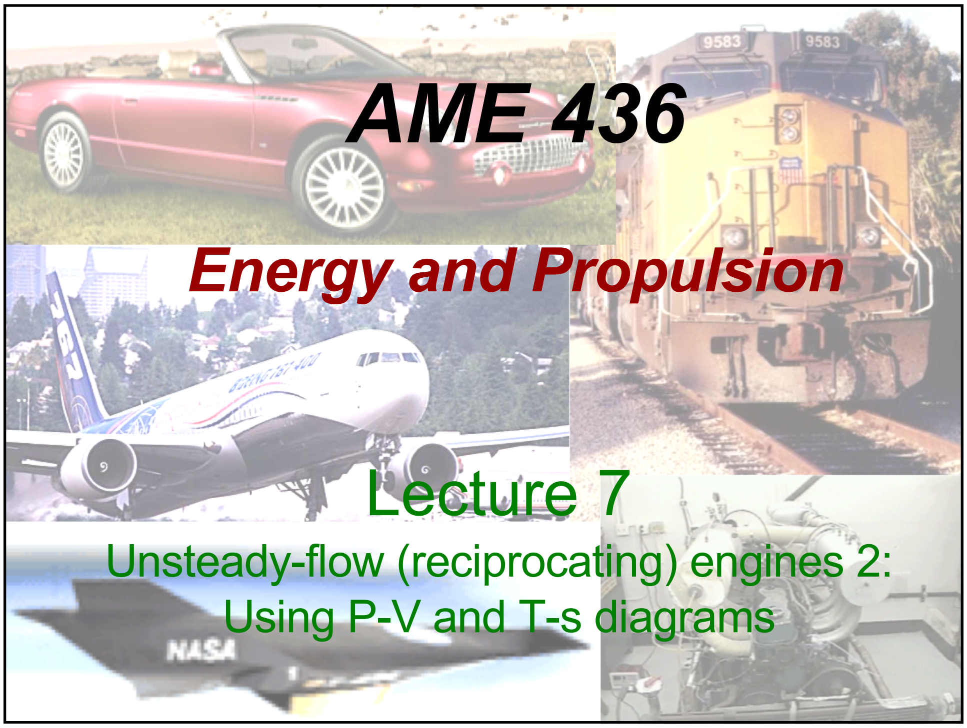


# AME 436

## Energy and Propulsion

### Lecture 7

### Unsteady-flow (reciprocating) engines 2: Using P-V and T-s diagrams

## Outline

USC Viterbi School of Engineering

- Air cycles
  - What are they?
  - Why use P-V and T-s diagrams?
- Using P-V and T-s diagrams for air cycles
  - Seeing heat, work and KE
  - Constant P and V processes
  - Inferring efficiencies
  - Compression & expansion component efficiencies
  - Correspondence between processes on P-V and T-s
  - Hints & tricks

## Air-cycles - what are they?

USC Viterbi School of Engineering

- Mostly we'll analyze "air cycles" - working fluid is air (or other ideal gas) with constant properties
- Changes in $C_P$, $M_i$, $\gamma$, etc. due to changes in composition, temperature, etc. are neglected; simplifies analysis, leads to simple analytic expressions for efficiency, power, etc.
- Later we'll analyze "fuel-air cycles" (using GASEQ) where the changes in $C_P$, $M_i$, $\gamma$, etc. due to changes in composition, temperature, etc. are considered
- "Fuel-air cycles" still can't account for slow burn, heat loss, etc. since it's still a thermodynamic analysis that tells us nothing about reaction rates, burning velocities, etc.
- P-V and T-s diagrams provide a clear visual representation of cycles



## Why use P-V diagrams?

USC Viterbi School of Engineering

- Pressure (P) vs. time and cylinder volume (V) vs. time are easily measured in reciprocating-piston engine experiments
- $\int PdV$ = work
- $\int PdV$ over whole cycle
  = net work transfer + net change in KE + net change in PE
  = net heat transfer
- Heat addition is usually modeled as constant P or constant V, so show as straight lines on P-V diagram
- Note that V on P-V diagram is cylinder volume (units $m^3$)
  - NOT a property of the gas (it's a property of the cylinder!)
  - Specific volume ($v = V/m$) (units $m^3/kg$) IS a property of the gas
  - We need to use V not v since cycle work = $\int PdV$
  - Can't use v since mass in the cylinder (m) changes during intake / blowdown / exhaust

