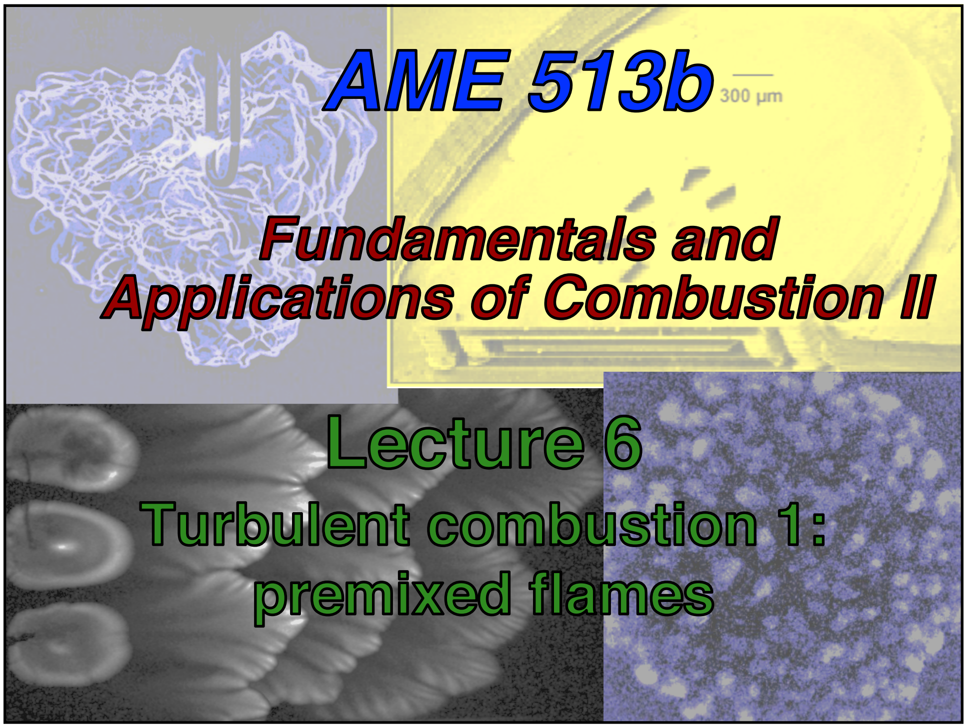

AME 513b

Fundamentals and Applications of Combustion II

Lecture 6

Turbulent combustion 1: premixed flames

•1

Turbulent combustion (Lecture 1)

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- Motivation
- Basics of turbulence
- Premixed-gas flames
  - Turbulent burning velocity
  - Regimes of turbulent combustion
  - Flamelet models
  - Non-flamelet models
  - Flame quenching via turbulence
  - Case study I: "Liquid flames" (turbulence without thermal expansion)
  - Case study II: Flames in Hele-Shaw cells (thermal expansion without turbulence)

•2

Motivation

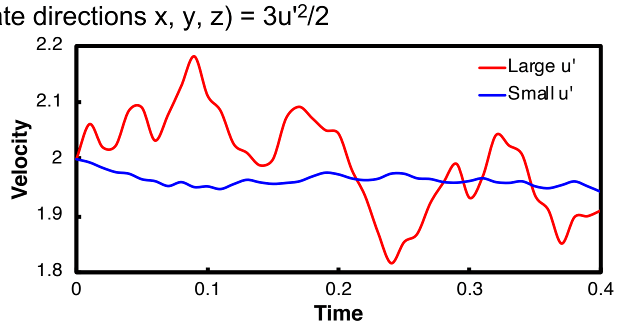
- Almost all flames used in practical combustion devices are turbulent because turbulent mixing increases burning rates, allowing more power/volume
- Even without forced turbulence, if the Rayleigh number $gd^3/\alpha\nu$ is larger than about 10^6 ($g = 10^3$ cm/s^2, $\alpha \approx \nu \approx 1$ cm^2/s $\Rightarrow d > 10$ cm), turbulent flow will exist due to buoyancy
- Examples
  - Premixed turbulent flames
    - » Gasoline-type (spark ignition, premixed-charge) internal combustion engines
    - » Stationary gas turbines (used for power generation, not propulsion)
  - Nonpremixed flames
    - » Diesel-type (compression ignition, nonpremixed-charge) internal combustion engines
    - » Gas turbines
    - » Most industrial boilers and furnaces

Basics of turbulence

- Very old but very good reference: Tennekes: "A First Course in Turbulence" (ISBN-10: 0262200198)
- Job 1: need a measure of the strength of turbulence
- Define turbulence intensity (u') as rms fluctuation of instantaneous velocity u(t) about mean velocity ($\overline{u}$)

$$(u')^2 \equiv \lim_{T\to\infty}\left[\frac{1}{T}\int_0^T \left(u(t)-\overline{u}\right)^2 dt\right];\quad \overline{u} \equiv \lim_{T\to\infty}\left[\frac{1}{T}\int_0^T u(t)dt\right]$$

- Kinetic energy of turbulence = mass*u'^{2}/2; KE per unit mass (total in all 3 coordinate directions x, y, z) = 3u'^{2}/2



Basics of turbulence

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- Job 2: need a measure of the length scale of turbulence
- Define integral length scale (L_I) as
  - A measure of size of largest eddies
  - Largest scale over which velocities are correlated
  - Typically related to size of system (tube or jet diameter, grid spacing, …)

$$L_I(x) \equiv \int_0^\infty A(x,r)dr; \;\; A(x,r) \equiv \frac{\overline{[u(x)-\bar{u}][u(x+r)-\bar{u}]}}{u'(x)u'(x+r)}$$

Here the overbars denote spatial (not temporal) averages

  - A(r) is the autocorrelation function at some time t
  - Note A(0) = 1 (fluctuations around the mean are perfectly correlated at a point)
  - Note A(∞) = 0 (fluctuations around the mean are perfectly uncorrelated if the two points are very distant)
  - For truly random process, A(r) is an exponentially decaying function $A(r) = \exp(-r/L_I)$

Basics of turbulence

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- In real experiments, generally know u(t) not u(x) - can define time autocorrelation function $A(x,\tau)$ and integral time scale τ_I at a point x

$$\tau_I(x) \equiv \int_0^\infty A(x,\tau)d\tau; \;\; A(x,\tau) \equiv \frac{\overline{[u(x,t)-\bar{u}(x)][u(x,t+\tau)-\bar{u}(x)]}}{u'(x)^2}$$

Here the overbars denote temporal (not spatial) averages

- With suitable assumptions $L_I = (8/\pi)^{1/2} u'\tau_I$
- Define integral scale Reynolds number $Re_L \equiv u'L_I/\nu$ (ν = kinematic viscosity)
- Note generally $Re_L \neq Re_{flow} = Ud/\nu$; typically $u' \approx 0.1U$, $L_I \approx 0.5d$, thus $Re_L \approx 0.05\, Re_{flow}$

Basics idea of turbulence

- Large scales
  - Viscosity is unimportant (high Re_L) - inertial range of turbulence
  - Through nonlinear interactions of large-scale features (e.g. eddies), turbulent kinetic energy cascades down to smaller scales
  - Interaction is NOT like linear waves: (e.g. waves on a string) that can pass through each other without changing or interacting in an irreversible way $$\frac{\partial^2 a}{\partial t^2} = c^2 \frac{\partial^2 a}{\partial x^2}$$
  - Interaction is described by inviscid Navier-Stokes equation + continuity equation

$$\frac{\partial(\rho\vec{u})}{\partial t} + \boxed{(\rho\vec{u}\cdot\vec{\nabla})\vec{u}} = -\vec{\nabla}P \qquad \frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u}) = 0$$

nonlinear term

- Smaller scales: viscosity dissipates KE generated at larger scales
- Steady-state: rate of energy generation at large scales = rate of dissipation at small scales
- Example: stir water in cup at large scale, but look later and see small scales - where did they come from???

Energy dissipation

- Why is energy dissipated at small scales?
  - Viscous term in NS equation ($\nu\nabla^2 u$) at scale x ~ $\nu u'/(x)^2$
  - Convective term $(u\cdot\nabla)u \sim (u')^2/x$
  - Ratio convective / viscous ~ $u'x/\nu = Re_x$
  - Viscosity more important as Re_x (thus x) decreases
- Define energy dissipation rate (ε) as energy dissipated per unit mass $(u')^2$ per unit time (τ_l) by the small scales
  - $\varepsilon \sim (u')^2/\tau_l \sim (u')^2/(L_l/u') = C(u')^3/L_l$ (units Watts/kg = m^2/s^3)
  - With suitable assumptions $C \approx 3.1$
- Note to obtain $u'/S_L = 10$ for stoichiometric hydrocarbon-air mixture (S_L = 40 cm/s) with typical L_l = 5 cm, $\varepsilon \approx 4000$ W/kg
- Is this a lot of power or energy?
  - Turbulent flame speed $S_T \approx u'$
  - Across distance L_l, heat release per unit mass is $Y_f Q_R$, time is L_l/S_T
  - Heat release per unit mass per unit time $\dot{Q} = Y_f Q_R S_T / L_l$ $\approx (0.068)(4.5 \times 10^7$ J/kg$)(4$ m/s$)/(0.05$ m$) \approx 2.4 \times 10^8$ W/kg
  - Dissipation rate / heat generation rate $\varepsilon/\dot{Q} \approx 1.6 \times 10^{-5}$
  - Turbulent kinetic energy / thermal energy = $(3/2\, u'^2)/Y_f Q_R \approx 8 \times 10^{-6}$!
  - Answer: NO, very little power or energy

Kolmogorov universality hypothesis (1941)

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- *In inertial range of scales, the kinetic energy* (E) *per unit mass per wavenumber* (k) (k = 2π/λ, λ = wavelength) *depends only on k and ε*
- Use dimensional analysis, E = $(m/s)^2/(1/m) = m^3/s^2$, k = m^{-1}
- E(k) ~ $\varepsilon^a k^b$ or $m^3/s^2 \sim (m^2/s^3)^a(1/m)^b \Rightarrow a = 2/3, b = -5/3$
- E(k) = $C\varepsilon^{2/3}k^{-5/3}$ (constant C from experiments or numerical simulations) (e.g. C = 1.606, Yakhot and Orzag, 1986)
- Total kinetic energy (K) over a wave number range ($k_1 < k < k_2$)

$$K = \int_{k_1}^{k_2} E(k)dk = \int_{k_1}^{k_2} 1.606\varepsilon^{2/3}k^{-5/3}dk = 2.409\varepsilon^{2/3}(k_1^{-2/3} - k_2^{-2/3})$$

- Characteristic intensity at scale L = u'(k) = $(2K/3)^{1/2} = 0.687\varepsilon^{1/3}L^{1/3}$
- Cascade from large to small scales (small to large k) ends at scale where dissipation is important
  - Smallest scale in the turbulent flow
  - This scale (Kolmogorov scale) can depend only on ε and ν
  - Dimensional analysis: wave number $k_{Kolmogorov} \sim (\varepsilon/\nu^3)^{1/4}$ or length scale $L_K = C(\nu^3/\varepsilon)^{1/4}$, C ≈ 11
  - $L_K/L_I = C(\nu^3/\varepsilon)^{1/4}/L_I = C(\nu^3/(3.1u'^3/L_I))^{1/4}/L_I = 14\ (u'L_I/\nu)^{-3/4} = 14\ Re_L^{-3/4}$
  - Time scale $t_K \sim L_K/u'(L_K) \sim (\nu/\varepsilon)^{1/2}$

•9

Kolmogorov universality hypothesis (1941)

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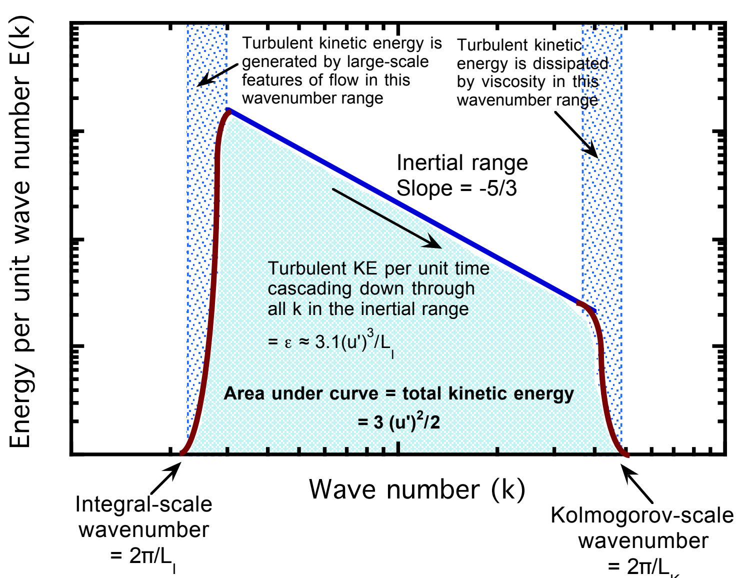

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Application of Kolmogorov scaling

- Taylor scale (L_T): scale at which average strain rate occurs
  - $L_T \equiv$ u'/(mean strain)
  - $L_T \sim u'/(t_K)^{-1} \sim u'/((\nu/\varepsilon)^{1/2})^{-1} \sim u'\nu^{1/2}/(u'^3/L_I)^{1/2} \sim L_I Re_L^{-1/2}$
  - Mean turbulent strain rate $\Sigma_T \sim 1/t_K \sim (\varepsilon/\nu)^{1/2} \sim (u'/L_I)Re_L^{1/2}$
  - With suitable assumptions $\Sigma_T \approx 0.157(u'/L_I)Re_L^{1/2}$
- Turbulent viscosity ν_T
  - Molecular gas dynamics: $\nu \sim$ (velocity of particles)(length particles travel before changing direction)
  - By analogy, at scale k, $\nu_T(k) \sim u'(k)(1/k)$
  - Total effect of turbulence on viscosity: at scale L_I, $\nu_T \sim u'L_I$
  - Usually written in the form $\nu_T = CK^2/\varepsilon$, $C \approx 0.084$, thus $\nu_T/\nu = 0.061\ Re_L$
- Turbulent thermal diffusivity $\alpha_T = \nu_T/Pr_T$
  - Pr changes from molecular value (gases ≈ 0.7, liquids 10 - 10,000; liquid metals 0.05) at $Re_L = 0$ to $Pr_T \approx 0.72$ as $Re_L \to \infty$ (Yakhot & Orzag, 1986)
  - Results in $\alpha_T/\alpha = 0.061\ Re_L$ as $Re_L \to \infty$ (similar to viscosity)

