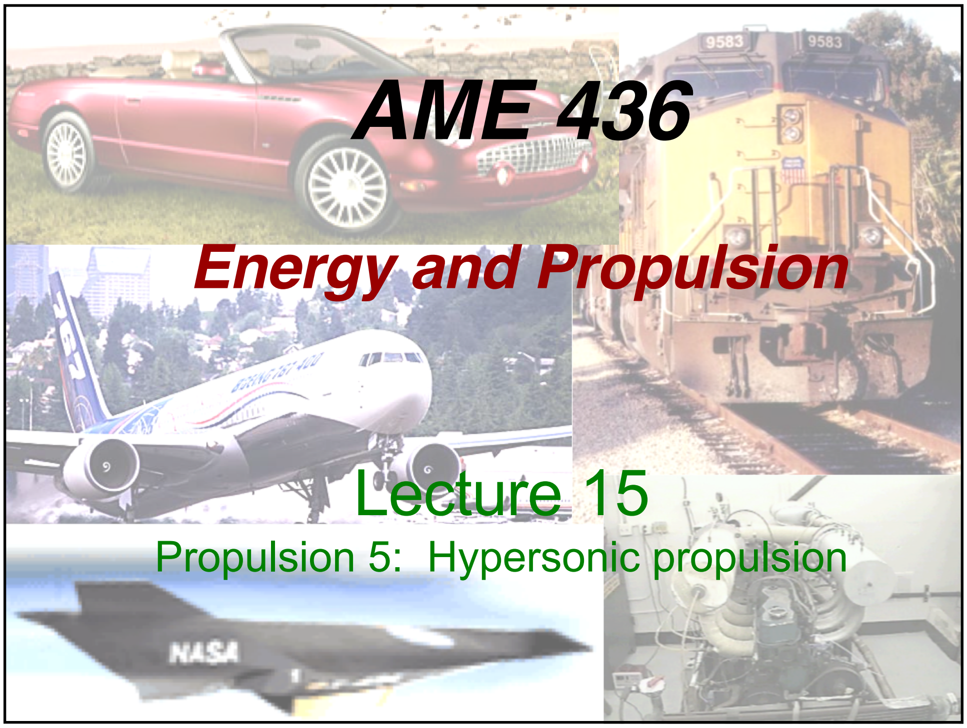


AME 436

Energy and Propulsion

Lecture 15

Propulsion 5: Hypersonic propulsion

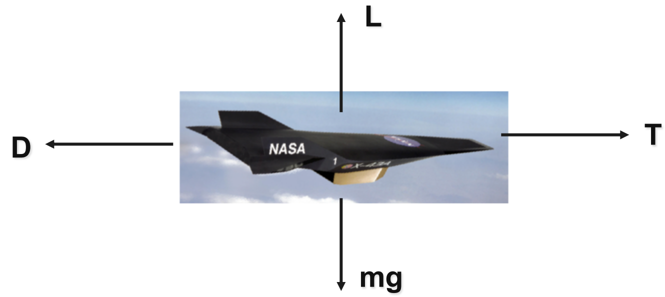
USC Viterbi School of Engineering

Outline

- Why hypersonic propulsion?
- What's different about it?
- “Conventional” ramjet (heat addition at $M << 1$)
- Heat addition in compressible flows at $M \neq 0$ (shortened version of what I didn’t cover in Lecture 12)
- AirCycles4Hypersonics.xls spreadsheet
- “Scramjet” cycles and performance

Hypersonic propulsion - motivation

- Why use air even if you're going to space?
  - Carry only fuel, not fuel + O_2, while in atmosphere
    - » 8x mass savings (H_2-O_2), 4x (hydrocarbons)
    - » Actually more than this when ln( ) term in Breguet range equation is considered
  - Use aerodynamic lifting body rather than ballistic trajectory
    - » Ballistic: need Thrust/weight > 1
    - » Lifting body, steady flight: Lift (L) = weight (mg); Thrust (T) = Drag (D), Thrust/weight = L/D > 1 for any decent airfoil, even at hypersonic conditions



What's different about hypersonic propulsion?