## *Why use T-s diagrams?*

USC Viterbi School of Engineering

- Idealized compression & expansion modeled as constant S
  - $dS \geq \delta Q/T$ (an important consequence of the 2nd Law)
  - For adiabatic process $\delta Q = 0$, for reversible = (not >) sign applies, thus $dS = 0$
  - Note that $dS = 0$ still allows for any amount of work transfer (W) to or from the gas
  - Note 2nd law does NOT lead to $dS \geq \delta Q/T - \delta W/T$ because there is no entropy change due to work transfer
- For reversible process, $\int TdS = Q$, thus area under T-S curves show amount of heat transferred
- $\int TdS$ over whole cycle = net heat transfer = net work transfer + net change in KE + net change in PE
- T-S diagrams show the consequences of non-ideal compression or expansion ($dS > 0$)
- For ideal gases, $\Delta T$ ~ heat transfer or work transfer or $\Delta KE$
- Efficiency can be determined by breaking any cycle into Carnot-cycle "strips," each strip (i) having $\eta_{th,i} = 1 - T_{L,i}/T_{H,i}$



## *T-S & P-V for control mass: work, heat & KE*

USC Viterbi School of Engineering

- For an ideal gas with constant $C_P$ & $C_v$:

  $\Delta h = C_p\Delta T$, $\Delta u = C_v\Delta T$

  1st Law for a control mass with $\Delta PE = 0$ (OK assumption for IC engines)

  $dE = d(U + KE) = d[m(u + KE)] = \delta q - \delta w$

  (using "KE" as in "Kinetic Energy" rather than "$u^2/2$" avoid confusion with u {internal energy})

  $q = Q/m$; $w = W/m$ (heat or work transfer per unit mass)
- If no work transfer ($\delta w = 0$) or KE change ($\Delta KE = 0$), $du = C_v dT = \delta q \Rightarrow q_{1\to 2} = C_v(T_2 - T_1)$
- If no heat transfer ($\delta q = 0$) or $\Delta KE$ $du = C_v dT = \delta w \Rightarrow w_{1\to 2} = -C_v(T_2 - T_1)$
- If no work or heat transfer ($\delta q = \delta w = 0$) $\Delta KE = KE_2 - KE_1 = -C_v(T_2 - T_1)$

$\Rightarrow$ For a control mass containing an ideal gas with constant $C_v$, $\Delta T$ ~ heat transfer - work transfer - $\Delta KE$

- Note that the 2nd law was not invoked, thus the above statements are true for any process, reversible or irreversible

## T-s & P-v for control volume: work, heat & KE

USC Viterbi School of Engineering

- 1st Law for a control volume, steady flow, with $\Delta PE = 0$

$$0 = \dot{Q} - \dot{W} + \dot{m}\left[(h_{in} - h_{out}) + (KE_{in} - KE_{out})\right]$$

- For ideal gas with constant $C_P$, $dh = C_P dT \Rightarrow h_2 - h_1 = C_P(T_2 - T_1)$
- If 2 = outlet, 1 = inlet, and noting $\dot{Q}/\dot{m} = q_{1\to 2}; \dot{W}/\dot{m} = w_{1\to 2}$

$$h_2 - h_1 = q_{1\to 2} - w_{1\to 2} - (KE_2 - KE_1)$$

- If no work transfer KE change ($w_{1\to 2} = \Delta KE = 0$)
  $dh = C_P dT = dq \Rightarrow q_{1\to 2} = C_P(T_2 - T_1)$
- If no heat transfer ($dq = 0$) or $\Delta KE$
  $dh = C_P dT = \delta w \Rightarrow w_{1\to 2} = -C_P(T_2 - T_1)$
- If no work or heat transfer
  $\Delta KE = KE_2 - KE_1 = -C_P(T_2 - T_1)$

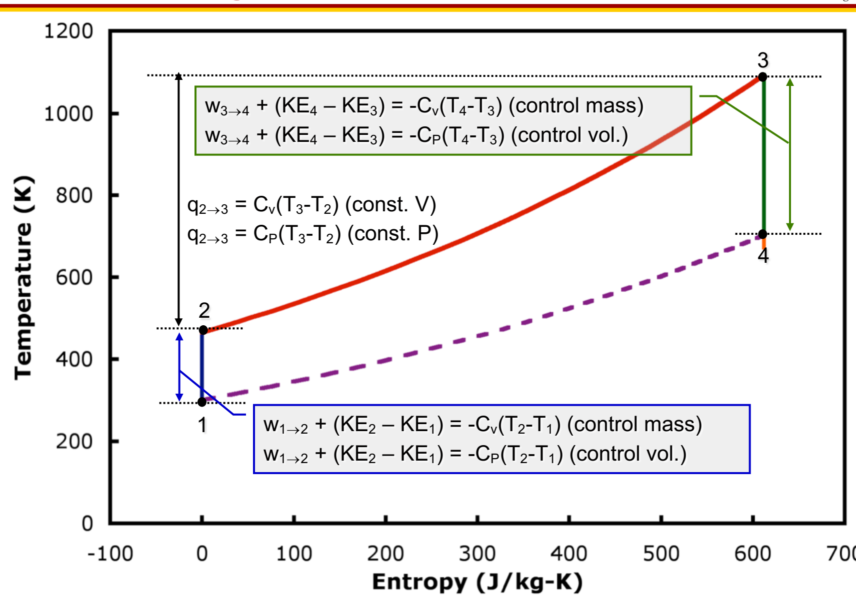
$\Rightarrow$ For a control volume containing an ideal gas with constant $C_P$, $\Delta T$ ~ heat transfer - work transfer - $\Delta KE$
(Same statement as control mass with $C_P$ replacing $C_v$)

