Non-premixed flames - review

- Nonpremixed: no inherent propagation rate (unlike premixed flames where propagation rate = $S_L$)
- No inherent thickness $\delta$ (unlike premixed flames where thickness $\sim \alpha/S_L$) - in nonpremixed flames, determined by equating convection time $\delta/U = \Sigma^{-1}$ to diffusion time $\delta^2/\alpha \gg \delta \sim (\alpha/\Sigma)^{1/2}$
- Have to mix first then burn
- Burning occurs near stoichiometric contour where reactant fluxes in stoichiometric proportions (otherwise surplus of one reactant)
- Burning must occur near highest T since $\omega \sim \exp(-E/RT)$ is very sensitive to temperature (like premixed flames)
- Simplest approach: "mixed is burned", i.e. $\omega >> \Sigma$
- Stoichiometric mixture fraction $Z_{st} = \frac{1}{1+S}$ where $S = \frac{v_m M_{st} Y_{st,max}}{v_f M_f Y_{o,x,max}}$

$\nu = \text{stoichiometric coefficient}, M = \text{molecular weight}, Y = \text{mass fraction}$
Nonpremixed turbulent jet flames

- Laminar: $L_t \sim U_0 d_0^2/D$
- Turbulent (Hottel and Hawthorne, 1949)
  - $D \sim u'L_t; u' \sim U_0; L_i \sim d_0 \Rightarrow L_i \sim d_0$ (independent of Re)
  - High $U_0 \Rightarrow$ high $u' \Rightarrow$ Da small – flame "lifts off" near base (why base first?)
  - Still higher $U_0$ - more of flame lifted
  - When lift-off height = flame height, flame "blows off" (completely extinguished)
Scaling of turbulent jets

- Hinze (1975)
- Determine scaling of mean velocity ($\bar{u}$), jet width ($r_{\text{jet}}$) and mass flux through jet ($\bar{m}$) with distance from jet exit ($x$)
- Assume $u'(x) \sim \bar{u}(x), L_i(x) \sim r_{\text{jet}}(x)$
- Note that mass flux is not constant due to entrainment!

Conservation of momentum flux ($Q$):

$$ Q = \dot{m}\bar{u} = (\rho A_{\text{jet}})\bar{u} \sim \rho \bar{u}^2 r_{\text{jet}}^2 = \text{constant} \Rightarrow \bar{u} \sim (Q / \rho r_{\text{jet}}^2)^{1/2} $$

Kinetic energy flux ($K$) (not conserved - dissipated!)

$$ K = 1/2 \dot{m}\bar{u}^2 \sim (\rho A_{\text{jet}})\bar{u}^2 \sim \rho \bar{u}^3 r_{\text{jet}}^2 \sim Q\bar{u} \Rightarrow dK / dx = Q(d\bar{u} / dx) $$

KE dissipation

$$ \frac{dK}{dx} = \frac{dK}{dm} \frac{dm}{dx} \sim \frac{\bar{u}^3}{L_i} d(\rho \cdot \text{Volume}) \sim \frac{\rho \bar{u}^3}{r_{\text{jet}}} d\left(\pi r_{\text{jet}}^2 x\right) \sim \frac{\rho \bar{u}^3}{r_{\text{jet}}} r_{\text{jet}}^2 \sim \frac{Q\bar{u}}{r_{\text{jet}}} $$
Scaling of turbulent jets

Combining \( \frac{dK}{dx} \sim \overline{Q} \) and \( \frac{dK}{dx} \sim -\overline{Q} \frac{\rho}{r_{jet}} \), we get \( \overline{K} \sim \frac{\rho}{r_{jet}} \frac{d\overline{R}}{dx} \). Combining these with the previous page, we have \( \overline{K} \sim \sqrt{\frac{Q}{\rho r_{jet}^2}} = \sqrt{\frac{Q}{\rho}} \frac{1}{r_{jet}} \) and \( \frac{d\overline{R}}{dx} \sim \sqrt{\frac{Q}{\rho r_{jet}^2}} \).