- Stagnation temperature T_t - measure of total energy (thermal + kinetic) of flow - is really large even before heat addition - materials problems

$$T_t = T\left(1 + \frac{\gamma - 1}{2}M^2\right)$$

- Stagnation pressure - measure of usefulness of flow (ability to expand flow) is really large even before heat addition - structural problems

$$P_t = P\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\gamma/\gamma-1}$$

- Large P_t means no mechanical compressor needed at large M
- Why are T_t and P_t so important? Recall (Lecture 12) isentropic expansion to $P_e = P_a$ (optimal exit P yielding maximum thrust) yields

$$u = \sqrt{\frac{2\gamma}{\gamma - 1}RT_t\left(1 - \left(\frac{P_9}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

- … but it's difficult to add heat at high M without major loss of stagnation pressure

What's different about hypersonic propulsion?

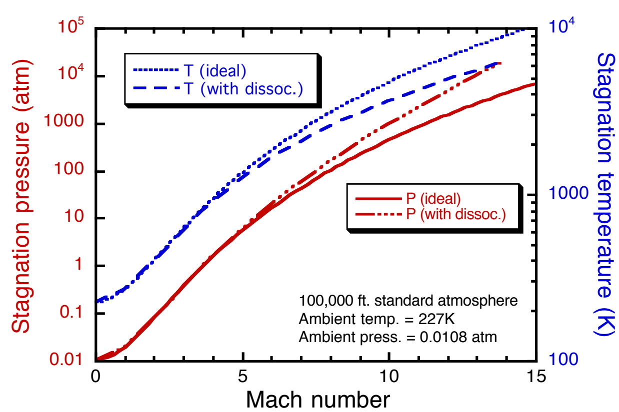
USC Viterbi School of Engineering

- High temperatures: γ not constant, also molecular weight not constant - dissociation - use GASEQ (http://www.gaseq.co.uk) to compute stagnation conditions
- Example calculation: standard atmosphere at 100,000 ft
  - T_1 = 227K, P_1 = 0.0108 atm, c_1 = 302.7 m/s, h_1 = 70.79 kJ/kg (atmospheric data from http://www.digitaldutch.com/atmoscalc/)
  - Pick $P_2 > P_1$, compress isentropically, note new T_2 and h_2
  - 1st Law: $h_1 + u_1^2/2 = h_2 + u_2^2/2$; since $u_2 = 0$, $h_2 = h_1 + (M_1c_1)^2/2$ or $M_1 = [2(h_2-h_1)/c_1^2]^{1/2}$
  - Simple relations ok up to $M \approx 7$
  - Dissociation not as bad as might otherwise be expected at ultra high T, since P increases faster than T
- Limitations of these estimates
  - Ionization not considered
  - Stagnation temperature relation valid even if shocks, friction, etc. (only depends on 1st law) but stagnation pressure assumes isentropic flow
  - Calculation assumed adiabatic deceleration - radiative loss (from surfaces and ions in gas) may be important

What's different about hypersonic propulsion?

USC Viterbi School of Engineering

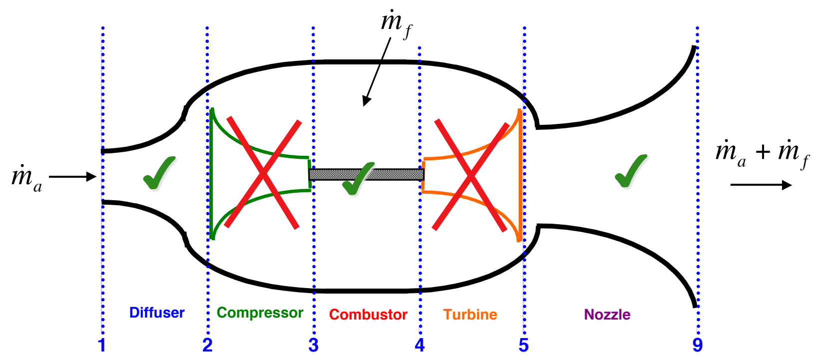
WOW!	HOT	WARM	COLD
5000K	3000K	1000K	200K
$N+O+e^-$	N_2+O	N_2+O_2	N_2+O_2