Reynolds-Averaged Navier-Stokes (RANS)

- How to model if Direct Numerical Simulation (DNS) too costly?
- Many approaches, most popular is RANS – starting with Navier-Stokes equations and averaging over time:

$$\text{Continuity: } \frac{\partial\rho}{\partial t}+\frac{\partial(\rho u_x)}{\partial x}+\frac{\partial(\rho u_y)}{\partial y}=0;\ \text{x-momentum: } \frac{\partial(\rho u_x)}{\partial t}+u_x\frac{\partial(\rho u_x)}{\partial x}+u_y\frac{\partial(\rho u_x)}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$\text{Mean and flucutating components: } u_x=\bar{u}_x+u'_x(t); u_y=\bar{u}_y+u'_y(t); \rho=\bar{\rho}+\rho'(t); P=\bar{P}+P'(t)$$

$$\text{Continuity: } \frac{\partial(\bar{\rho}+\rho')}{\partial t}+\frac{\partial(\rho\bar{u}_x)}{\partial x}+\frac{\partial(\rho u'_x)}{\partial x}+\frac{\partial(\rho\bar{u}_y)}{\partial y}+\frac{\partial(\rho u'_y)}{\partial y}=0$$

$$\langle\text{Time average}\rangle:\ \langle\bar{u}_x\rangle=\bar{u}_x; \langle u'_x\rangle=0 \text{ etc.} \Rightarrow \frac{\partial\bar{\rho}}{\partial t}+\frac{\partial(\rho\bar{u}_x)}{\partial x}+\frac{\partial(\rho\bar{u}_y)}{\partial y}=0$$

- same as non-averaged since linear

$$\text{x-momentum, } \rho=\text{const: } \frac{\partial\bar{u}_x}{\partial t}+\bar{u}_x\frac{\partial\bar{u}_x}{\partial x}+\bar{u}_y\frac{\partial\bar{u}_x}{\partial y}=-\frac{1}{\rho}\frac{\partial\bar{P}}{\partial x}-\frac{\partial}{\partial x}\left(\langle u'_x u'_x\rangle+\langle u'_x u'_y\rangle\right)$$

- nonlinear convection terms generate new viscous-like terms: ***Reynolds Stresses***
- 3 terms in 2D, 6 terms in 3D
- Special case - isotropic turbulence: $k-\varepsilon$
- Must be modeled: **closure problem**

Turbulent premixed combustion - motivation

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- Study of premixed turbulent combustion is important because
  - Turbulence increases mean flame propagation rate (S_T) compared to S_L
  - If this trend increased *ad infinitum*, arbitrarily lean mixtures (low S_L) could be burned arbitrarily fast by using sufficiently high u', but too high u' leads to extinction - nixes that idea
  - Theories don't agree with experiments nor with each other
  - Direct numerical simulations difficult at high u'/S_L (>> 1)
- Applications to
  - Automotive engines
  - Gas turbine designs - possible improvements in emission characteristics with lean premixed combustion compared to non-premixed combustion (remember why?)

•13

Turbulent burning velocity

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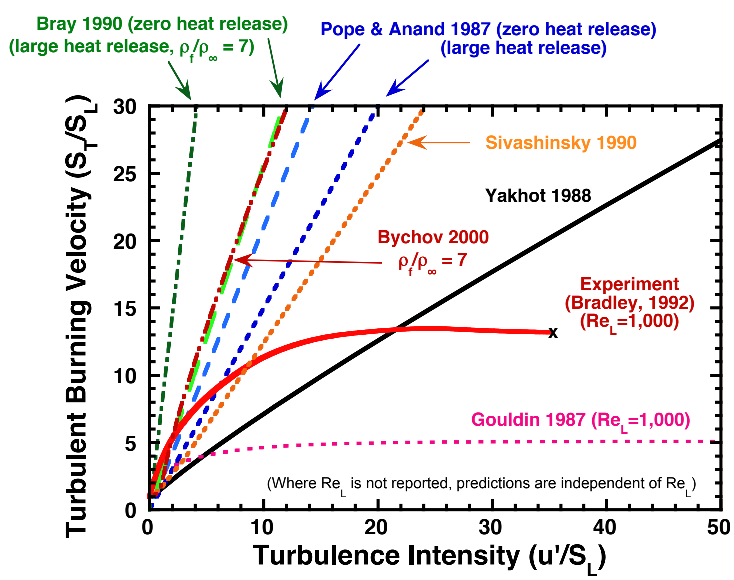

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Turbulent burning velocity

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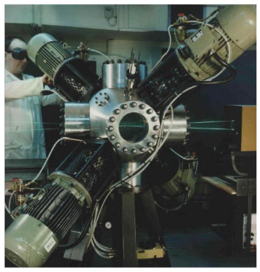
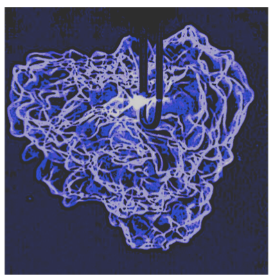
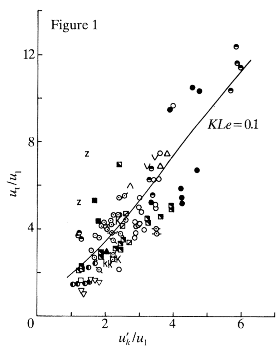
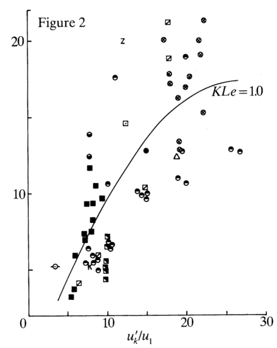
- Experimental results compiled by Bradley et al. (1992) - very smoothed data from many sources, e.g. fan-stirred bomb









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Turbulent burning velocity

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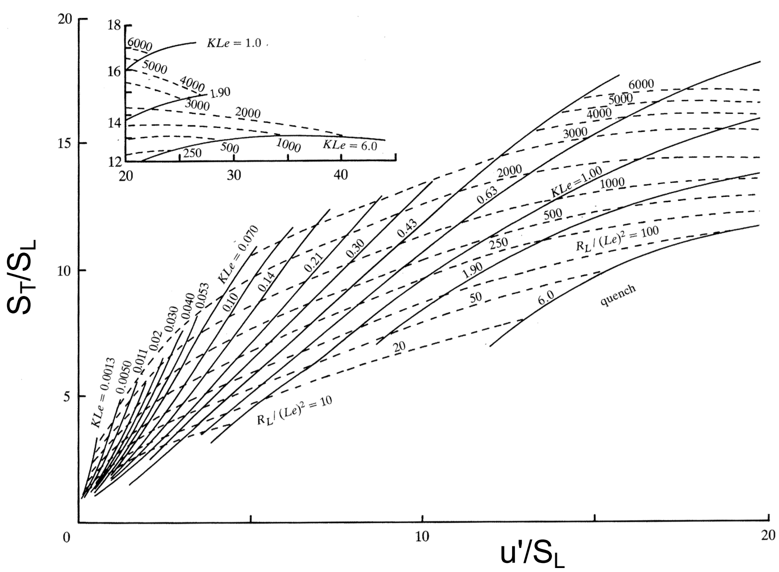
Bradley et al. (1992)



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Characteristics of turbulent flames

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- Most models based on physical description of Damköhler (1940)
- Behavior depends on Karlovitz number (Ka)

$$Ka \equiv \frac{\text{Mean strain rate}}{\text{Mean chemical rate } (\omega)} \approx \frac{\Sigma}{S_L^2/\alpha} = \frac{\Sigma\alpha}{S_L^2}$$

- For Kolmogorov turbulence

$$Ka_T = \frac{\Sigma\alpha}{S_L^2} = \frac{0.157\left(u'/L_I\right)\sqrt{\mathrm{Re}_L}}{S_L^2}\frac{u'^2}{u'^2}\frac{\alpha}{\nu}\nu = 0.157\,\mathrm{Re}_L^{-1/2}\left(\frac{u'}{S_L}\right)^2$$

(note this assumes $\alpha/\nu = 1/Pr \approx 1$, OK for gases but not liquids!)

  - Alternatively, $Ka_T = (\delta/L_K)^2 \sim (S_L/\alpha)/(\mathrm{Re}_L^{-3/4}L_I)^2 \sim \mathrm{Re}_L^{-1/2}(u'/S_L)^2$
    - same scaling results based on length scales
- Sometimes Damköhler number (Da) is reported instead - two definitions, based on integral or Taylor time scale
  - $Da_1 = \omega\,\tau_L \sim (S_L^2/\alpha)(L_I/u') \sim \mathrm{Re}_L(u'/S_L)^{-2}$
  - $Da_2 = \omega\,\tau_{Taylor} \sim (S_L^2/\alpha)(\mathrm{Re}_L^{-1/2}L_I/u') \sim \mathrm{Re}_L^{1/2}(u'/S_L)^{-2} \sim 1/Ka_T$

Regimes of turbulent combustion

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- $\mathrm{Re}_L < 1$: laminar flames
- Ka < 1: "Huygens propagation," thin fronts wrinkled by turbulence but internal structure of laminar flame unchanged
  - $u'/S_L < 1$: weakly wrinkled laminar flamelets
  - $u'/S_L > 1$: strongly "corrugated" flamelets
- $Ka > \mathrm{Re}^{1/2}$: "Distributed reaction zones" – all scales of turbulence fit inside flame front – when turbulent flame thickness $\delta_T > L_I$

$$\frac{\delta_T}{L_I} \sim \frac{\frac{\alpha_T}{S_T}}{L_I} \sim \frac{\frac{\alpha_T}{\sqrt{\alpha_T\omega_T}}}{L_I} \sim \frac{\frac{\alpha_T}{\sqrt{\alpha_T\left(S_L^2/\alpha\right)}}}{L_I} \sim \sqrt{\frac{\alpha_T}{\alpha}}\frac{\alpha}{S_L L_I} \sim \sqrt{\mathrm{Re}_L}\frac{u'}{S_L}\frac{\nu}{u' L_I}$$

$$\sim \frac{1}{\sqrt{\mathrm{Re}_L}}\frac{u'}{S_L} \sim \left(\frac{1}{\mathrm{Re}_L^{1/4}}\frac{u'}{S_L}\right)\frac{1}{\mathrm{Re}_L^{1/4}} \sim \frac{\sqrt{Ka}}{\mathrm{Re}_L^{1/4}};\quad \delta_T > L_I \Rightarrow Ka > C\sqrt{\mathrm{Re}_L}$$

Regimes of turbulent combustion

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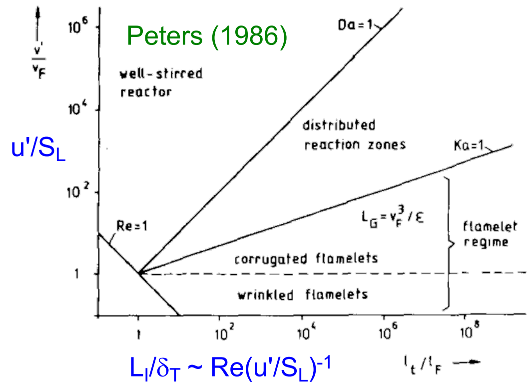
- "Distributed reaction zones" often erroneously called "well stirred reactor" which is an apparatus regime (where δ_T > apparatus size) not a combustion regime; if apparatus is large enough, a front will always exist (δ_T < apparatus size)
- 1 < Ka < $Re^{1/2}$: "Broadened" flames (Ronney & Yakhot, 1992) or "thin reaction zones" (Peters, 1999)
  - Smaller scales fit within "broadened" flame front
  - Larger scales wrinkle "broadened" front
- "Borghi diagram" – regimes of behavior



Characteristics of turbulent flames

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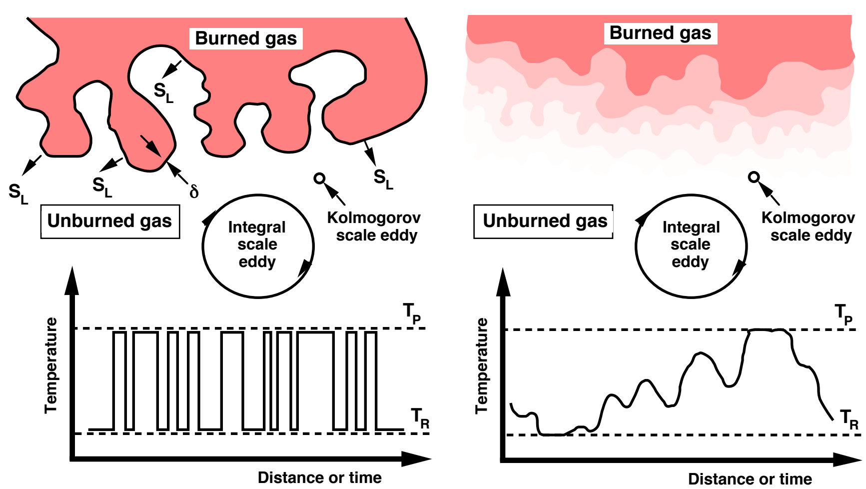

"Thin" flame (Re » 1, Ka « 1)

"Distributed" flame (Re » 1, Ka » 1)