- Again true for any process, reversible or irreversible



## T-s & P-v diagrams: work, heat & KE

USC Viterbi School of Engineering




Case shown: Const. Vol. heat input, $r_c = r_e = 3$, $\gamma = 1.4$, $\Delta T_{comb} = fQ_R/C_v = 628K$, $P_1 = 0.5$ atm

## T-s & P-v diagrams: work, heat, KE & PE

USC Viterbi School of Engineering

- How do I know that work shown on the P-V (via ∫PdV) diagram is the same as that shown (via $C_v\Delta T$) on the T-s diagram? As an example, for isentropic compression ($PV^\gamma$ = constant)

$$W_{2\to3} = \int_2^3 P\,dV = \int_2^3 \frac{P_2V_2^\gamma}{V^\gamma}dV = -\frac{P_2V_2^\gamma}{\gamma-1}\left(\frac{1}{V_3^{\gamma-1}} - \frac{1}{V_2^{\gamma-1}}\right)$$

$$= -\frac{P_2V_2^1}{\gamma-1}\left(\frac{V_2^{\gamma-1}}{V_3^{\gamma-1}} - \frac{V_2^{\gamma-1}}{V_2^{\gamma-1}}\right) = -\frac{mRT_2}{\gamma-1}\left(\left(\frac{V_2}{V_3}\right)^{\gamma-1} - 1\right)$$

$$= -mC_vT_2\left(\left(\left(\frac{T_3}{T_2}\right)^{\frac{1}{\gamma-1}}\right)^{\gamma-1} - 1\right) = -mC_vT_2\left(\frac{T_3}{T_2} - 1\right) = -mC_v\left(T_3 - T_2\right)$$



## T-s & P-v diagrams: work, heat, KE & PE

USC Viterbi School of Engineering

- Going back to the 1st Law again

$$dE = \delta Q - \delta W \Rightarrow \oint dE = \oint \delta Q - \oint \delta W$$

- Around a closed path, since E = U + KE + PE; since U is a property of the system, $\oint dU = 0$, thus around a closed path, i.e. a complete thermodynamic cycle (neglecting PE again)

$$\oint \delta Q = \oint \delta W + \Delta KE$$

But wait - does this mean that the thermal efficiency

$$\eta_{th} = \frac{\oint \delta W + \Delta KE}{\oint \delta Q} = 1?$$

No, the definition of thermal efficiency is

$$\eta_{th} = \frac{\text{What you get}}{\text{What you pay for}} = \frac{\oint \delta W + \Delta KE}{\text{Heat in}} = \frac{\text{Heat in} - \text{heat out}}{\text{Heat in}} < 1$$

- For a reversible process, δQ = TdS, thus δq = Tds and

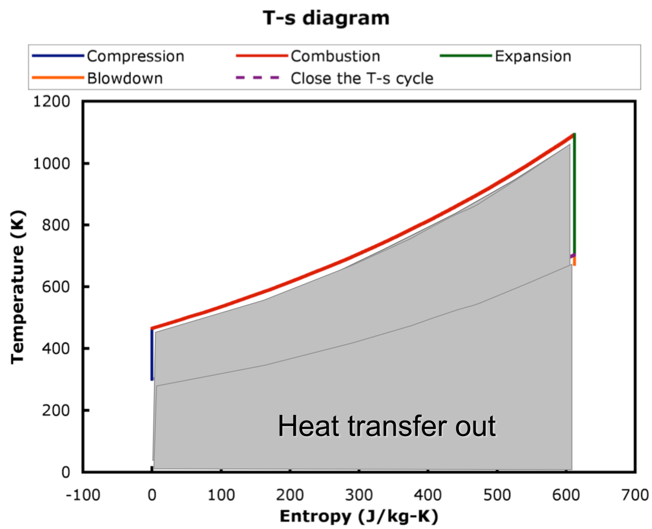
$$\oint Tds = \oint \delta q = \oint \delta w + \Delta KE/m$$