"Conventional" ramjet

USC Viterbi School of Engineering

- Incoming air decelerated isentropically to M = 0 - high T, P
- No compressor needed, so only parameters are M_1
- Heat addition at M = 0 - no loss of P_t - to max. allowable $T_4 = \tau_\lambda T_1$
- Expand to $P_9 = P_1$
- Doesn't work well at low M - P_t/P_1 & T_t/T_1 low - Carnot efficiency low
- As M increases, P_t/P_1 and T_t/T_1 increases, cycle efficiency increases, but if M too high, limited ability to add heat (T_t close to T_{max}) - high efficiency but less thrust

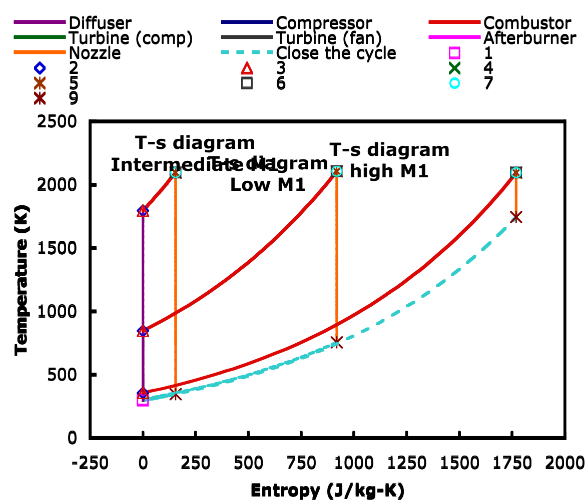


"Conventional" ramjet - effect of M_1

USC Viterbi School of Engineering

- "Banana" shaped cycles for low M_1, tall skinny cycles for high M_1, "fat" cycles for intermediate M_1

Basic ramjet
$\tau_\lambda = 7$



"Conventional" ramjet example

USC Viterbi School of Engineering

- Example: $M_1 = 5$, $\tau_\lambda = 12$, $\gamma = 1.4$
- Initial state (1): $M_1 = 5$
- State 2: decelerate to $M_2 = 0$

$$T_2 = T_1\left(1+\frac{\gamma-1}{2}M_1^2\right) = T_1\left(1+\frac{1.4-1}{2}5^2\right) = 6T_1$$

$$P_2 = P_1\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1} = P_1\left(1+\frac{1.4-1}{2}5^2\right)^{1.4/1.4-1} = 529.1P_1$$

- State 4: add at heat const. P; $M_4 = 0$, $P_4 = P_2 = 529.1P_1$, $T_4 = 12T_1$
- State 9: expand to $P_9 = P_1$

$$P_4 = 529.1P_1 = P_9\left(1+\frac{\gamma-1}{2}M_9^2\right)^{\gamma/\gamma-1} = P_1\left(1+\frac{1.4-1}{2}M_9^2\right)^{1.4/1.4-1} \Rightarrow M_9 = 5.00$$

$$T_4 = 12T_1 = T_9\left(1+\frac{\gamma-1}{2}M_9^2\right) = T_9\left(1+\frac{1.4-1}{2}5^2\right) \Rightarrow T_9 = 2T_1$$

"Conventional" ramjet example

USC Viterbi School of Engineering

- Specific thrust (ST) (assume FAR << 1)

$$Thrust = \dot{m}_a\left[\left(1+FAR\right)u_9 - u_1\right] + \left(P_9 - P_1\right)A_9; FAR << 1, P_9 = P_1$$

$$\Rightarrow ST = \frac{Thrust}{\dot{m}_a c_1} = \frac{u_9}{c_1} - \frac{u_1}{c_1} = \frac{u_9}{c_9}\frac{c_9}{c_1} - M_1 = M_9\sqrt{\frac{T_9}{T_1}} - M_1 = 5\sqrt{2} - 5 = 2.07$$