Improved (?) Borghi diagram

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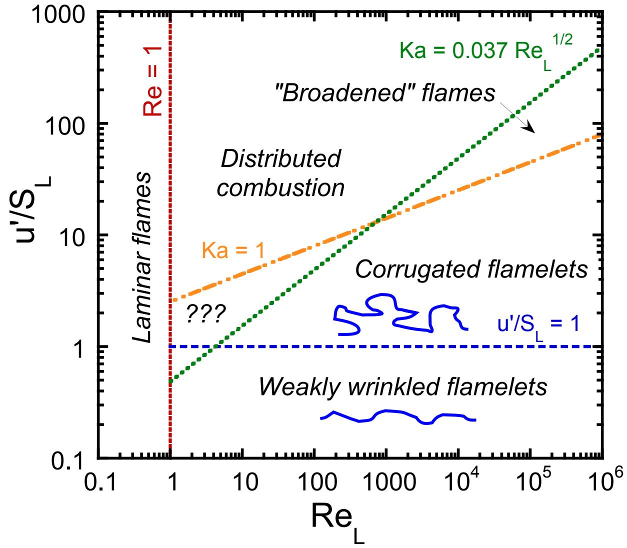

Turbulent combustion regimes

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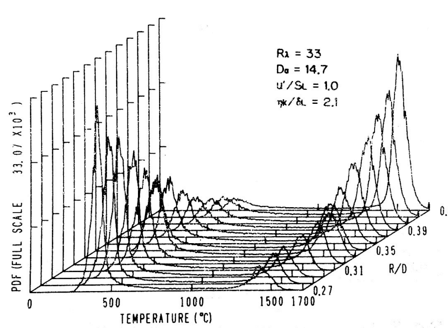
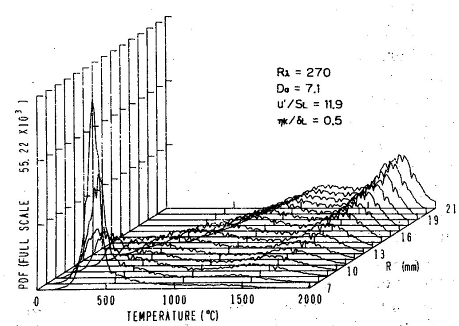
- Comparison of flamelet and distributed combustion (Yoshida, 1988)





Flamelet: temperature is either T_∞ or T_{ad}, never between, and probability of product increases downstream (towards the rear of the above figure)

Distributed: significant probability of temperatures between T_∞ or T_{ad}, probability of intermediate T peaks in middle of flame

Modeling of premixed turbulent flames

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- Most model employ assumptions not satisfied by real flames, e.g.
  - Adiabatic (sometimes ok)
  - Homogeneous, isotropic turbulence over many L_I (never ok)
  - Low Ka or high Da (thin fronts) (sometimes ok)
  - Lewis number = 1 (sometimes ok, e.g. CH_4-air)
  - Constant transport properties (never ok, ≈ 25x increase in ν and α across front!)
  - u' doesn't change across front (never ok, thermal expansion across flame generates turbulence) (but viscosity increases across front, decreases turbulence, effects sometimes almost cancel out)
  - Constant density (never ok!)

Flamelet modeling

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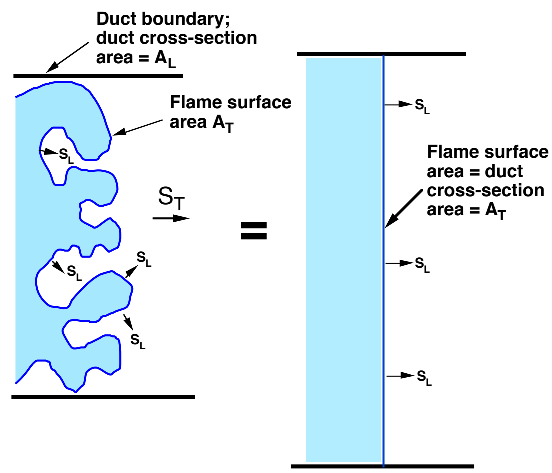
- Damköhler (1940): in Huygens propagation regime, flame front is wrinkled by turbulence but internal structure and S_L are unchanged
- Propagation rate S_T due only to area increase via wrinkling:

  $S_T/S_L = A_T/A_L$



Flamelet modeling

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- Low u'/S_L: weakly wrinkled flames
  - $S_T/S_L = 1 + (u'/S_L)^2$ (Clavin & Williams, 1979) - standard for many years
  - Actually Kerstein and Ashurst (1994) showed this is valid only for periodic flows - for random flows $S_T/S_L - 1 \sim (u'/S_L)^{4/3}$
- Higher u'/S_L: strongly wrinkled flames
  - Schelkin (1947) - A_T/A_L estimated from ratio of cone surface area to base area; height of cone ~ u'/S_L; result

$$S_T/S_L \approx \sqrt{1+\left(2u'/S_L\right)^2} \approx \sqrt{2}\left(u'/S_L\right) \text{ at high } u'/S_L$$

  - Other models based on fractals, probability-density functions, etc., but mostly predict $S_T/S_L \sim u'/S_L$ at high u'/S_L with the possibility of "bending" or quenching at sufficiently high Ka ~ $(u'/S_L)^2$

Flamelet modeling

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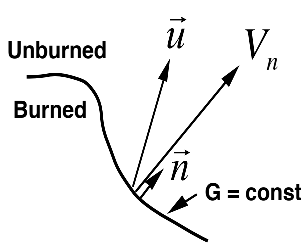
- Many models based on scalar kinematic "G-equation" (Kerstein et al. 1987) – looks like typical convection equation with weird source term
- Any curve of G = C is a flame front (G < C unburned; G > C burned)
- Flame advances by self-propagation and advances or retreats by convection

$$\vec{n} = \text{unit normal to front} = \frac{-\left(\frac{\partial G}{\partial x}\hat{i} + \frac{\partial G}{\partial y}\hat{j}\right)}{\sqrt{\left(\frac{\partial G}{\partial x}\right)^2 + \left(\frac{\partial G}{\partial y}\right)^2}} = \frac{-\nabla G}{\left|\vec{\nabla} G\right|}$$



$$V_n = \text{flame speed in lab frame} = \frac{\partial G/\partial t}{\left|\vec{\nabla} G\right|}$$

$$V_n - \vec{u} \bullet \vec{n} = \text{flame speed in lab frame - flow speed} = S_L$$

$$\frac{\partial G/\partial t}{\left|\vec{\nabla} G\right|} + \frac{\vec{u} \bullet \vec{\nabla} G}{\left|\vec{\nabla} G\right|} = S_L \text{ or } \boxed{\frac{\partial G}{\partial t} + \vec{u} \bullet \vec{\nabla} G = S_L \left|\vec{\nabla} G\right|}$$

Flamelet modeling

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- G-equation formulation, renormalization group (Yakhot, 1988) (successive scale elimination); invalid (but instructive) derivation:

$$S_T(1)/S_L = 1+\left(u'(1)/S_L\right)^2 \text{ at smallest scale}$$

$$S_T(2)/S_T(1) = 1+\left(u'(2)/S_T(1)\right)^2 \text{ at next scale, where } S_T(1) \text{ is substituted for } S_L(2)$$

$$S_T(i+1)/S_T(i) = 1+\left(u'(i+1)/S_T(i)\right)^2 \text{ in general}$$

$$S_T(n)/S_L = \left(S_T(1)/S_L\right)\left(S_T(2)/S_T(1)\right)...\left(S_T(n)/S_T(n-1)\right)$$

$$S_T(n)/S_L = \left(1+\left(u'(1)/S_L\right)^2\right)\left(1+\left(u'(2)/S_T(1)\right)^2\right)...\left(1+\left(u'(n)/S_T(n-1)\right)^2\right)$$

$$S_T(n)/S_L = \exp\left[\ln\left(1+\left(u'(1)/S_L\right)^2\right)\right]\exp\left[\ln\left(1+\left(u'(2)/S_T(1)\right)^2\right)\right]...$$

$$\text{Since } \ln(1+\varepsilon) \approx \varepsilon,\ S_T(n)/S_L = \exp\left[\left(u'(1)/S_L\right)^2\right]\exp\left[\left(u'(2)/S_T(1)\right)^2\right]...$$

$$\text{Eventually } \frac{S_T}{S_L} \approx \exp\left(\frac{\left(u'/S_L\right)^2}{\left(S_T/S_L\right)^2}\right) \approx \frac{\left(u'/S_L\right)}{\sqrt{\ln\left(u'/S_L\right)}} \text{ at high } u'/S_L$$

Fractal models of flames

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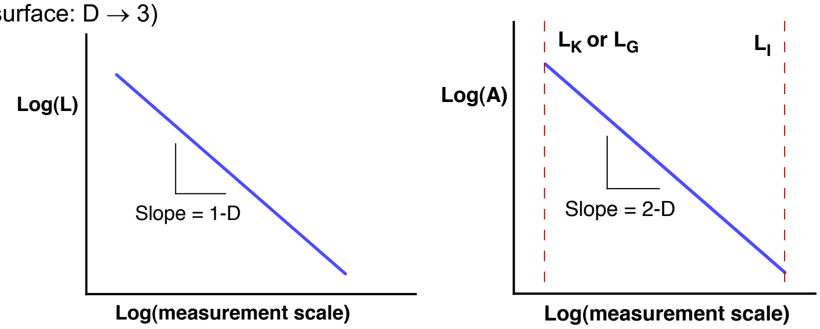
- What is the length (L) is the coastline of Great Britain? Depends on measurement scale: L(10 miles) < L(1 mile)
- Plot log(L) vs. log(measurement scale); if it's a straight line, it's a fractal with dimension (D) = 1 - slope
- If object is very smooth (e.g. circle), D = 1
- If object has self-similar wrinkling on many scales, D > 1 - expected for flames since Kolmogorov turbulence exhibits scale-invariant properties
- If object is very rough, D $\rightarrow$ 2
- If object 3D, everything is same except length $\rightarrow$ area, D $\rightarrow$ D + 1 (space-filling surface: D $\rightarrow$ 3)



Generic 2D object   3D flame surface

Fractal models of flames

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- Application to flames (Gouldin, 1987)
  - Inner (small-scale) and outer (large-scale) limits ("cutoff scales")
  - Outer: L_I; inner: L_K or L_G (no clear winner…)
  - L_G = "Gibson scale," where $u'(L_G) = S_L$
  - Recall $u'(L) \sim \varepsilon^{1/3}L^{1/3}$, thus $L_G \sim S_L{}^3/\varepsilon$
  - Since $\varepsilon \sim u'^3/L_I$ thus $L_G \sim L_I(S_L/u')^3$
- $S_T/S_L = A_T/A_L = A(L_{inner})/A(L_{outer})$
- For 3-D object, $A(L_{inner})/A(L_{outer}) = (L_{outer}/L_{inner})^{2-D}$
- What is D? Kerstein (1988) says it should be 7/3 for a flame in Kolmogorov turbulence (consistent with experiments), but Haslam & Ronney (1994) show it's 7/3 even in non-Kolmogorov turbulence!
- With D = 7/3, $L_{inner} = L_K$, $S_T/S_L = (L_K/L_I)^{2-7/3} \sim (Re_L{}^{-3/4})^{-1/3} \sim Re_L{}^{1/4}$
- With D = 7/3, $L_{inner} = L_G$, $S_T/S_L = (L_G/L_I)^{2-7/3} \sim ((u'/S_L)^{-3})^{-1/3} \sim u'/S_L$
- Consistent with some experiments (like all models…)

Effects of thermal expansion

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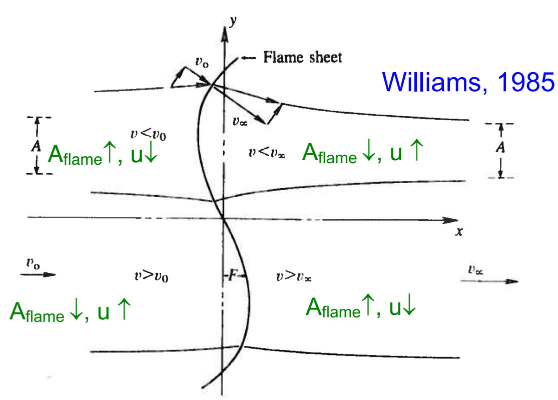
- Excellent review of flame instabilities: Matalon, 2007
- Thermal expansion (thus velocity change) across flame affects turbulence properties thus flame properties - exists even when no forced turbulence - *Darrieus-Landau instability*
- Some models (e.g. Pope & Anand, 1987) predict thermal expansion decreases S_T/S_L for fixed u'/S_L but most predict an increase

FLOW →



Williams, 1985

Effects of thermal expansion

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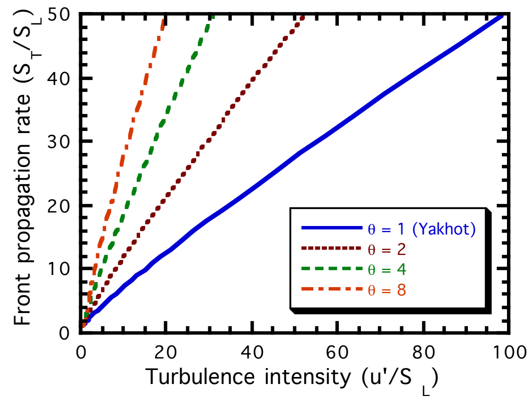
- Byckov (2000):

$$\frac{S_T}{S_L} = \exp\left(\frac{4\theta^3\left(1+2\left(S_T/S_L\right)^{-2}\right)}{\left[\theta+1+2\theta(\theta-1)\left(S_T/S_L\right)^{-2}\right]^2+8\theta^2\left(S_T/S_L\right)^{-2}}\frac{\left(u'/S_L\right)^2}{\left(S_T/S_L\right)^2}\right); \quad \theta \equiv \frac{\rho_\infty}{\rho_f} \approx \frac{T_{ad}}{T_\infty} > 1$$

