$

$$Y_2 = \frac{\partial^2 y_0^\pm}{\partial x^2}(0, z, \tau) \cdot \frac{\eta^2}{2}$$

$$+ \frac{\partial y_1^\pm}{\partial x}(0, z, \tau) \cdot \eta$$

$$+ y_2^\pm(0, z, \tau) + \text{(transc.)} \qquad \text{(A-17)}$$

in which the coefficients of powers of η were found by matching (A-16) and (A-17) with the β expansions of the outer solutions $\theta^\pm(x, z, \tau)$ and $y^\pm(x, z, \tau)$. When the differential equations for Θ_2 and Y_2 are added, the production term disappears. If attention is restricted to the first two terms of an ϵ expansion of the last relation, one obtains

$$\left(1 - \epsilon \frac{\partial \Phi}{\partial \tau}\right) \frac{\partial}{\partial \eta}(\Theta_1 + Y_1) + \epsilon \cdot \frac{\partial^2 \Phi}{\partial z^2}$$

$$\cdot \frac{\partial}{\partial \eta}\left(\Theta_1 + \frac{Y_1}{l_0}\right) = \frac{\partial^2}{\partial \eta^2}\left(\Theta_2 + \frac{Y_2}{l_0}\right.$$

$$\left. - \frac{l_1}{l_0^2} Y_1\right). \qquad \text{(A-18)}$$

Once (A-18) is integrated between $\eta = \pm\infty$, (A-4), (A-5), (A-16), (A-17), and (35), equation (36) can be used to give the following two relations:

$$\left(1 - \epsilon \frac{\partial \Phi}{\partial \tau}\right)\left(\frac{\partial \theta_0^-}{\partial x} + \frac{\partial y_0^-}{\partial x}\right)_{x=0}$$

$$= \left(\frac{\partial^2 \theta_0^-}{\partial x^2} + \frac{1}{l_0}\frac{\partial^2 y_0^-}{\partial x^2}\right)_{x=0}$$

$$- \left(\frac{\partial^2 \theta_0^+}{\partial x^2} + \frac{1}{l_0}\frac{\partial^2 y_0^+}{\partial x^2}\right)_{x=0} \qquad \text{(A-19)}$$

(coming from the identification of the factors of η), and

$$0 = \left[\frac{\partial \theta_1}{\partial x}\right]_{x=0^+}^{x=0^-} + \frac{1}{l_0}\left[\frac{\partial y_1}{\partial x}\right]_{x=0^+}^{x=0^-}$$

$$- \frac{l_1}{l_0^2}\left[\frac{\partial y_0}{\partial x}\right]_{x=0^+}^{x=0^-} \qquad \text{(A-20)}$$

(coming from the constant terms). Equation (A-20), which corresponds to (41), gives the zeroth order of the ϵ expansion of $\theta_1(0)$. Indeed, using (27), (28), and (A-12) in (A-20), one obtains for the stationary case:

$$\bar\theta_1(0) = -H_0 \qquad \text{(A-21)}$$

Equations (A-21) and (A-14) give $2L_0 l_0 e^{-H}0 = 1$, which is (16).

It is worthwhile to notice that (A-20) and (A-11) correspond, up to the first two terms of a β expansion, to the following jump relation through the reactive surface:

$$\left[\frac{\partial \theta}{\partial x}\right]_{0^+}^{0^-} + \frac{1}{Le}\left[\frac{\partial y}{\partial x}\right]_{0^+}^{0^-} = 0. \qquad (A-22)$$

NOMENCLATURE

A	limiting reactant
B	preexponential factor, (3)
C_p	specific heat at constant pressure
D	diffusion coefficient of A into the mixture
E	activation energy, (3)
e	2.718...
$H^{(i)}(l_1)$	functions of l_1, defined in the text (see Figs. 1 and 2)
H	reduced heat-loss coefficient, (12)
$i = \sqrt{-1}$	imaginary unit number, (19)
$Im(\ )$	imaginary part of ( )
k	dimensionless wave number [such that $(2\pi/k)\cdot(\lambda/\rho U C_p)$ is the wavelength]
k_c, $k_c^{(1)}$, $k_c^{(2)}$	critical values of k; see text and Fig. 3
K	heat-loss coefficient, (1)
L	eigenvalue for the steady planar nonadiabatic propagation, (11)
$L_{ad} = (1/2\ Le)\ (1 + 0(\beta^{-1}))$	adiabatic analog of L
$Le = (\lambda/\rho DC_p) = l_0 + \beta^{-1}l_1 + \cdots +$	Lewis number
R	universal gas constant
$Re(\ )$	real part of ( )
T	temperature
t	time
U	*non*adiabatic flame speed (steady planar case)
U_{ad}	adiabatic flame speed (steady planar case)