Then \( \overline{R} \sim \frac{d\overline{R}}{dx} \frac{dr_{jet}}{dx} \) \( \Rightarrow \) \( \frac{Q}{\rho r_{jet}^2} \) \( \sim \) \( \left( \frac{Q}{\rho} \right)^{1/2} \) \( x \). From previous page \( \frac{\rho}{r_{jet}} \) \( \sim \) \( \sqrt{\frac{Q}{\rho r_{jet}^2}} \), thus \( \frac{d\overline{R}}{dx} \sim \sqrt{\frac{Q}{\rho r_{jet}^2}} \).

\( \Rightarrow r_{jet} \sim x \) (experimental: spread \( \theta = \pm 12^\circ \)).

\[ Q \sim \rho \overline{R}_j r_{jet}^2 = \text{constant} \Rightarrow \overline{R} \sim (Q / \rho)^{1/2} x^{-1} \] \( \Rightarrow \overline{R} = \rho \overline{R}_j r_{jet}^2 \sim \rho \overline{R}_j \sim (Q / \rho)^{1/2} x \).

\[ \text{Re}_L = \frac{u' L_j}{v} \sim \frac{u' \overline{R}_j r_{jet}^2}{v} \sim \left( \frac{Q}{\rho} \right)^{1/2} x \] \( \Rightarrow \text{Re}_L \sim \left( \frac{Q}{\rho} \right)^{1/2} x \) \( \sim \text{constant} \sim \frac{U_d}{v} \).

Mean strain \( \sim \frac{u'}{L_j} \sim \frac{u'}{L_j} \text{Re}_{L}^{-1/2} \sim \frac{U_d}{L_j} \text{Re}_{L}^{-1/2} \sim \frac{U_d}{v} \) \( \sim \left( \frac{Q}{\rho} \right)^{1/2} x \) \( \sim \text{constant} \sim \frac{U_d}{v} \).

Liftoff of turbulent jets

- Scaling suggests mean strain \( \sim \text{Re}_d 3/2 \upsilon x^2 \).
- For fixed fuel & ambient atmosphere (e.g. air), strain rate at flamelet extinction (\( \Sigma_{ext} \)) \( \sim \text{constant} \).
- Liftoff height \( (x_{LO}) \sim (\upsilon / \Sigma_{ext})^{1/2} \text{Re}_d 3/4 \sim u_d 3/4 \upsilon d_0^{3/4} \).
- Experiments - closer to \( x_{LO} \sim u_d 3/4 \upsilon d_0 \), but how to make dimensionless?

Need time scale, i.e. \( x_{LO} \sim u_0 3/4 \upsilon / \omega \), where \( \omega = \frac{S_r}{\alpha} = \frac{S_r}{\upsilon} \), leading to \( x_{LO} S_r / \upsilon \sim u_0 / S_r \).

**AME 513b - Spring 2020 - Lecture 7 - Turbulent nonpremixed flames**
**Liftoff of turbulent jets**

- Similar trend \((x_{LO} \sim u_0 d_{p} / \omega)\) seen in other fuels, scaled by highest \(S_L\) of fuel/air mixture and fuel density ratio function \(g\)
  
  \[
g(\bar{\rho}) = 0.04 + 0.46 \bar{\rho} + 0.5 \bar{\rho}^2; \bar{\rho} = \rho_{fuel} / \rho_{air}
\]

**Lifted turbulent flame theories (Lyons, 2007)**

- Premixed flame theory: \(S_L \approx \bar{u}\) at \(x = x_{LO}\) (≈ edge-flame theory)
- Critical scalar dissipation (Peters & Williams, 1983): \(\omega \approx \) mean turbulent strain at \(x = x_{LO}\)
- Turbulent flame theory: \(S_T \approx \bar{u}\) at \(x = x_{LO}\) (Kalghatgi, 1984)
  - Which \(S_T\)? Flamelet or distributed? Which model?
  - Need to satisfy 2 conditions
    - \(S_T \approx \bar{u}\)
    - Stoichiometric mixture (maximum \(S_T\))
    - Stabilization point may not be at jet centerline
- What are predictions? Homework problem!
EDGE FLAMES - motivation