- Thus, for a reversible process, the area inside a cycle on a T-s diagram is equal to both (net work transfer + net KE) and net heat transfer

## T-s & P-v diagrams: work, heat & KE

USC Viterbi School of Engineering

- Animation: using T-s diagram to determine heat & work






## Constant P and V curves

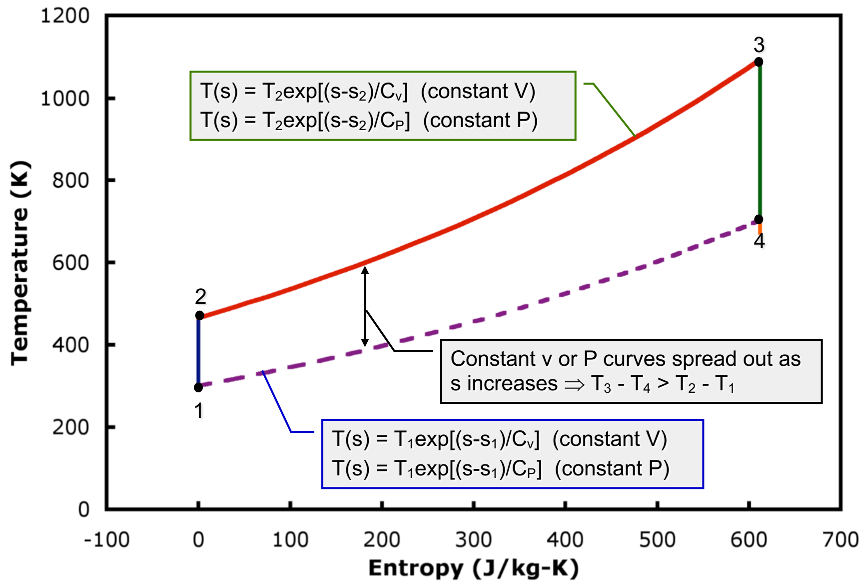
USC Viterbi School of Engineering

- For ideal gas with constant specific heats, review of thermodynamics (1st lecture) showed that

$$s_2 - s_1 = C_P \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right);\ s_2 - s_1 = C_V \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right)$$

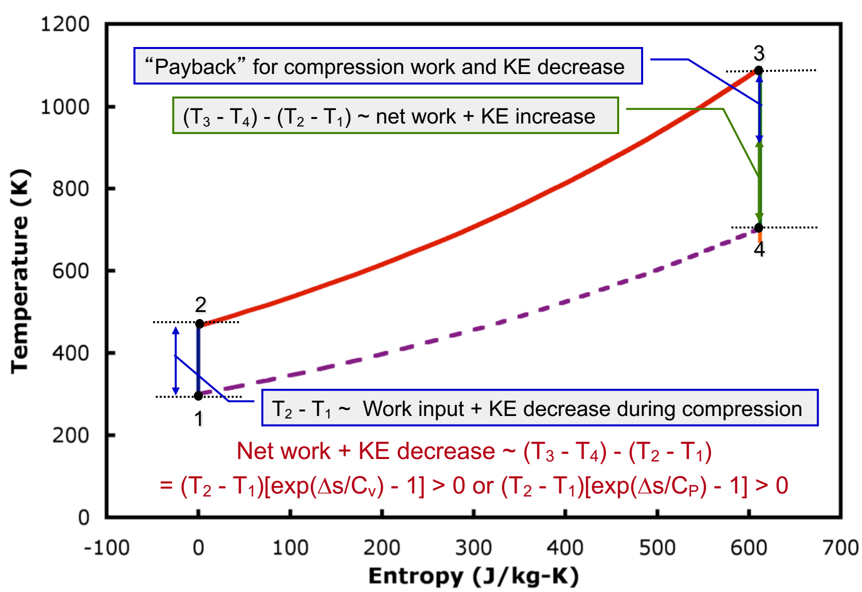
- If P = constant, $\ln(P_2/P_1) = 0 \Rightarrow T_2 = T_1\exp[(s_2-s_1)/C_P]$
- If V = constant, $\ln(V_2/V_1) = 0 \Rightarrow T_2 = T_1\exp[(s_2-s_1)/C_v]$
  $\Rightarrow$ constant P or V curves are exponentials on a T-S diagram
- Since constant P or V curves are exponentials, as S increases, the $\Delta T$ between two constant-P or constant-V curves increases; this ensures that compression work is less than expansion work for ideal Otto (const. V) or Brayton (const. P) cycles
- Since $C_P = C_v + R$, $C_P > C_v$ or $1/C_P < 1/C_v$, constant V curves rise faster than constant P curves on a T-s diagram
- Constant P or constant V lines cannot cross (unless they correspond to cycles with different $C_P$ or $C_V$)