- TSFC and overall efficiency

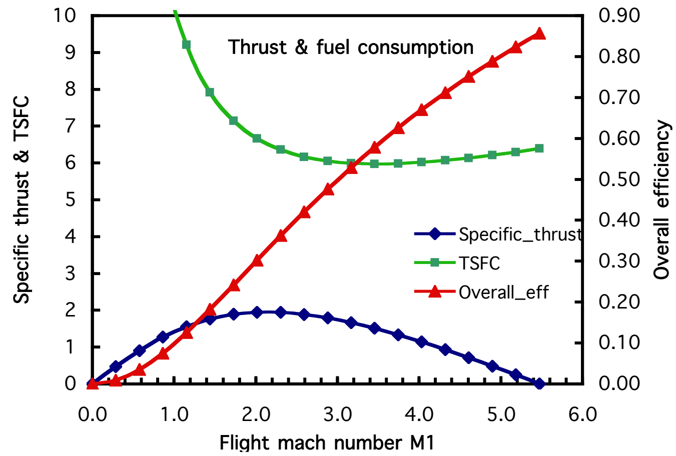
$$TSFC \equiv \frac{\text{Heat input}}{Thrust \cdot c_1} = \frac{\dot{m}_a C_P(T_{4t} - T_{2t})}{Thrust \cdot c_1} = \frac{\dot{m}_a c_1}{Thrust}\frac{C_P}{c_1^2}\left(T_{4t} - T_{2t}\right)$$

$$= \frac{1}{\text{ST}}\frac{[\gamma/(\gamma-1)]R}{\gamma R T_1}\left(12T_1 - 6T_1\right) = \frac{1}{2.07}\frac{1}{1.4-1}\left(12-6\right) = 7.24;$$

$$\eta_o = \frac{M_1}{TSFC} = \frac{5}{7.24} = 0.691$$

"Conventional" ramjet - effect of M_1

Basic ramjet
$\tau_\lambda = 7$



Scramjet (Supersonic Combustion RAMjet)

- What if $T_t > T_{max}$ allowed by materials or $P_t > P_{max}$ allowed by structure? Can't decelerate to M = 0!
- Need to mix fuel & burn supersonically, never allowing air to decelerate to M = 0
- Many laboratory studies, very few successful test flights (e.g. X-43 below)

Scramjet (Supersonic Combustion RAMjet)

USC Viterbi School of Engineering

- Australian project (HyShot):
  - http://hypersonics.mechmining.uq.edu.au/hyshot-about
- US project (X-43)
  - https://www.nasa.gov/centers/dryden/history/pastprojects/HyperX/index.html
  - Steady flight (thrust ≈ drag) achieved at $M_1 \approx 9.65$ at 110,000 ft altitude ($u_1 \approx 2934$ m/s = 6562 mi/hr)
  - 3.8 lbs. H_2 burned during 10 - 12 second test
  - Rich H_2-air mixtures ($\phi \approx 1.2 - 1.3$), ignition with silane (SiH_4, ignites spontaneously in air)
  - …but no information about the conditions at the combustor inlet, or the conditions during combustion (constant P, T, area, …?)
  - Real-gas stagnation temperatures 3300K (my model, slide 36: 3500K), surface temperatures up to 2250K (!)
- USAF X-51
  - http://www.af.mil/About-Us/Fact-Sheets/Display/Article/104467/x-51a-waverider/
  - Acceleration to steady flight achieved at $M_1 \approx 5$ at 70,000 ft for 140 seconds using hydrocarbon fuel

1D steady flow of ideal gases (Lecture 11)