- Same as Yakhot (1988) if no thermal expansion ($\theta = 1$)
- Also says for any θ, if $u'/S_L = 0$ then $S_T/S_L = 1$; probably not true



•31

Effects of thermal expansion

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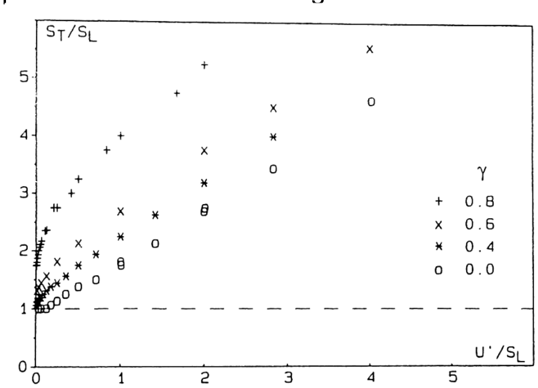
- Cambray and Joulin (1992): one-scale (not really turbulent) flow with varying thermal expansion $\gamma = 1 - \varepsilon = 1 - \rho_{ad}/\rho_\infty \approx 1 - T_\infty/T_{ad}$
  - S_T/S_L increases as γ increases (more thermal expansion)
  - If $\gamma \neq 0$, then S_T/S_L does not go to 1 as u'/S_L goes to zero - inherent thermal expansion induced instability and wrinkling causes $S_T/S_L > 1$ even at $u'/S_L = 0$
  - Effect of expansion-induced wrinkling decreases as u'/S_L increases



•32

Effects of thermal expansion

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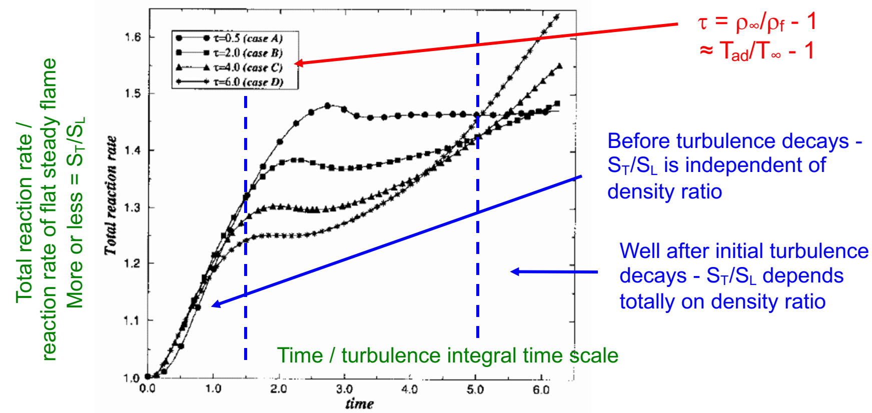
- Time-dependent computations in decaying turbulence suggest
  - Early times, before turbulence decays, turbulence dominates
  - Late times, after turbulence decays, buoyancy dominates
- Next lecture: thermal expansion effects on S_T with no 'turbulence', and turbulence effects on S_T with no thermal expansion



Boughanem and Trouvé, 1998

Initial u'/S_L = 4.0 (decaying turbulence); integral-scale Re = 18

Buoyancy effects

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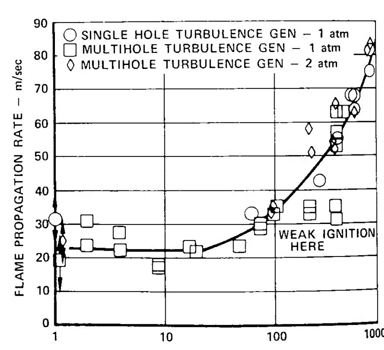
- Upward propagation: buoyantly unstable, should increase S_T compared to no buoyancy effects
- Buoyancy affects largest scales, thus scaling of buoyancy effects should depend on integral time scale / buoyant time scale = $(L_I/u')/(L_I/g)^{1/2}$ = $(gL_I/u'^2)^{1/2} = (S_L/u')(gL_I/S_L^2)^{1/2}$
- Libby (1989) (g > 0 for upward propagation)

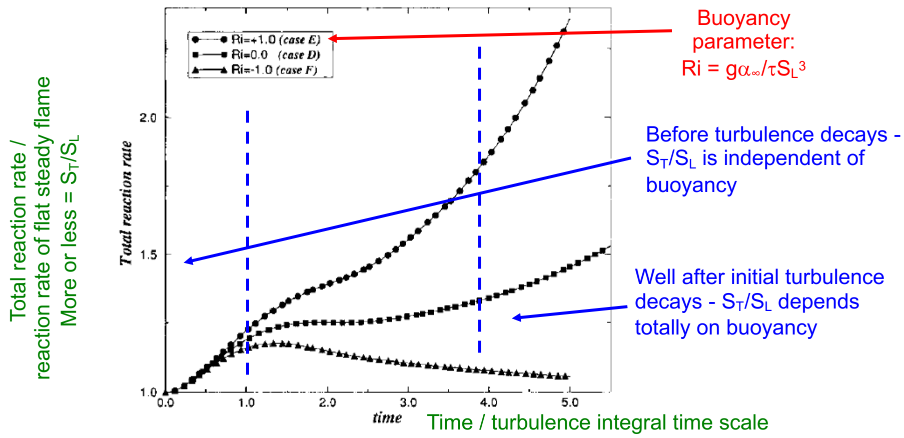
$$\frac{S_T}{S_L} = 2.15\frac{u'}{S_L} + \frac{1-\varepsilon}{0.867}\frac{gL_I}{S_L^2}; \varepsilon \equiv \frac{\rho_f}{\rho_\infty}$$

- No systematic experimental test to date
- Lewis (1970): experiments on flames in tubes in a centrifugal force field ($50\ g_o < g < 850\ g_o$), no forced turbulence: $S_T \sim g^{0.387}$, almost independent of pressure - may be same as $S \sim g^{1/2}$



Effects of buoyancy

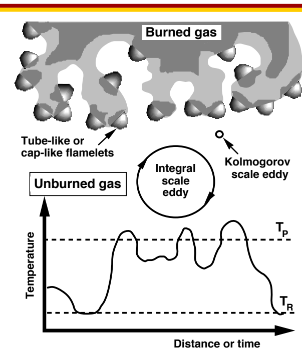
- Time-dependent computations in decaying turbulence suggest
  - Early times, before turbulence decays, turbulence dominates
  - Late times, after turbulence decays, buoyancy dominates - upward propagating: higher S_T/S_L; downward: almost no wrinkling, $S_T/S_L \approx 1$



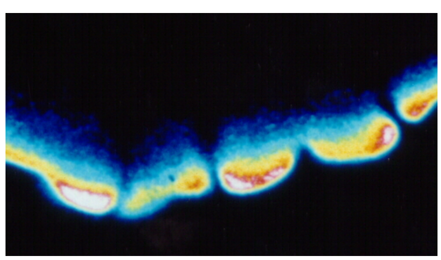
Boughanem and Trouve, 1998
Initial u'/S_L = 4.0 (decaying turbulence); integral-scale Re = 18

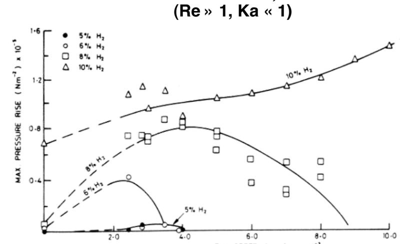
Lewis number effects

- Turbulence causes flame stretch & curvature
- Since both + and - curved flamelets exist, effects nearly cancel (Rutland & Trouve, 1993)
- Mean flame stretch is biased towards + stretch - benefits flames in low-Le mixtures
- H_2-air: extreme case, lean mixtures have cellular structures similar to flame balls - "Pac-Man combustion"
- Causes change in behavior at 8% - 10% H_2, corresponding to $\phi \approx 0.25$ (Al-Khishali *et al.*, 1983) - same as transition from flame ball to continuous flames in non-turbulent mixtures



"Broken" flamelets, Le << 1
(Re » 1, Ka « 1)





Distributed combustion modeling

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- Much less studied than flamelet combustion
- Damköhler (1940):

$$S_T/S_L \approx \sqrt{\omega_T D_T/\omega_L D_L} \approx \sqrt{D_T/D_L} = \sqrt{\nu_T/\nu_L}\sqrt{Sc_L/Sc_T}$$

$$\approx \sqrt{0.061 \mathrm{Re}_L}\sqrt{Sc_L/Sc_T} \approx A\sqrt{\mathrm{Re}_L}$$

$A \approx 0.25$ (gas); $A \approx 6.5$ (liquid)

- Assumption $\omega_T \approx \omega_L$ probably not valid for high β; recall

$$\beta \equiv \frac{T_{ad}}{\omega(T_{ad})}\frac{\partial\omega}{\partial T}\bigg|_{T=T_{ad}} = \frac{E}{RT_{ad}}$$

…but probably ok for small β

- Example: 2 equal volumes of combustible gas with E = 40 kcal/mole, 1 volume at 1900K, another at 2100K

$\omega(1900) \sim \exp(-40000/(1.987*1900)) = 3.73 \times 10^4$

$\omega(2100) \sim \exp(-40000/(1.987*2100)) = 1.34 \times 10^4$

Average = 2.55×10^4, whereas $\omega(2000) = 2.2 \times 10^4$ (16% difference)!

$\therefore$ Averaging over $\pm$ 5% T range gives 16% error!

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•37

Distributed combustion modeling

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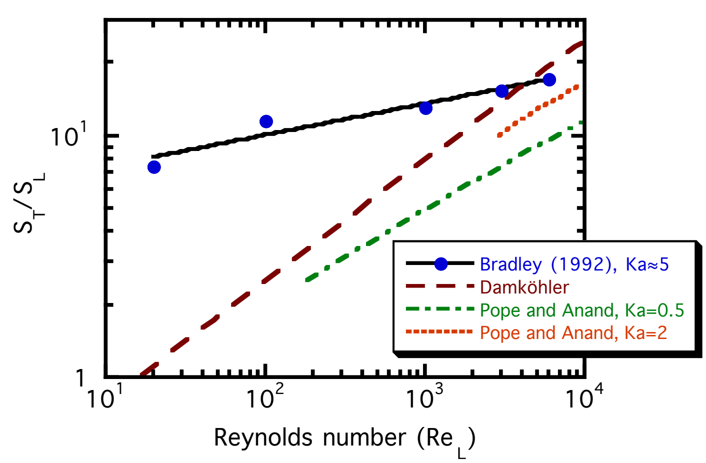
- Pope & Anand (1984)

Constant density, 1-step chemistry, $\beta \approx 12.3$:

$S_T/S_L \approx 0.3 (u'/S_L)[0.64 + \log_{10}(\mathrm{Re}_L) - 2 \log_{10}(u'/S_L)]$

- Close to Damköhler (1940) (?!) for relevant Re_L



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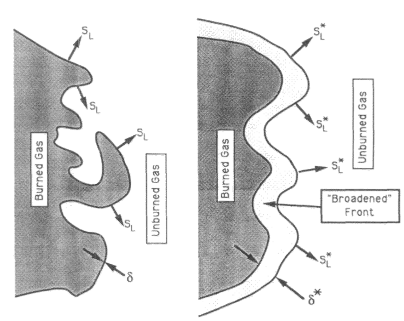
•38

Distributed combustion modeling

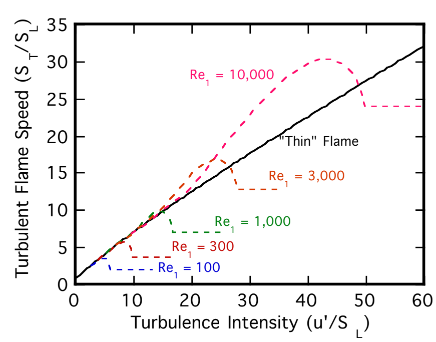
- When applicable? $\delta_T \geq L_I$
  - $\delta_T \approx 6\alpha_T/S_T$ (Ronney & Yakhot, 1992), $S_T/S_L \approx 0.25\ Re_L^{1/2}$
    $\Rightarrow$ Ka $\geq$ 0.037 $Re_L^{1/2}$ (based on cold-gas viscosity)
  - Experiments (Abdel-Gayed *et al.*, 1989)
    - » Ka ≥ 0.3: "significant flame quenching within the reaction zone" - independent of Re_L
  - Applicable range uncertain, but probably only close to quenching
- Experiment - Bradley (1992)
  - $S_T/S_L \approx 5.5\ Re^{0.13}$ - close to models over limited applicable range
- Implications for "bending"
  - Flamelet models generally $S_T \sim u'$
  - Distributed combustion models generally $S_T \sim \sqrt{u'}$
  - Transition from flamelet to distributed combustion - possible mechanism for "bending"

"Broadened" flames

- Ronney & Yakhot, 1992: If $Re^{1/2}$ > Ka > 1, some but not all scales (i.e. smaller scales) fit inside flame front and "broaden" it since $\alpha_T > \alpha$
- Combine flamelet (Yakhot) model for large (wrinkling) scales with distributed (Damköhler) model for small (broadening) scales
- Results show S_T/S_L "bending" and even decrease in S_T/S_L at high u'/S_L as in experiments



"Thin" flame "Broadened" flame



Quenching by turbulence

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- Low Da / high Ka: "thin-flame" models fail
  - "Distributed" combustion
  - "Quenching" at still higher Ka
- Quenching attributed to mass-extinguishment of flamelets by zero-mean turbulent strain

  ...but can flamelet quenching cause flame quenching?
- Hypothetical system: flammable mixture in adiabatic channel with arbitrary zero-mean flow disturbance
- 1st law: No energy loss from system $\Rightarrow$ adiabatic flame temperature is eventually reached
- 2nd law: Heat will be transferred to unburned gas
- Chemical kinetics: $\omega(T>T_H) >> \omega(T_L)$

  $\therefore$ Propagating front will always exist (???)