## Constant P and V curves



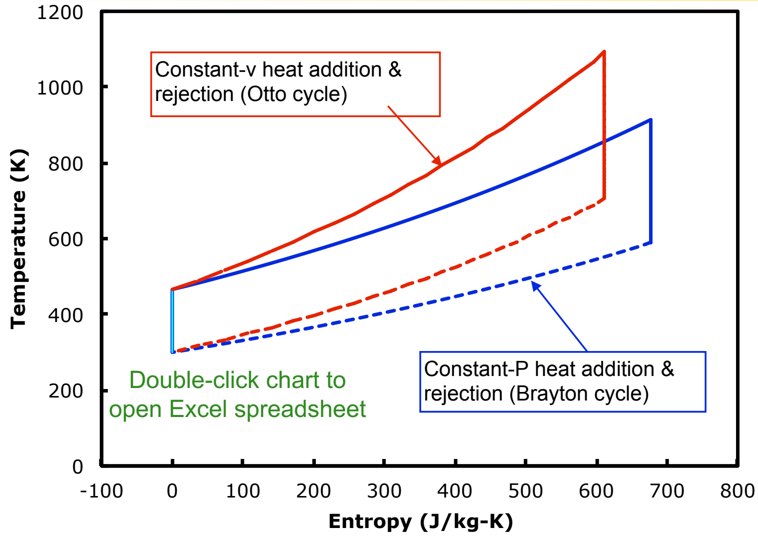



## Constant P and V curves



## *Constant P and V curves*

USC Viterbi School of Engineering




Constant-v curves are steeper than constant-P curves on the T-s

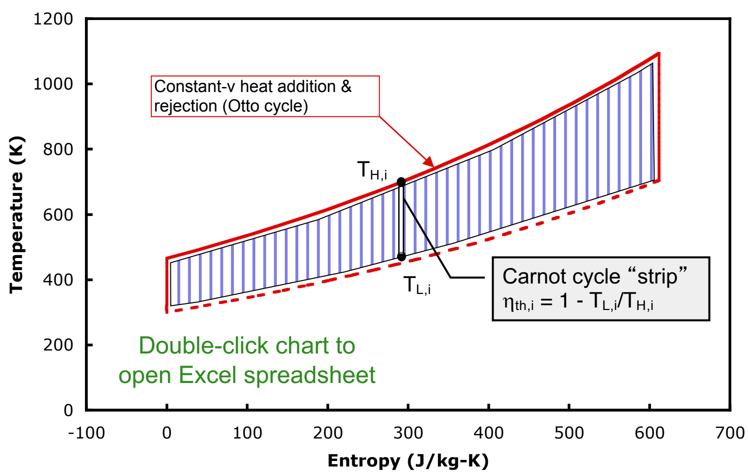
Both cases: $T_2/T_1$ = 1.552, $\gamma$ = 1.4, $fQ_R$ = 4.5 x $10^5$ J/kg, $P_1$ = 0.5 atm

The two cycles shown also have the same thermal efficiency ($\eta_{th}$)



## *Inferring efficiencies*

USC Viterbi School of Engineering




Carnot cycles appear as rectangles on the T-s diagram; any cycle can be broken into a large number of tall skinny Carnot cycle "strips," each strip (i) having $\eta_{th,i} = 1 - T_{L,i}/T_{H,i}$

## Compression & expansion efficiency

USC Viterbi School of Engineering

- If irreversible compression or expansion, $dS > \delta Q/T$; if still adiabatic ($\delta Q = 0$) then $dS > 0$
- Causes more work input (more $\Delta T$) during compression, less work output (less $\Delta T$) during expansion
- Define compression efficiency $\eta_{comp}$ & expansion efficiency $\eta_{exp}$

$$\eta_{comp} \equiv \frac{\text{Reversible adiabatic work input for given V or P ratio}}{\text{Actual work input required for same V or P ratio}}$$

$$= \frac{-C_V[T_1(V_1/V_2)^{\gamma-1} - T_1]}{-C_V(T_2 - T_1)} = \frac{(V_1/V_2)^{\gamma-1} - 1}{T_2/T_1 - 1}$$

(control mass, specified volume ratio)

$$\eta_{\exp} \equiv \frac{\text{Actual work output for given V or P ratio}}{\text{Reversible adiabatic work output for same V or P ratio}}$$

$$= \frac{-C_V(T_2 - T_1)}{-C_V[T_1(V_1/V_2)^{\gamma-1} - T_1]} = \frac{T_2/T_1 - 1}{(V_1/V_2)^{\gamma-1} - 1}$$

(control mass, specified volume ratio)