- Laminar flamelet model used to describe local interaction of premixed & nonpremixed flames with turbulent flow
- Various regions of turbulent flow will be above/below critical extinction strain ($\Sigma_{ext}$)

Issues
- Can flame “holes” or “edges” persist?
- Will extinguishment in high-$\Sigma$ region spread to other regions?
- Will burning region re-ignite extinguished region?
- Will $\Sigma$ at edge locate ($\Sigma_{edge}$) be at higher or lower strain than extinction strain for uniform flame ($\Sigma_{ext}$)?
- How is edge propagation rate affected by $\Sigma$?
- Lewis number effects?

Edge flames

- Flame propagates from a burning region to a non-burning region, or retreats into the burning region
- Could be premixed or non-premixed

Non-premixed edge-flame in a counterflow

Kim et al., 2006
Edge flames - nonpremixed - theory

- Daou and Linan, 1998
- Configuration as in previous slide
- No thermal expansion
- Non-dimensional stretch rate $\varepsilon = (\Sigma \alpha / S_L)^{1/2}$ or Damköhler number $\sim S_L^{2/\Sigma \alpha} \sim \varepsilon^{-2}$
  - $S_L$ = steady unstretched $S_L$ of stoichiometric mixture of fuel & air streams

Effects of stretch

- Low $\varepsilon$: flame advances, "triple flame" structure - lean premixed, rich premixed & trailing non-premixed flame branches
- Intermediate $\varepsilon$: flame advances, but premixed branches weaker
- High $\varepsilon$: flame retreats, only nonpremixed branch survives
- Note that stretch rate corresponding to zero edge speed is always less than the extinction stretch rate for the uniform (edgeless, infinite) flame sheet - edge flames always weaker than uniform flames - retreat in the presence of less strain than required to extinguish uniform flame
- Le effects: as expected - lower Le yields higher edge speeds, broader extinction limits

**AME 513b - Spring 2020 - Lecture 7 - Turbulent nonpremixed flames**

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No heat loss

<table>
<thead>
<tr>
<th>$\kappa = 0, \varepsilon = 0.2$</th>
<th>$\omega_{\text{max}} = 0.76$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa = 0, \varepsilon = 1.2$</td>
<td>$\omega_{\text{max}} = 0.76$</td>
</tr>
<tr>
<td>$\kappa = 0, \varepsilon = 2.7$</td>
<td>$\omega_{\text{max}} = 0.76$</td>
</tr>
</tbody>
</table>

With heat loss

<table>
<thead>
<tr>
<th>$\kappa = 0.04, \varepsilon = 0.2$</th>
<th>$\omega_{\text{max}} = 0.56$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa = 0.04, \varepsilon = 1.2$</td>
<td>$\omega_{\text{max}} = 0.47$</td>
</tr>
<tr>
<td>$\kappa = 0.04, \varepsilon = 2.1$</td>
<td>$\omega_{\text{max}} = 0.63$</td>
</tr>
</tbody>
</table>

Daou et al., 2002

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**AME 513b - Spring 2020 - Lecture 7 - Turbulent nonpremixed flames**
Edge flames - nonpremixed - theory

- Daou and Linan, 1998 ($l_F = \beta(Le_{Fuel} - 1)$)
- Thermal expansion effect (Ruetsch et al., 1995): $U/S_L \sim (\rho_{wall}/\rho)^{1/2}$ with proportionality constant shown

Edge flames - nonpremixed - with heat loss

- Daou et al., 2002
- $\kappa = \text{dimensionless heat loss} = 7.5\beta/Pe^2; Pe = S_Ld/\alpha$ (see Cha & Ronney, 2006)
- With heat loss: trailing non-premixed branch disappears at low $\epsilon$ - nonpremixed flame extinguishes because mixing layer thickness $\sim (\alpha/\Sigma)^{1/2}$ (thus volume, thus heat loss) increases while burning rate decreases
Experiments - nonpremixed edge-flames