V	material velocity
w	source term
$\hat{w}(x)$	x-dependent part of $(W - W(\bar{y}, \bar{\theta}))$, equation (22)
W	dimensionless source term (10)
x	dimensionless longitudinal coordinate, (6)
X	Cartesian coordinate, normal to the unperturbed flame front
$X_f(t, Z)$	equation of the reactive sheet, (5)
y_A	mass fraction of A
$y = y_A/y_{A_u}$	reduced mass fraction of A
$\hat{y}(x)$	x-dependent part of $y - \bar{y}$, equation (19)
$\bar{y}(x)$	y in the steady planar case, (14)
$Y_i(\eta, z, \tau)$	ith term of the inner β expansion of y, equation (A-1)
Z	transverse coordinate
$z = (Z\cdot\rho U C_p/\lambda)$	dimensionless transverse coordinate

Greek Symbols

$\alpha = [(T_b - T_u)/T_b]$	
$\beta = \alpha\ (E/RT_b)$	reduced activation energy
$\Gamma = \sqrt{1 + 4(\omega_0 + k_0^2)}$, $Re(\Gamma) > 1$, equation (47)	
$\delta = H_0 - 1 - (l_1/2)$, equation (49)	
Δ	Laplacian operator
∇	gradient
ϵ	dimensionless amplitude of front corrugations, (5)
$\eta = \beta x$	
$\theta = [(T - T_u)/(T_b - T_u)]$	reduced temperature
$\hat{\theta}(x)$	x-dependent part of $\theta - \bar{\theta}$, equation (19)
$\bar{\theta}(x)$	θ in the steady planar case, (13)
$\Theta_i(\eta, z, \tau)$	ith term of the inner β expansion of θ, equation (A-1)
$\Lambda = (2\pi/k)$	wavelength in $(\lambda/\rho U C_p)$ units
λ	thermal conductivity, (1)
$\nu^{\pm}$	coefficients defined by (26)
$\pi = 3.1415...$	
ρ	density
$\sigma = 1 - H_0$, equation (50)	
$\tau = t\ \rho U^2 C_p/\lambda$	dimensionless time, (6)
$\phi(Z, t) = (X_f + Ut)\cdot\epsilon^{-1}$	corrugation of the flame front, (5)

$\Phi(z, \tau)$ dimensionless corrugation of the flame front

$\chi^{\pm}$ coefficients defined by (26)

ω amplification coefficient, (19)

Subscripts

$()_b$ refers to the adiabatic flame temperature T_b

$()_j, (j = 0, 1, 2, ...)$ characterizes the jth term of a β expansion

$()_m$ corresponds to the maximum amplification coefficient $(Re(\omega_0))_m$

$()_u$ refers to the unburned mixture

Superscripts

$()^*$ refers to the extinction regime $(H_0 = 1)$

$()^-$ or $()^+$ refer to the upstream and downstream outer zones, respectively

$(\hat{\ })$ refers to x-dependent part of perturbations

$(\bar{\ })$ refers to the steady planar case

REFERENCES

1. Spalding, D. B. *Proc. Roy. Soc. (Lond.)* A240:83 (1957).
2. Adler, J., *Combust. Flame* 7:39 (1963).
3. Bush, W. B., and Fendell, F. E., *J. Fluid Mech.* 56(1) (1971).
4. Liñan, A., *Acta Astronaut.* 1:1007 (1974).
5. Joulin, G., and Clavin, P., *Acta Astronaut.* 3:223 (1976); Stabilité d'Une Flamme Prémélangée Non Adiabatique, paper presented at 6th International Colloquium on Gasdynamics of Explosions and Reactive Systems, Stockholm, Sweden, August 22-26, 1977.
6. Buckmaster, J., *Combust. Flame* 26:151 (1976).
7. Landau, L. D., *Acta Physicochim. (USSR)* 19:77 (1944).
8. Darrieus, G., Propagation d'Un Front de Flamme. Essai de Théorie de Vitesse Arnomale de Deflagration par Développement Spontanné de la Turbulence, paper presented at 6th International Congress of Applied Mathematics, Paris, 1946.
9. Markstein, G. H., *Nonsteady Flame Propagation,* Pergamon Press, New York, 1964.
10. Eckhaus, W., *J. Fluid. Mech.* 10:80 (1961).
11. Sivashinsky, G. I., *Acta Astronaut.* 18:253 (1973). *Combust. Sci. Technol.* 15:137 (1977).
12. Lewis, B., and Von Elbe, G., *Combustion, Flames and Explosions of Gases,* Academic Press, New York, 1951.
13. Barenblatt, G. I., *Priklad. Mekh. Tekh. Fiz.* N2 (1962).
14. Williams, F. A., *Combustion Theory,* Addison-Wesley, Reading, Mass., 1965.
15. Joulin, G., Stabilité d'Une Flamme Adiabatique au Voisinage de la Stoechiométrie (Modèle Thermique-Diffusif), unpublished CNRS report, 1977-1978.
16. Clavin, P., and Williams, F. A., *J. Fluid Mech.* (in press).