## Compression & expansion efficiency

USC Viterbi School of Engineering

$$\eta_{comp} = \frac{-C_V[T_1(P_2/P_1)^{(\gamma-1)/\gamma} - T_1]}{-C_V(T_2 - T_1)} = \frac{(P_2/P_1)^{(\gamma-1)/\gamma} - 1}{T_2/T_1 - 1}$$

(control mass, specified pressure ratio)

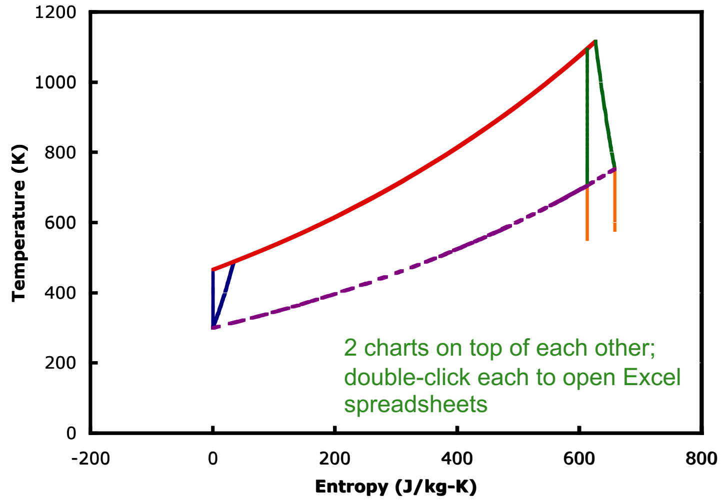
$$\eta_{\exp} = \frac{-C_V(T_2 - T_1)}{-C_V[T_1(P_2/P_1)^{(\gamma-1)/\gamma} - T_1]} = \frac{T_2/T_1 - 1}{(P_2/P_1)^{(\gamma-1)/\gamma} - 1}$$

(control mass, specified pressure ratio)

- Control volume: replace u (internal energy) with h (enthalpy) and thus replace $C_v$ with $C_P$, but it cancels out so definitions are same
- These relations give us a means to quantify the efficiency of an engine component (e.g. compressor, turbine, …) or process (compression, expansion) as opposed to the whole cycle

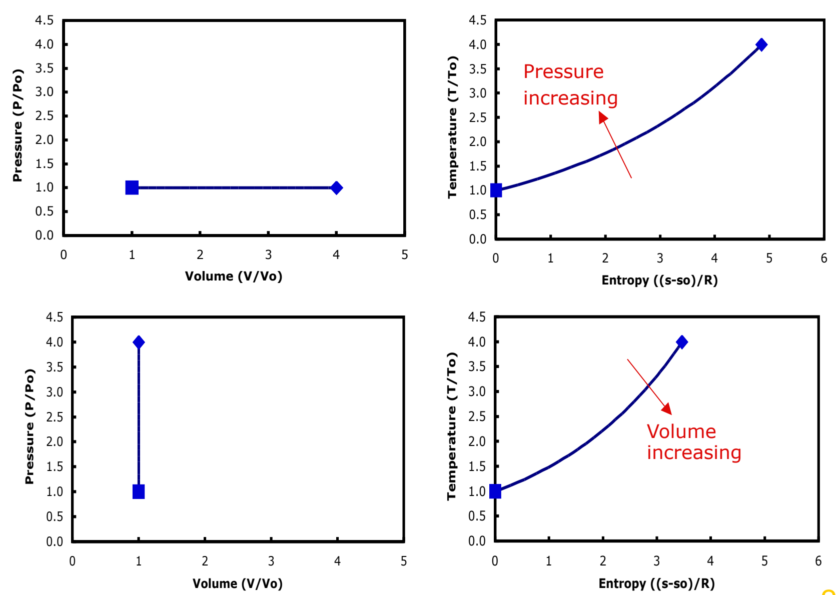
## Compression & expansion efficiency

- Animation: comparison of ideal Otto cycle with non-ideal compression & expansion
- Same parameters as before but with $\eta_{comp} = \eta_{exp} = 0.9$