Quenching by turbulence

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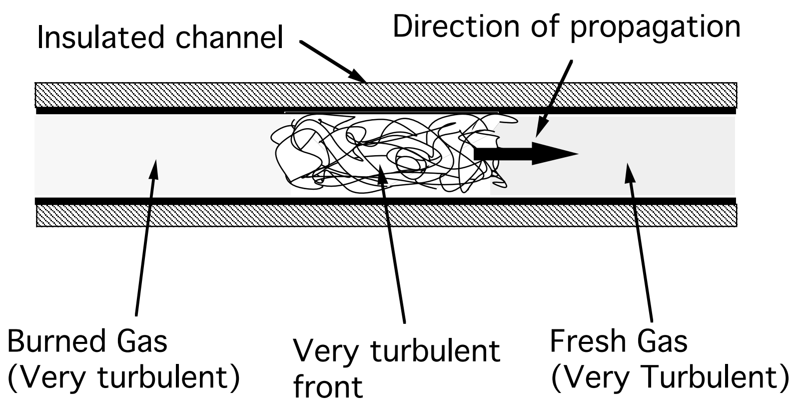

IDEALIZED TURBULENT COMBUSTION APPARATUS

Turbulent flame quenching in gases

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- "Liquid flame" experiments suggest flamelet quenching does not cause flame quenching
- Poinsot *et al.* (1990): Flame-vortex interactions
  - Adiabatic: vortices cause temporary quenching
  - Non-adiabatic: permanent flame quenching possible
- Giovangigli and Smooke (1992) no intrinsic flammability limit for planar, steady, laminar flames
- So what causes quenching in "standard" experiments?
  - Radiative heat losses?
  - Ignition limits?

Turbulent flame quenching in gases

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- High Ka (distributed combustion):
  - Turbulent flame thickness (δ_T) >> δ_L
  - Larger δ_T $\Rightarrow$ more heat loss (~ δ_T), S_T increases less
  - $\Rightarrow$ heat loss more important at high Ka

$$\frac{Q_{loss}}{Q_{gen}} \approx \frac{1}{\beta} \approx \frac{4\sigma a_p (T_f^4 - T_\infty^4)}{\rho_\infty S_T C_P \Delta T / \delta_T}; S_T \approx 5.5 \mathrm{Re}_L^{0.13} S_L; \delta_T \approx 6\alpha_T / S_T$$

$\Rightarrow$ $Ka_q \approx 0.038\ Re_L^{0.76}$ for typical L_I = 0.05 m

  - Also: large $\delta_T \Rightarrow$ large ignition energy required

$$E \geq \frac{4\pi}{3}\delta_T^3 \rho_R C_P \Delta T \Rightarrow Ka_q \approx 6.6 \left(\frac{E}{\rho_R C_P \Delta T L_I^3} \right)^{1/3} \mathrm{Re}_L^{-0.24}$$

$\Rightarrow$ $Ka_q \approx 2.3\ Re_L^{-0.24}$

- 2 limit regimes qualitatively consistent with experiments

Turbulent flame quenching in gases

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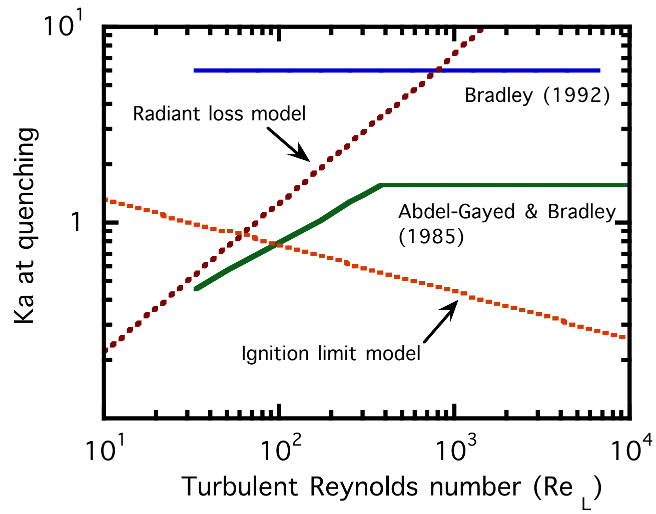

Constant density "flames" - motivation

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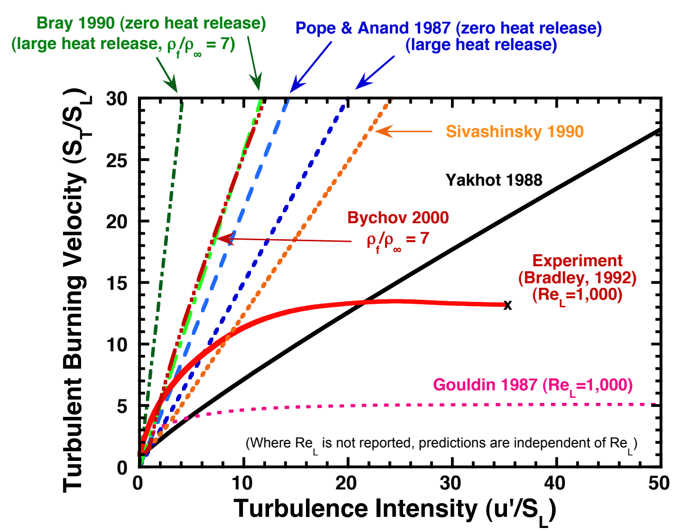
- Models of premixed turbulent combustion don't agree with experiments nor each other!



"Liquid flame" idea

- See Epstein and Pojman, 1998
- Use propagating acidity fronts in aqueous solution
- Studied by chemists for 100 years
- Generic form
  A + nB → (n+1)B - *autocatalytic*
- $\Delta\rho/\rho << 1$ - no self-generated turbulence
- $\Delta T \approx 3$ K - no change in transport properties
- Zeldovich number $\beta \approx 0.05$ vs. 10 in gas flames
  Aqueous fronts not affected by heat loss!!!
- Large Schmidt number [= $\nu/D \approx 500$ (liquid flames) vs. ≈ 1 (gases)] - front stays "thin" even at high Re

$$Ka \sim \frac{u'/L_T}{S_L^2/D} \sim \frac{\nu}{u'L_I}\frac{L_I}{L_T}\frac{u'^2}{S_L^2}\frac{D}{\nu} \sim \mathrm{Re}_L^{-1/2}\left(\frac{u'}{S_L}\right)^2 Sc^{-1}$$

Gaseous vs. liquid flames

- Most model employ assumptions not satisfied by real flames
  - Adiabatic (gas flames: sometimes ok) (Liquid flames TRUE!)
  - Homogeneous, isotropic turbulence over many L_I (gas flames: never ok) (Liquid flames: can use different apparatuses where this is more nearly true)
  - Low Ka or high Da (thin fronts) (gas flames: sometimes ok) (Liquid flames: more often true due to higher Sc)
  - Lewis number = 1 (gas flames: sometimes ok, e.g. CH_4-air) (Liquid flames: irrelevant since heat transport not a factor in propagation)
  - Constant transport properties (gas flames: never ok, ≈ 25x increase in ν and α across front!) (Liquid flames: TRUE)
  - u' doesn't change across front (gas flames: never ok, thermal expansion across flame generates turbulence) (but viscosity increases across front, decreases turbulence, sometimes almost cancels out) (Liquid flames: TRUE)
  - Constant density (gas flames: never ok!) (Liquid flames: true, although buoyancy effects still exist due to small density change)
- Conclusion: liquid flames better for testing models!

Approach - chemistry

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- Simpler chemistry than gaseous flames
- Color-changing or fluorescent pH indicators
- Original: arsenous acid - iodate system

$IO_3^- + 5I^- + 6H^+ \rightarrow 3\ I_2 + 3\ H_2O$

$H_3AsO_3 + I_2 + H_2O \rightarrow 2\ I^- + 2\ H^+ + H_3AsO_4$

$IO_3^- + 3\ H_3AsO_3 \rightarrow I^- + 3\ H_3AsO_4$

... autocatalytic in iodide (I^-)

- Later: iodate-hydrosulfite system

$IO_3^- + 6\ H^+ + 6e^- \rightarrow I^- + 3\ H_2O$

$S_2O_4^{-2} + 4\ H_2O \rightarrow 6\ e^- + 8\ H^+ + 2\ SO_4^{-2}$

$IO_3^- + S_2O_4^{-2} + H_2O \rightarrow I^- + 2\ SO_4^{-2} + 2\ H^+$

  - Simple solutions
  - Non-toxic
  - "Lightning fast" (up to 0.05 cm/sec)

•49

Comparison of gaseous & liquid flames

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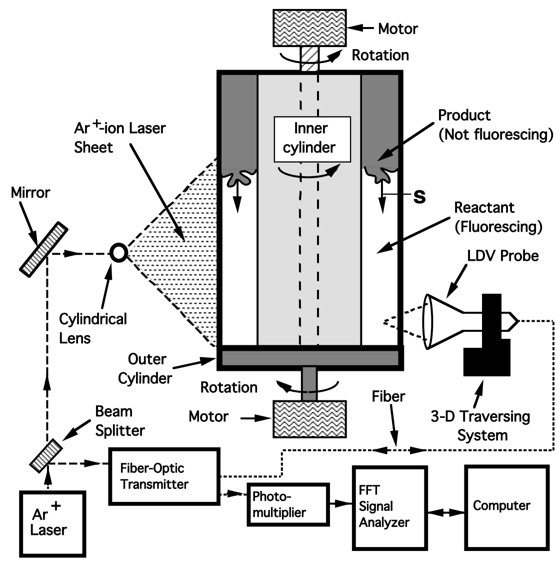
Property	*Stoichiometric hydrocarbon-air flame*	*Autocatalytic chemical front*
Reaction mechanism	Many-step, chain-branching	Two-step, straight-chain
S_L	40 cm/sec	0.03 cm/sec
$\beta = E/RT_{ad}$	10	0.05
$\Delta\rho/\rho_f$	6	0.0003
$\Delta\nu/\nu_R$	25	0.02
Sc	1	500
Impact of heat loss	Critical	Irrelevant
Ease of LIF imaging	Tough ($$$)	Trivial

•50

Taylor-Couette apparatus



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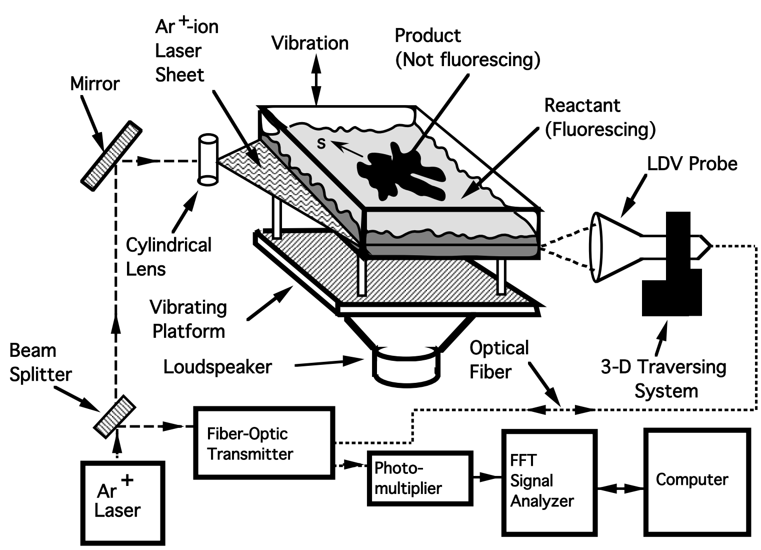
Capillary-wave apparatus



•52

Results - flow characteristics

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- Ronney *et al.*, 1995
- Taylor-Couette, counter-rotating, "featureless turbulence" regime
  - ≈ homogeneous except near walls
  - Gaussian velocity histograms
  - Time autocorrelation (τ_a) nearly exponential
  - $L_I \equiv \sqrt{(8/\pi)}u'\tau_a \approx$ 1/2 cylinder gap
- Capillary wave
  - Mean velocity ≈ 0, u' ≈ constant across dish except near walls
  - u' ~ z
  - u' ≡ average over z - interpret as if 2-d
- Vibrating grid (Shy *et al.*, 1996)
  - Fairly homogeneous & isotropic in central region
  - Kolmogorov-like spectrum

Results - flow characteristics

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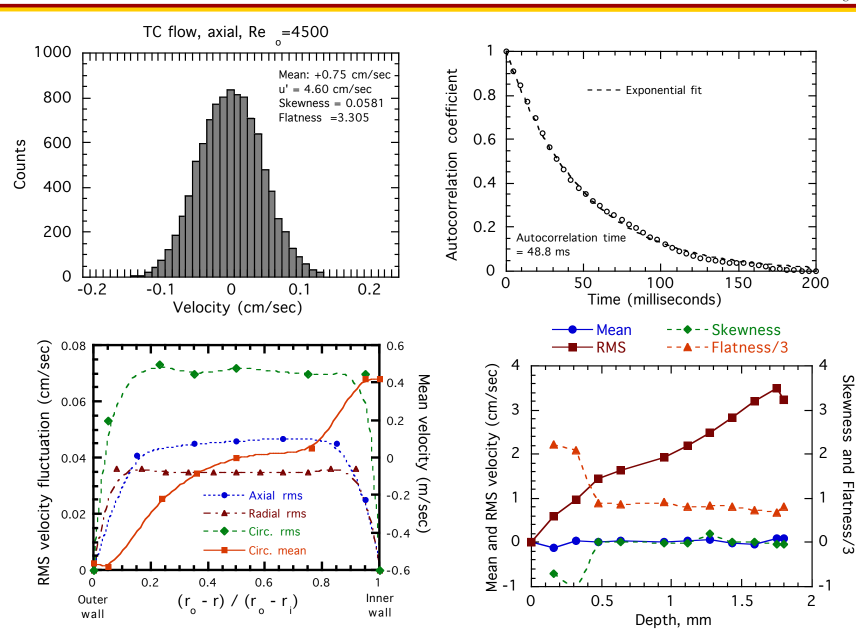