- Cha & Ronney (2006)
- Use parallel slots, not misaligned slots
- Use jet of $N_2$ to "erase" flame, then remove jet to watch advancing edge, or establish flame, then zap with $N_2$ jet to obtain retreating flame

Edge flames - nonpremixed - experiment

Propagating

Retreating

"Tailless"
Edge flames - nonpremixed - experiment

- Behavior very similar to model predictions with heat loss - negative at low or high $\varepsilon$, positive at intermediate $\varepsilon$ with $U_{edge}/S_L \approx 0.7(\rho_{fuel}/\rho_{fuel})^{1/2}$ (prediction: $\approx 1.8$)
- Propagation rates $\approx$ same for different gaps when scaled by strain rate $\Sigma$ except at low $\Sigma$ (thus low $V_{rel}$) where smaller gap $\Rightarrow$ more heat loss (higher $\kappa$)

![Graph showing behavior of $U_{edge}/S_L$ vs. $\varepsilon$ for different $d$ values.](image)

**Dimensional $U_{edge}$ vs. stretch ($\Sigma$) can be mapped into scaled $U_{edge}$ vs. scaled stretch rate ($\varepsilon$) for varying mixtures.**
- Little effect of mixture strength when results scaled by $\varepsilon$, except at low $\Sigma$, where weaker mixture $\Rightarrow$ lower $S_L$ $\Rightarrow$ higher $\kappa$

![Graph showing dimensional and dimensionless $U_{edge}$ vs. $\varepsilon$ for different $d$ values.](image)

Numbers refer to molar $N_2$ dilution when fuel and oxidant streams are mixed in stoichiometric proportions ($CH_4/O_2:N_2 = 1:2:1$).
Edge flames - nonpremixed - experiment

- High stretch ($\varepsilon$) or heat loss ($\kappa$) causes extinction - experiments surprising consistent with experiments, even quantitatively
- Very difficult to see "tailless" flames in experiments
  - Model assumes volumetric heat loss (e.g., radiation) - mixing layer thickness can increase indefinitely, get weak non-premixed branch that can extinguish when premixed branches do not
  - Experiment - heat loss mainly due to conduction to jet exits, mixing layer thickness limited by jet spacing

Experiment (Cha & Ronney, 2006)  
Prediction (Daou et al., 2002)

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Edge flames - nonpremixed - experiment

- Lewis number effects VERY pronounced - at low $Le$, MUCH higher $U_{edge}$ than mixture with same $S_L$ (thus $\kappa$) with $Le \approx 1$
- Opposite trend for $Le > 1$

Low $Le$ (CH$_4$-O$_2$-CO$_2$)  
($Le_{fuel} = 0.74; Le_{O2} = 0.86$)

High $Le$ (C$_3$H$_8$-O$_2$-N$_2$)  
($Le_{fuel} = 1.86; Le_{O2} = 1.05$)
U̇_edge not at all symmetric with respect to Z_{st} = 0.5!
- Due to shift in reaction zone (radical production) from O_2 to fuel side as Z_{st} increases; radicals don’t need to “swim upstream” & cross stagnation plane to reach and decompose O_2
- Lewis number effects for large fuel molecules – non-monotonic trend
- More easily decomposed fuels (n-butane vs. iso-butane) have smaller fragments, lower effective Le, higher U̇_edge

Note strain rate Σ = σ here! Song et al., 2006

Low Le, high Z_{st}: diffusive-thermal (cellular) instability (Chen et al., 1992; Kim et al., 1996)

What happens in H_2-O_2-N_2 edge-flames? High Z_{st} should have higher U̇_edge due to Le effects; is there also a chemical effect?