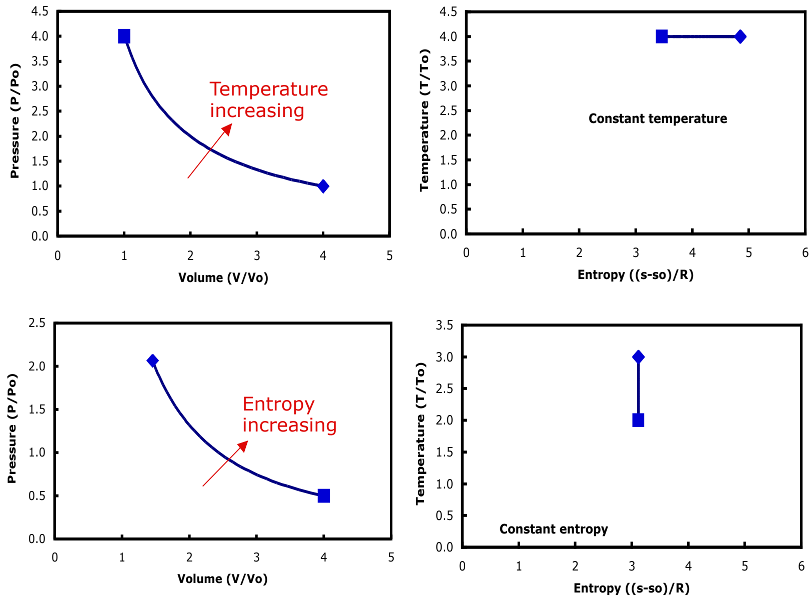



## Correspondence between P-V & T-s



## Correspondence between P-V & T-s

USC Viterbi School of Engineering



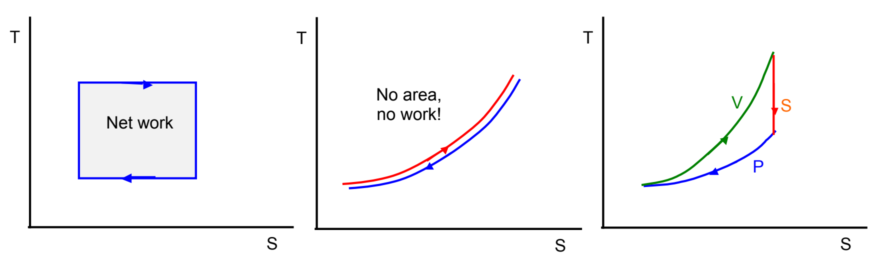



## Example

USC Viterbi School of Engineering

- Why do internal combustion engines compress before burning? Is it possible to produce work or thrust without compression?

To generate positive area (thus net work) on a T-s diagram (left), the pre-heat-addition process must be more nearly vertical than the heat addition process, otherwise there is no area thus no work (middle). The best way to do this is with isentropic compression followed by heat addition (left) - but it's not required. You could have (for example) constant-V heat addition followed by isentropic expansion back to ambient P (right); work would be generated (although at lower efficiency) without a compressor

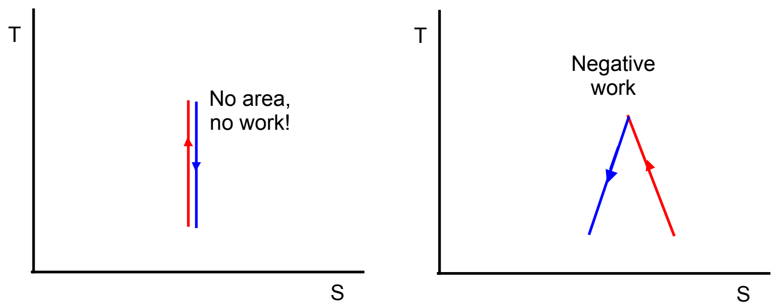


## Example

USC Viterbi School of Engineering

- Why is it necessary to add heat to generate work or thrust?

Without heat transfer, dq = 0 and thus (for reversible cycles) TdS = 0, thus $\oint T\,dS = \oint P\,dV$ = Net work = 0 (left figure). If the process is irreversible, TdS < 0, thus $\oint T\,dS$ < 0 and thus = $\oint P\,dV$ = Net work < 0 (right figure).



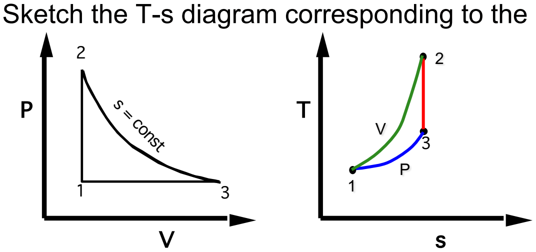



## Example

USC Viterbi School of Engineering

Consider the "Lenoir cycle" engine shown on the P-V diagram

a) Sketch the T-s diagram corresponding to the P-V diagram

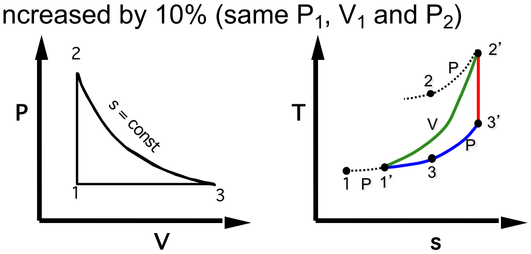



*1 → 2 const. V, increasing P ⇒ heat addition, s increases*

*2 → 3 const. s, decreasing P ⇒ expansion, T decreases*

*3 → 1 const. P, decreasing v ⇒ heat rejection, s decreases*

b) Sketch modified P-V and T-s diagrams if the initial temperature is increased by 10% (same $P_1$, $V_1$ and $P_2$)