Results - liquid flames

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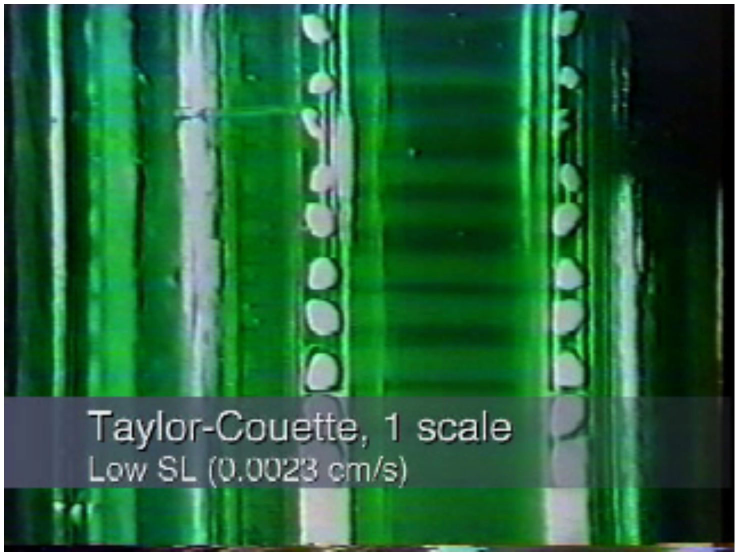

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Results

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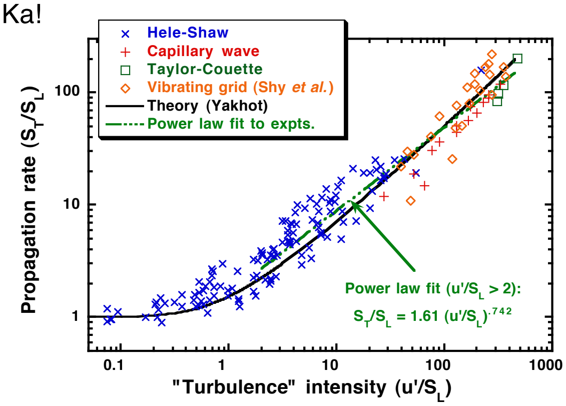
- Thin "sharp" fronts at low Ka (< 5)
- Thick "fuzzy" fronts at high Ka (> 10)
- No global quenching observed, *even at Ka > 2500 !!!*
- High Da - S_T/S_L in 4 different flows consistent with Yakhot model with no adjustable parameters
- High Ka - S_T/S_L lower than at low Ka - consistent with Damköhler model over 1000x range of Ka!



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Results - liquid flames - propagation rates

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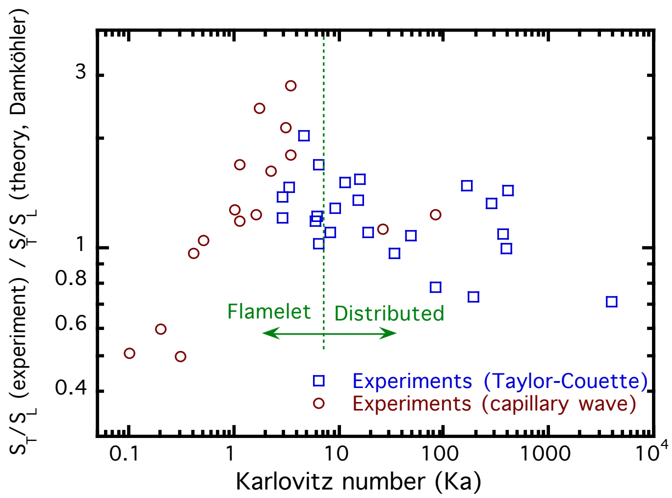
- Data on S_T/S_L in distributed combustion regime (high Ka) consistent with Damköhler's model - *no adjustable parameters*



Ronney et al., 1995

Front propagation in one-scale flow

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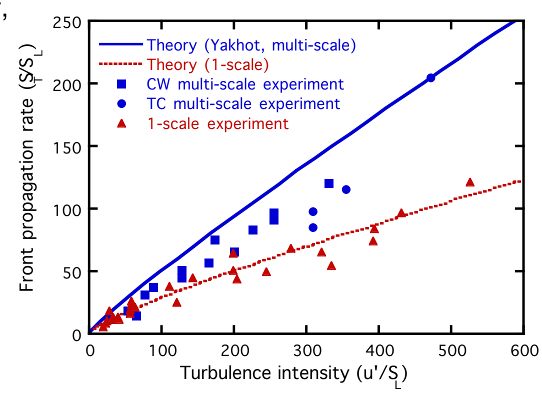
- Turbulent combustion models not valid when energy concentrated at one spatial/temporal scale
- Experiment - Taylor-Couette flow in "Taylor vortex" regime (one-scale)
- Result - S_T/S_L lower in TV flow than in turbulent flow but consistent with model for one-scale flow (Shy et al., 1992) probably due to "island" formation & reduction in flame surface (Joulin & Sivashinsky, 1991)

$$S_T/S_L = \exp\left(\frac{u'/S_L}{S_T/S_L}\left(1-\exp\left(-\frac{u'/S_L}{S_T/S_L}\right)\right)\right)$$



Fractal analysis in CW flow

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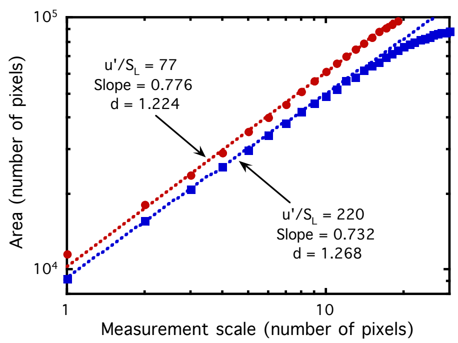
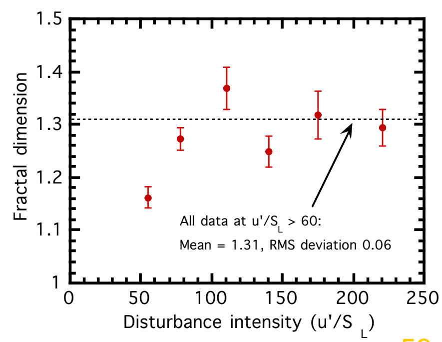
- Haslam and Ronney (1995) - fractal-like behavior exhibited with D ≈ 1.35 (⇒ 2.35 in 3-d) independent of u'/S_L - same as gaseous flame front, passive scalar in CW flow
- Problem - why is d seemingly independent of
  - Propagating front vs. passively diffusing scalar
  - Velocity spectrum
  - Constant or varying density & transport properties
  - 2-d object or planar slice of 3-d object





•59

Self-generated wrinkling due to instabilities

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- What about self-generated "turbulence" due to inherent instabilities of flames not subjected to forced turbulence?
- First step: linear stability analysis of flat, steady flame
- Basic goal of linear stability analysis: determine growth rate of instability (σ, units 1/time) as a function of disturbance wavelength (λ) or wavenumber ($k = 2\pi/\lambda$)
- Many types of instabilities may occur
  - Thermal expansion (Darrieus-Landau, DL)
  - Rayleigh-Taylor (buoyancy-driven, RT)
  - Viscous fingering (Saffman-Taylor, ST) in narrow channels when viscous fluid displaced by less viscous fluid
  - Diffusive-thermal (DT) (Lewis number)
  - Joulin & Sivashinsky (1994) - combined effects of DL, ST, RT & heat loss (but no DT effect - no damping at small wavelength λ)

•60

Self-generated flame wrinkling

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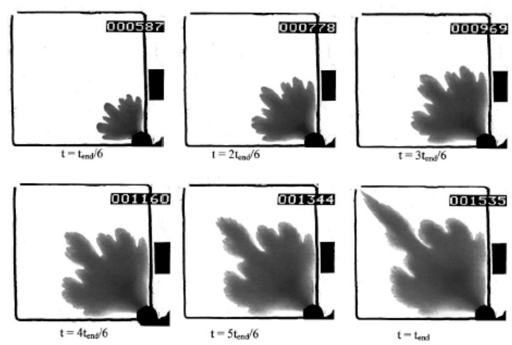
- Si & Ronney (2020?)
- Use Hele-Shaw cell
  - Flow between closely-spaced parallel plates
  - Described by linear 2-D equation (Darcy's law)
  - 1000's of references
- Measure
  - Propagation rates
  - Wrinkling wavelengths



Petitjeans et al. (1999) - displacement of viscous glycern-water mixture (white) by less viscous water-dye mixture (dark) injected in lower-right corner

Background – DL + ST + RT

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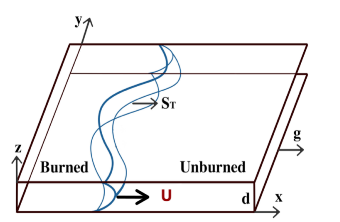
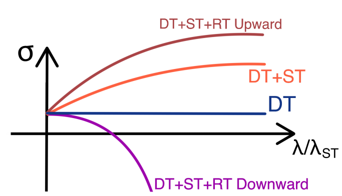
- Joulin & Sivashinsky (1994): linear stability analysis of flame propagation in HS cells including DL, RT & ST
- U = local propagation rate (not necessarily S_L due to curvature in 3[rd] dimension & Le effects)
- Dispersion relation: growth rate (ω) vs wavenumber of transverse wrinkles (k)
  - Friction coefficient f = $12\mu/d^2$
  - Characteristic wavelength for ST: $\lambda_{ST} = (\pi/6)\rho_\infty U d^2/\mu_{av}$
  - Ω: dimensionless growth rate (JS Parameter)
  - F & G: ST & RT parameters





$$\Omega^2 + (1+\Lambda)\Omega - \frac{1-\varepsilon^2}{4\varepsilon}\left[1 + \frac{\varepsilon}{1-\varepsilon}(F+G)\Lambda\right] = 0; \Omega \equiv \frac{\omega(1+\varepsilon)}{2kU}; \Lambda \equiv \frac{\lambda}{\lambda_{ST}} = \frac{f_{av}}{\rho_\infty U k}$$

$$F \equiv \frac{f_{ad} - \varepsilon f_\infty}{\varepsilon f_{av}}; G \equiv \frac{(1-\varepsilon)\rho_\infty g}{f_{av} U}; \varepsilon \equiv \frac{\rho_{ad}}{\rho_\infty} \approx \frac{T_\infty}{T_{ad}}; f_{av} \equiv \frac{f_{ad} + f_\infty}{2}$$

Background – DL + ST + RT

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- DL only ($\varepsilon \neq 1$, G = 0, $f_u = f_b = 0 \Rightarrow \Lambda = 0$)

$$\Omega^2 + \Omega - \frac{1-\varepsilon^2}{4\varepsilon} = 0 \Rightarrow \Omega = \frac{1}{2}\left(\sqrt{\frac{1+\varepsilon-\varepsilon^2}{\varepsilon}} - 1\right) > 0$$

  - All flames are unstable ($\Omega > 0$) due to thermal expansion effects; no preferred wavelength since $\Omega \sim \omega/k$ = constant
- RT only ($\varepsilon \to 1$, G ≠ 0, $f_u = f_b = 0 \Rightarrow \Lambda = 0$)

$$\Omega^2 + \Omega - \frac{1}{2}G\Lambda = 0 \Rightarrow \Omega^2 + \Omega - \frac{1}{4}Fr_\lambda^{-1} = 0; Fr_\lambda \equiv \frac{\pi}{1-\varepsilon}\frac{U^2}{g\lambda}$$

$$\Rightarrow \Omega = \frac{1}{2}\left(\sqrt{1+Fr_\lambda^{-1}} - 1\right)$$

  - All upward-propagating flames (g > 0) are unstable ($\Omega > 0$) and growth rate is larger for larger wavelengths (larger λ)
  - Downward-propagating flames (g < 0) are stable ($\Omega < 0$) for small wavelengths but produce oscillatory instabilities ($Im(\Omega) \neq 0$) for larger wavelengths

Background – DL + ST + RT

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- ST only ($\varepsilon \to 1$, G = 0, $f_\infty \neq f_{ad} \neq 0 \Rightarrow F \neq 0, \Lambda \neq 0$)

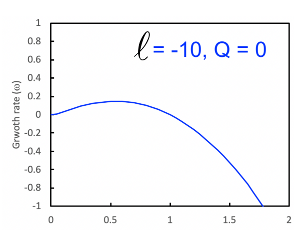
$$\Omega^2 + (1+\Lambda)\Omega - \frac{1}{2}F\Lambda = 0 \Rightarrow \Omega = \frac{\sqrt{1+2\Lambda(1+F)+\Lambda^2} - \sqrt{1+2\Lambda+\Lambda^2}}{2} > 0$$

$$\text{Small } F\text{: } \Omega \approx \frac{F}{2}\frac{\Lambda}{1+\Lambda}; \Omega \to 0 \text{ for } \Lambda \to 0,\ \Omega \to \frac{F}{2} \text{ for } \Lambda \to \infty$$

  - All flames in confined channels are unstable ($\Omega > 0$) due to viscosity increase across front (F > 0); Ω increases with increasing wavelength
- DT (Joulin & Clavin, 1979) (not covered by JS analysis) including heat loss term Q:

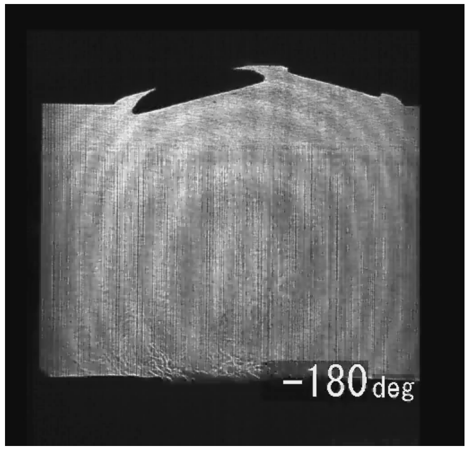
$$(1-\Gamma)\left[\frac{Q}{2}(\Gamma+1) - \Gamma^2\right] = \frac{\ell}{2}(1-\Gamma+2\omega)$$

$$\Gamma \equiv \sqrt{1+4(\omega+k^2)}; \ell \equiv \beta(Le-1)$$



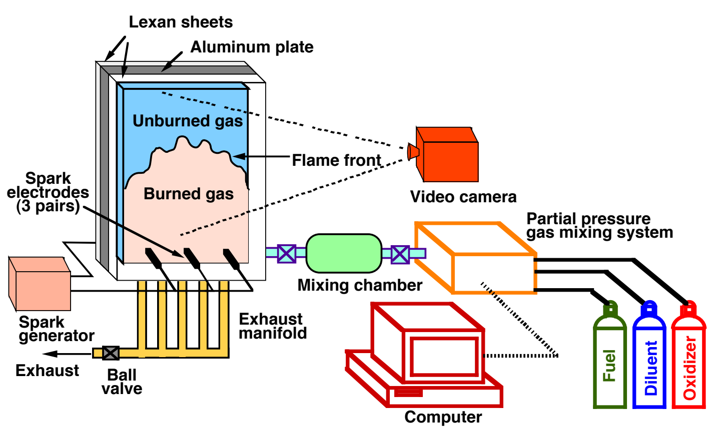
Self-generated flame wrinkling

- Practical applications to combustion
  - Spark-ignition engines at time of combustion (below)
  - Flame propagation in cylinder crevice volumes



Video courtesy
Prof. Yuji Ikeda,
Kobe University

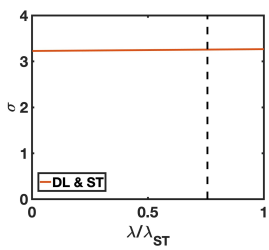
Hele-Shaw apparatus

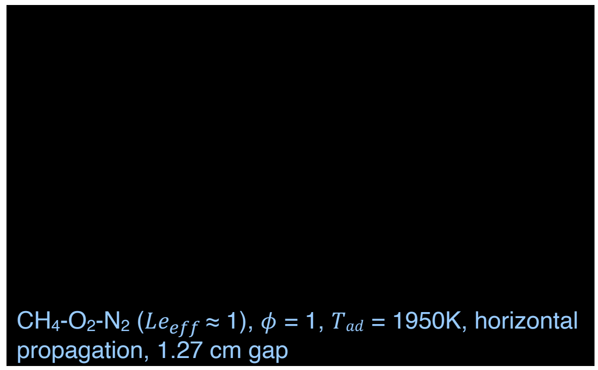


- Aluminum frame sandwiched between Lexan windows
- 40 cm x 60 cm x 1.27 or 0.635 or 0.318 cm test section
- H_2, CH_4 & C_3H_8 fuel, N_2 & CO_2 diluent - affects Le, Peclet #
- Upward, horizontal, downward orientation
- Spark ignition (3 locations)

Hele-Shaw videos - "baseline" case

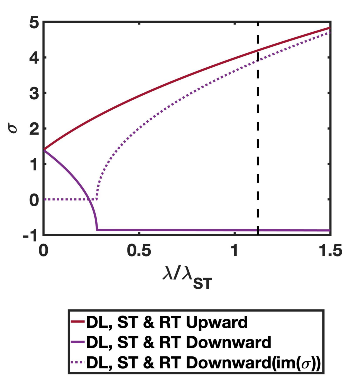
- Le ≈ 1, G = 0: classic cusp shape caused by DL
- No characteristic wavelength preference

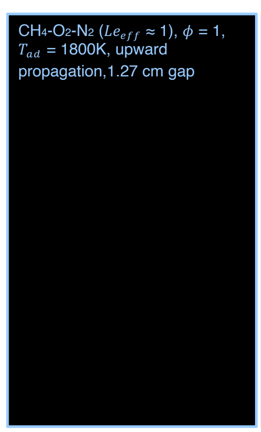


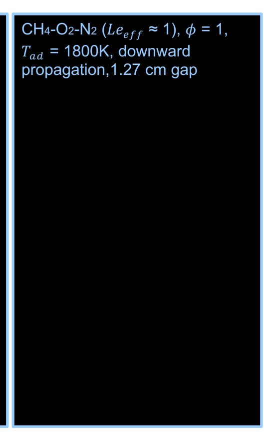


Hele-Shaw videos – G > 0 & G < 0

- Le ≈ 1, G > 0: larger wrinkle and faster propagation speed for upward vs. downward propagating flame due to RT
- Le ≈ 1, G < 0: Oscillatory "sloshing" motion for downward propagating flame due to imaginary component of σ







Hele-Shaw videos – large F

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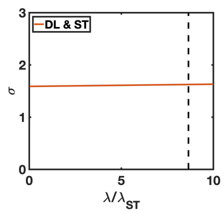
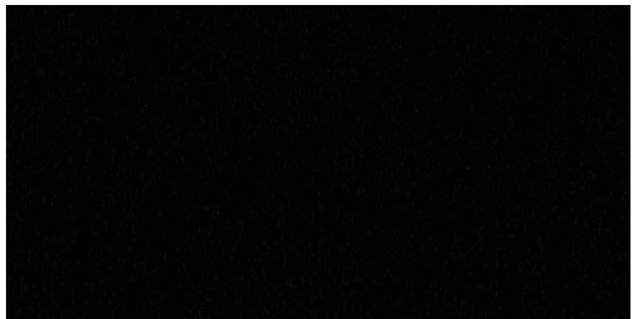
- Le ≈ 1, large F (due to thin cell, *i.e.*, small d)
  - Cell width << λ_{ST} so ST mode dominates
  - 1 large wrinkle ("tulip flame") fills entire cell





CH_4-O_2-N_2 (Le_{eff} ≈ 1), ϕ = 1, T_{ad} = 2100K, horizontal propagation, 0.318 cm gap

Hele-Shaw videos – Le > 1, G = 0

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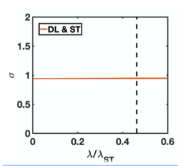
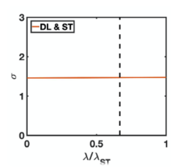
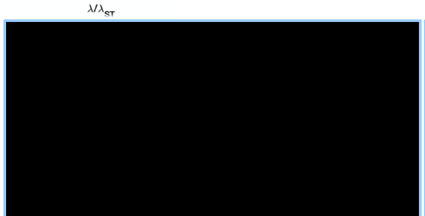
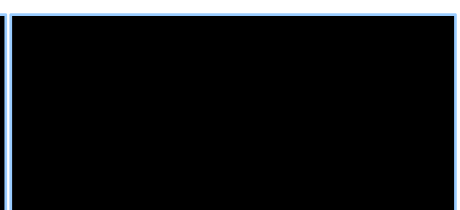
- Less wrinkling in flames with $Le_{eff} > 1$ than in CH_4-O_2-N_2 flame due to DT







H_2-O_2-N_2 (Le_{eff} ≈ 1.3), ϕ = 2.0, T_{ad} = 1260K, horizontal propagation, 1.27 cm gap



C_3H_8-O_2-N_2 (Le_{eff} ≈ 1.6), ϕ = 0.5, T_{ad} = 1850K, horizontal propagation, 1.27 cm gap

Hele-Shaw videos – Le < 1, G = 0

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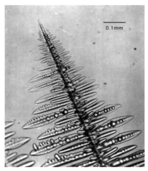
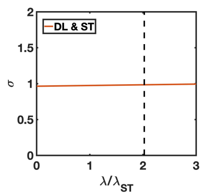
- Cellular structure caused by DT
- Large scale: "sawtooth" structure instead of classic cusp shape





H_2-O_2-N_2 (Le_{eff} ≈ 0.6), ϕ = 0.8, T_{ad} = 1300K, horizontal propagation, 1.27 cm gap

Hele-Shaw videos – Le < 1, G = 0

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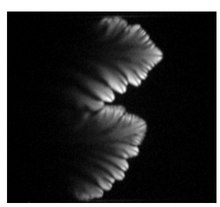
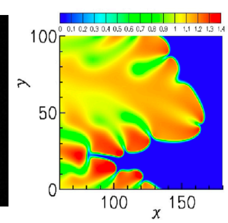
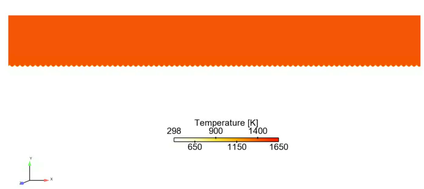
- Similar dendric structure observed in simulations:
  - Fernandez *et al.* (2018) – Hele-Shaw simulation (DL+ST+DT), one-step chemistry
  - Berger *et al.* (2019) – 2D simulation (DL+DT), detailed chemistry
- Dendric structure is result of DL and DT interaction; ST and detailed chemistry not required



Expt. (Le ≈ 0.3, ε ≈ 0.25)



Galisteo et al., 2018



Berger et al. (2019)

Hele-Shaw videos – Le < 1, very non-adiabatic

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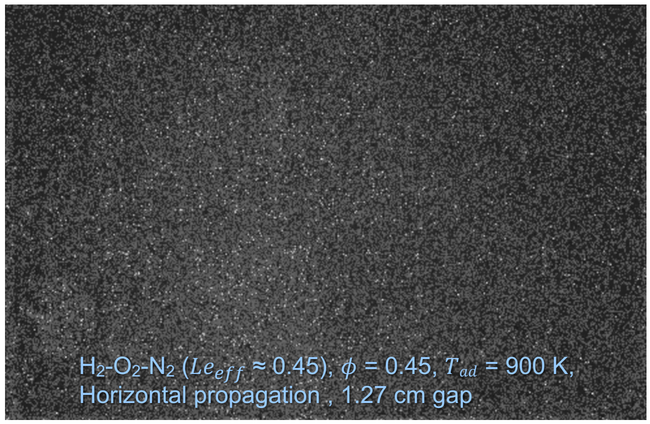
- DL (thermal expansion) is suppressed in low T_{ad} (slow S_L) flame due to heat losses – no dendric structure



Hele-Shaw - summary of qualitative observations

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- Le ≈ 1: wrinkled cusped structure
  - Upward propagation - more large-scale wrinkling
  - Downward propagation – large-scale wrinkling suppressed, but "sloshing" motion observed
  - High F – transition to "tulip" flame
  - Consistent with Joulin-Sivashinsky predictions
- Le > 1: similar to $Le_{eff} \approx 1$ cases, but less small-scaled wrinkling due to DL instability
- Le < 1: small-scale DT cellular structures
  - Large-scale wrinkling changes from cusped to angular dendrite-like structures, which requires only DL & DT to observe in simulations; ST, RT and detailed chemistry not required
  - Very low flame temperature: changes in ρ and μ are suppressed due to heat losses, only DT retained
- *For practical range of conditions, buoyancy & diffusive-thermal effects cannot prevent wrinkling due to viscous fingering & thermal expansion*

Hele-Shaw results - propagation rates

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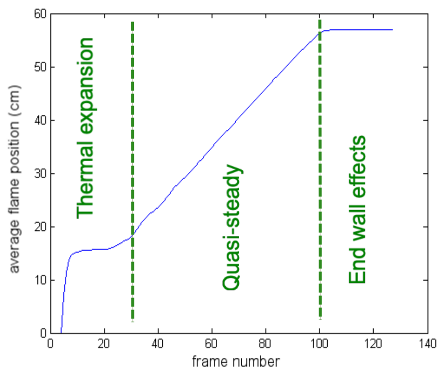
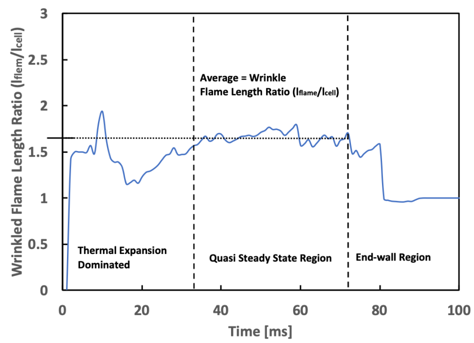
- 3-stage propagation
  - Thermal expansion - most rapid
  - Quasi-steady
  - Near-end-wall - slowest - large-scale wrinkling suppressed
- Quasi-steady propagation rate (S_T) always larger than S_L - typically $3S_L$ even though $u'/S_L = 0$!