Zhou et al., 2019
Strong Le effects on strained flames cause unreasonably large $U_{\text{edge}}$ & $\sigma_{\text{ext}}$

- $S_{\text{i}}$ hardly changes with Le, but $\sigma_{\text{ext}}$ sure does
- How to incorporate Le effects? Use $\varepsilon = \sigma_{\text{ext}} / S_{\text{L,eff}} = (\alpha \sigma_{\text{ext}})^{1/2}$

$L = \frac{U_{\text{edge}} / S_{\text{L,eff}}}{U_{\text{edge}} \sqrt{\alpha \sigma_{\text{ext}}}}$

**Laminar burning velocity**

- Low and intermediate $Z_{\text{st}}$ show monotonically decreasing with increasing $\varepsilon$, but high $Z_{\text{st}}$ shows opposite trend due to lower effective Le
- Proposed scaling with $\sigma_{\text{ext}}$ works – all $Z_{\text{st}}$ (thus all Le) show similar trends

Zhou et al., 2019
Mode I: advancing continuous flame
Mode II: retreating continuous flame
Mode III: advancing broken flame
Mode IV: stationary broken flame

Bizarre bifurcation point at $Z_{st} \approx 0.6$, $\varepsilon \approx 1$

5 possible outcomes depending on the "compass direction"!

Edge flames – nonpremixed – low Le

Computed reaction rate contours for an advancing broken flame with $Le=0.33$
(Thatcher & Dold, CTM 1999)

Zhou et al., 2019
With increasing dilution or strain, broken flamelets become more sparse and edge-flame changes from propagating (Mode III) to stationary (Mode IV).

\[
Z_{st} = 0.80 \quad \sigma = 100/s \quad H_2/ O_2/ N_2 = 2/1/18
\]

\[
Z_{st} = 0.80 \quad \sigma = 100/s \quad H_2/ O_2/ N_2 = 2/1/18.5
\]

\[
Z_{st} = 0.80 \quad \sigma = 100/s \quad H_2/ O_2/ N_2 = 2/1/19
\]

Zhou et al., 2019

Computation 1D extinction limits for continuous flames are similar to experiments.

Bifurcation at \(Z_{st} \approx 0.6\) allows highly-strained flames at higher \(Z_{st}\) & \(\sigma\) to exist as broken flames only.

Limit conditions for continuous flames is same as transition to broken flames.

Flames break only if they have no other choice for survival!

Zhou et al., 2019
**Edge flames - premixed - theory**

- Similar response of $U_{\text{edge}}$ to stretch as nonpremixed flames
- Again larger $U_{\text{edge}}$ for low $Le$
- Again 2nd extinction limit at low strain rates when heat losses present

![Graph](image)

Daou & Linan, CNF 1999  
Daou et al., CTM 2003

**Edge flames – twin premixed - images**

- Lewis number effects very evident – curvature effects on leading edge

![Images](image)

Clayton et al., 2019
Edge flames – twin premixed - $U_{\text{edge}}$

- Propagation rates scale with $(\rho_f/\rho_b)^{1/2}$ not $(\rho_f/\rho_b)^{1/2}$
- Again 2 extinction limits, high & low strain rates

Lewis number effects similar to nonpremixed edge flames

Clayton et al., 2019
Regimes of behavior similar to nonpremixed edge flames

![Graph showing non-dimensional flame thickness vs heat loss factor with regimes of behavior marked: advancing, retreating, extinction, tailless flames.]

**Experiments**
Clayton et al., 2019

**Model predictions**
Daou et al., CTM 2003

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**Edge flames – single premixed - images**

- **CH₄-Air, Le = 1, d = 7.5 mm**
  - $\sigma = 18.7 \, s^{-1}$
  - $\sigma = 80 \, s^{-1}$
  - $\sigma = 160 \, s^{-1}$

- **CH₄-O₂-CO₂, Le < 1, d = 5 mm**
  - $\sigma = 16 \, s^{-1}$
  - $\sigma = 60 \, s^{-1}$
  - $\sigma = 160 \, s^{-1}$

- **C₃H₈-Air, Le > 1, d = 7 mm**
  - $\sigma = 28.6 \, s^{-1}$
  - $\sigma = 50 \, s^{-1}$
  - $\sigma = 78.6 \, s^{-1}$

Clayton et al., 2019
**References**

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