*P - V plot unchanged (but higher T means less mass processed)*

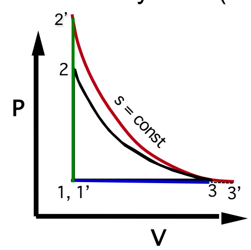
*Higher T, same P ⇒ $s_1$, $s_2$ increases*

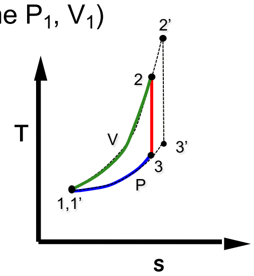
*Still need 3 → 1 const. P*

## Example

USC Viterbi School of Engineering

c) Sketch modified P-V and T-s diagrams if the "compression ratio" $V_3/V_1$ is increased by 20% (same $P_1$, $V_1$)



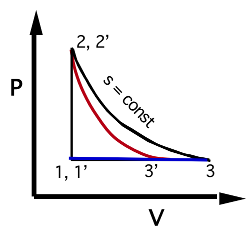



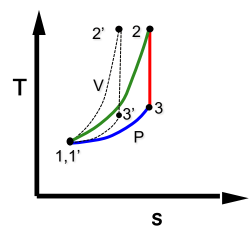

*1 → 2 const. V, $P_2$ larger than base cycle*

*On P-V, curves of const. T slope less steeply than const. s, thus $T_3' > T_3$*

*Still need 3 → 1 const. P*

d) Sketch modified P-V and T-s diagrams if a gas with higher $\gamma$ is used (same $P_1$, $V_1$, $P_2$, R)




*$PV^\gamma$ curves steeper if $\gamma$ increases*

*$C_V = R/(\gamma-1)$; if R = const. and $\gamma$ increases, $C_V$ decreases, thus const.-v curves on T – s are steeper*

*$T_2$ doesn't change since $V_1 = V_2$, $P_1$, $P_2$ and R don't change*



## Using T-s and P-v diagrams - summary

USC Viterbi School of Engineering

- Thermodynamic cycles as they occur in IC engines are often approximated as a series of processes occurring in an ideal gas
- T-S and P-V diagrams are very useful for inferring how changes in a cycle affect efficiency, power, peak P & T, etc.
- The $\Delta$T (on T-S diagrams) and areas (both T-S & P-V) are very useful for inferring heat & work transfers

## Using T-s and P-v diagrams - summary

USC Viterbi School of Engineering

- Each process (curve or line) on a T-s or P-v diagram has 3 parts
  - An initial state
  - A process (const. P, V, T, S, area, etc.)
  - A final state
    - » For compression and expansion processes in piston engines, a specified V (i.e., a particular compression or expansion ratio)
    - » For compressors in propulsion cycles, a specified pressure ratio
    - » For turbines in propulsion cycles, a specified temperature that makes the work output from the turbine equal the work required to drive compressor and/or fan
    - » For diffusers in propulsion cycles, a specified Mach number
    - » For nozzles in propulsion cycles, the pressure after expansion (usually ambient pressure)
    - » For heat addition processes, either a specified heat input = $\int Tds$ (i.e. a mixture having a specified FAR and $Q_R$) thus a given area on the T-s diagram, or a specified temperature (i.e. for temperature limited turbines in propulsion cycles)
    - » The constant P and constant V exponential curves on the T-S diagram are very useful for determining end states



## Using T-s and P-V diagrams - summary

USC Viterbi School of Engineering

- Three or more processes combine to make a complete cycle
- When drawing P-V or T-S diagrams, ask yourself
  - What is the P, V, T and S of the initial state? Is it different from the baseline case?
  - For each subsequent process
    - » What is the process? Is it the same as the baseline cycle, or does it change from (for example) reversible to irreversible compression or expansion? Does it change from (for example) constant pressure heat addition to heat addition with pressure losses?
    - » When the process is over? Is the target a specified pressure, volume, temperature, heat input, work output, etc.?
    - » In gas turbine cycles, be sure to make work output of turbines = work input to compressors and fans in gas
- Be sure to close the cycle by having (for reciprocating piston cycles) the final volume = initial volume or (for propulsion cycles) (usually) the final pressure = ambient pressure