Propagation rates - CH_4/air, horizontal

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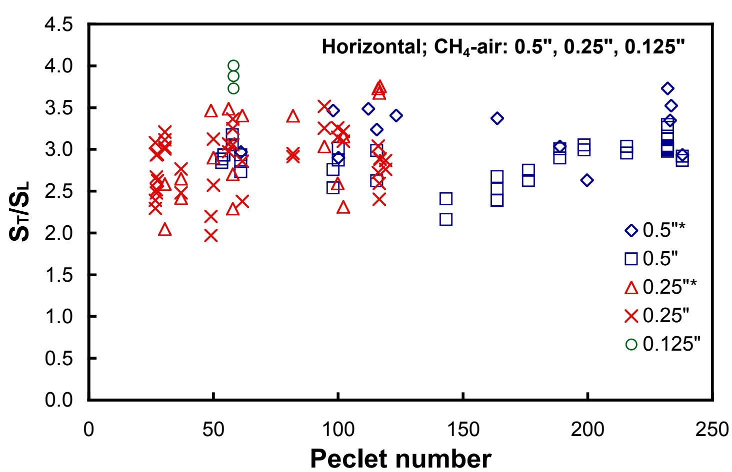
- Horizontal, CH_4-air ($Le \approx 1$): $S_T/S_L \approx 3$
- Independent of $Pe = S_L w/\alpha \Rightarrow$ independent of heat loss
- Slightly higher S_T/S_L for thinner cell despite lower Pe (greater heat loss) (for reasons to be discussed later…)



Propagation rates - C_3H_8-air, horizontal

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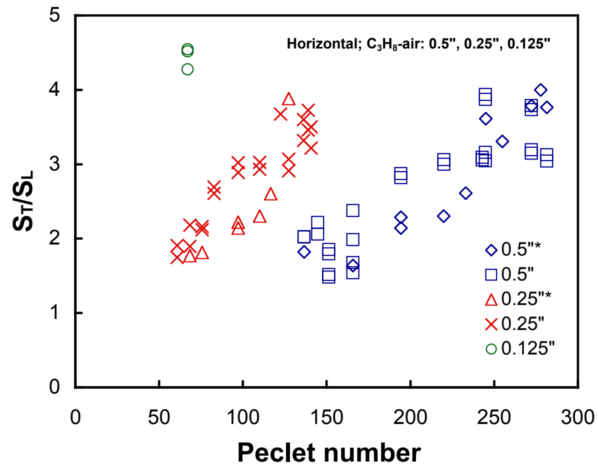
- Horizontal, C_3H_8-air: very different trend from CH_4-air - S_T/S_L depends significantly on Pe & cell thickness (why? next slide…)
- STILL slightly higher S_T/S_L for thinner cell despite lower Pe (greater heat loss)



•77

Propagation rates - C_3H_8-air, re-plotted

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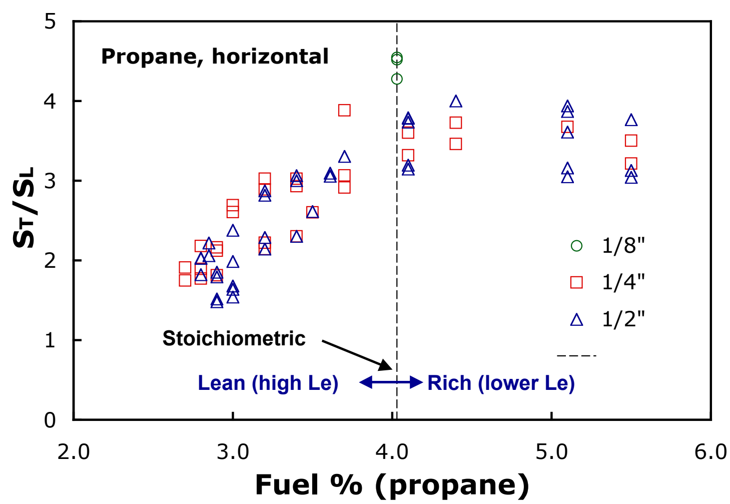
- C_3H_8-air: Le ≈ 1.7 (lean), lower S_T/S_L
- C_3H_8-air: Le ≈ 0.9 (rich) S_T/S_L ≈ independent of Pe, similar to CH_4-air



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Propagation rates - CH_4-O_2-CO_2 (low Le)

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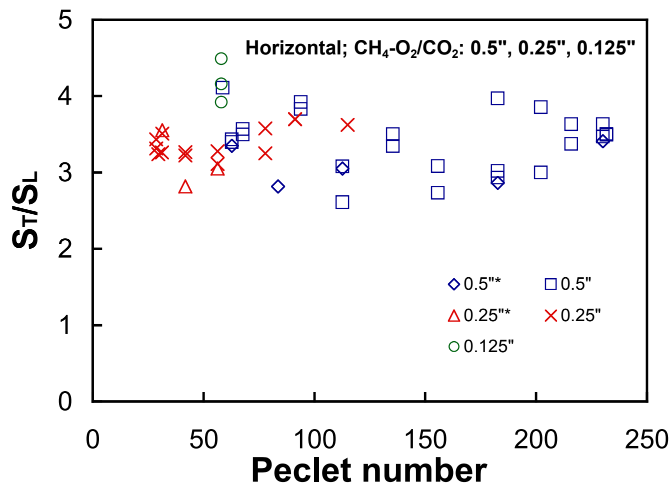
- Horizontal, CH_4-O_2-CO_2 (Le ≈ 0.7): similar to CH_4-air, no effect of Pe but slightly higher average S_T/S_L: 3.5 vs. 3.0, narrow cell again slightly higher



Propagation rates - orientation effect

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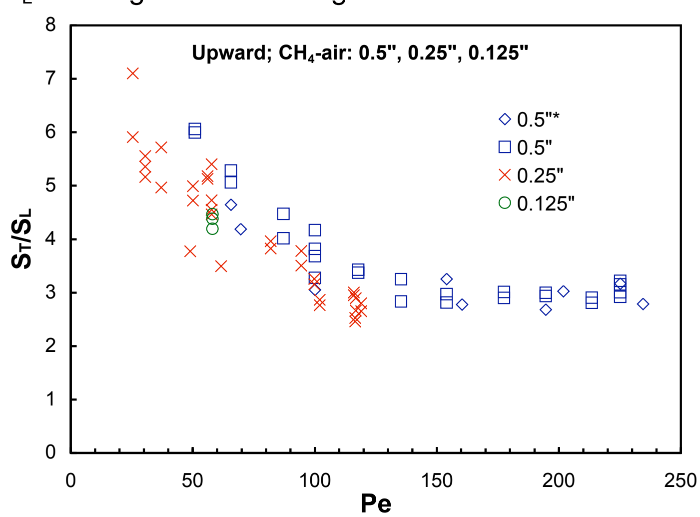
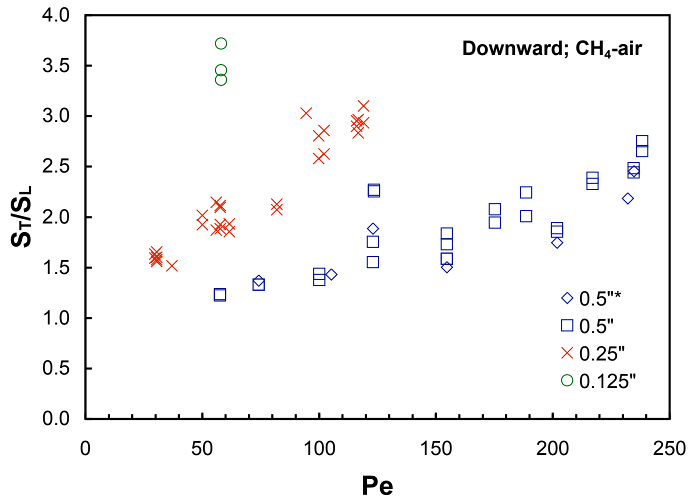
- Upward - S_T/S_L ⇓ as Pe ⇑ (S_L increases, decreasing benefit of buoyancy); highest propagation rates
- S_T/S_L converges to ≈ 3 at large Pe – same as horizontal



Results - orientation effect

- Downward - S_T/S_L ⇑ as Pe ⇓ (decreasing penalty of buoyancy); lowest propagation rates - but Pe isn't whole story…
- S_T/S_L converges to ≈ 3 at large Pe



•81

Scaling analysis

- How to estimate "driving force" for flame wrinkling?
- Hypothesis: use linear growth rate (ω) of Joulin-Sivashinsky analysis divided by wavenumber (k) (i.e. phase velocity ω/k) scaled by S_L as a dimensionless growth rate
  - Analogous to a "turbulence intensity")
  - Use largest value of growth rate, corresponding to longest half-wavelength mode that fits in cell, i.e., $k^* = (2\pi/L)/2$ (L = width of cell = 39.7 cm)
  - "Small" L, i.e. L < ST length = $(\pi/6)(\rho_u U w^2/\mu_{av})$
    - » DL dominates - ω/k = constant
    - » Propagation rate should be independent of L
  - "Large" L, i.e. L > $(\pi/6)(\rho_u U w^2/\mu_{av})$
    - » ST dominates - ω/k increases with L
    - » Propagation rate should increase with L
  - Baseline condition: (6.8% CH_4-air, S_L = 15.8 cm/s, w = 12.7 mm): ST length = 41 cm > L - little effect of ST

•82

Scaling analysis

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- ST length smaller (thus more important) for slower flames and smaller w - but these conditions will cause flame quenching - how to get smaller ST length without quenching?
- ST length = w $(\pi/6)(\mu_u/\mu_{av})(1/Pr)Pe$ for fixed cell width, minimum Pe ≈ 40 set by quenching - easier to get smaller ST length without quenching in thinner cells

•83

Results - orientation effect revisited

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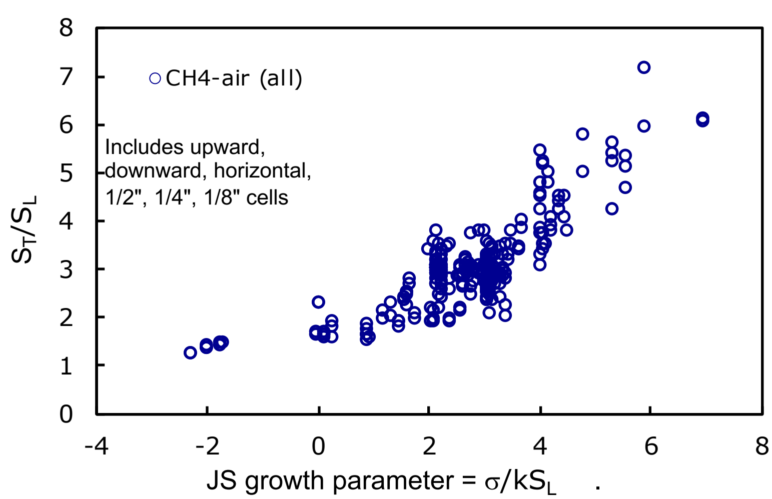
- Results scale reasonably well with JS growth parameter which is basically u'/S_L, with $S_T/S_L \approx 1 + u'/S_L$



•84

Effect of JS parameter

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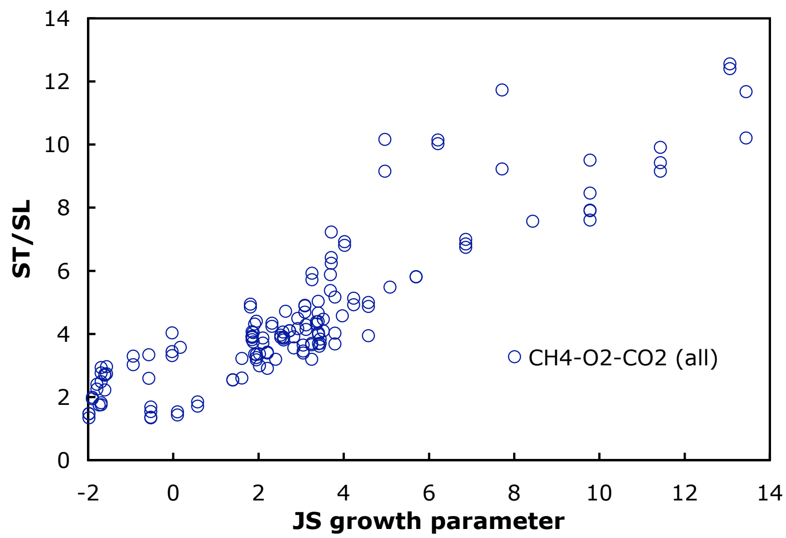
- Very similar for CH_4-O_2-CO_2 mixtures …



Effect of JS parameter

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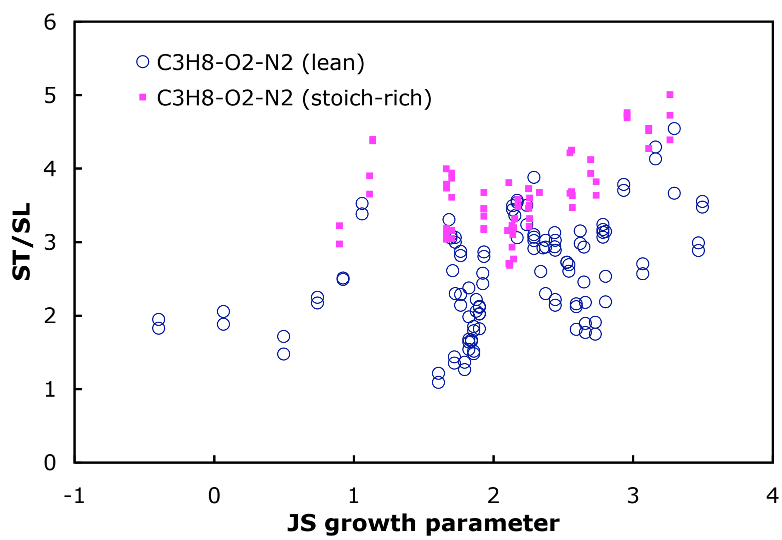
- … but propane far less impressive



Conclusions

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- Flame propagation in quasi-2D Hele-Shaw cells shows effects of
  - Thermal expansion - always present
  - Viscous fingering - narrow channels, long wavelengths
  - Buoyancy - destabilizing/stabilizing at long wavelengths for upward/downward propagation
  - Lewis number – affects behavior at small wavelengths but propagation rate & large-scale structure unaffected
  - Heat loss (Peclet number) – little effect since need only order $1/\beta$ reduction in temperature (thus density ratio) due to heat loss to cause extinction, but need order 1 change in expansion ratio to cause significant change in flow

Remark

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- Most experiments are conducted in open flames (Bunsen, counterflow, ...) - gas expansion relaxed in 3rd dimension
- … but most practical applications in confined geometries, where *unavoidable thermal expansion (DL) & viscous fingering (ST) instabilities cause propagation rates ≈ 3 S_L even when heat loss, Lewis number & buoyancy effects are negligible*
- DL & ST effects may affect propagation rates substantially even when strong turbulence is present - generates wrinkling up to scale of apparatus
  - $(S_T/S_L)_{Total} = (S_T/S_L)_{Turbulence} \times (S_T/S_L)_{ThermalExpansion}$?

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