USC Viterbi School of Engineering

- Assumptions
  - Ideal gas, steady, quasi-1D
  - Constant C_P, C_v, $\gamma \equiv C_P/C_v$
  - Unless otherwise noted: adiabatic, reversible, constant area
  - Note since 2nd Law states $dS \geq \delta Q/T$ (= for reversible, > for irreversible), reversible + adiabatic ⇒ isentropic ($dS = 0$)
- Governing equations
  - Equations of state $h_2 - h_1 = C_P(T_2 - T_1)$
    $P = \rho RT$; $S_2 - S_1 = C_p \ln(T_2/T_1) - R\ln(P_2/P_1)$
  - Isentropic ($S_2 = S_1$) (where applicable): $P_2/P_1 = (T_2/T_1)^{\gamma/(\gamma-1)}$
  - Mass conservation: $\dot{m} = \rho_1 u_1 A_1 = \rho_2 u_2 A_2$
  - Momentum conservation, constant area duct (see lecture 11):
    $$AdP + \dot{m}du + C_f(\rho u^2/2)Cdx = 0$$
    - » C_f = friction coefficient; C = circumference of duct
    - » No friction: $P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$
  - Energy conservation: $h_1 + u_1^2/2 + q - w = h_2 + u_2^2/2$
    q = heat input per unit mass = fQ_R if due to combustion
    w = work output per unit mass

1D steady flow of ideal gases (Lecture 11)

USC Viterbi School of Engineering

- Types of analyses: everything constant except…
  - Area (isentropic nozzle flow) – already covered
  - Entropy (shock) - skip
  - Momentum (Fanno flow) (constant area with friction) - skip
  - Diabatic (q ≠ 0) - several possible assumptions - covered here
    - » Constant area A (Rayleigh flow) (useful if limited by space)
    - » Constant T (useful if limited by materials) (sounds weird, heat addition at constant T…)
    - » Constant P (useful if limited by structure)
- Products of analyses
  - Stagnation temperature
  - Stagnation pressure
  - Mach number = u/c = u/(γRT)$^{1/2}$ (recall c = sound speed at local conditions in the flow (NOT at ambient condition!))
  - From this, can get exit velocity u_9, exit pressure P_9 and thus thrust

Heat addition at const. Area (Rayleigh flow)

USC Viterbi School of Engineering

- Mass, momentum, energy, equation of state all apply
- Reference state ( )*: use M = 1 (not a throat in this case!)
- Energy equation not useful except to calculate heat input ($q = C_p(T_{2t} - T_{1t})$)

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2;\ u_1^2 = \gamma R T_1;\ u_2^2 = \gamma R T_2;\ \rho_1 = P_1 / RT_1;\ \rho_2 = P_2 / RT_2$$

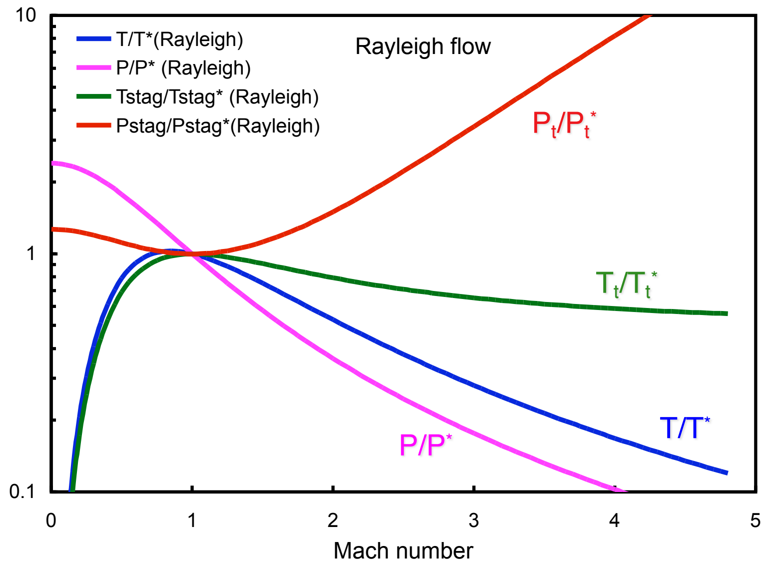
$$\text{Combine: } \frac{P_2}{P_1} = \frac{1+\gamma M_2^2}{1+\gamma M_1^2};\ \text{use reference state ( )}^* \text{ where M = 1: } \frac{P}{P^*} = \frac{1+\gamma}{1+\gamma M^2}$$

Mass conservation with A = const.: $\rho_1 u_1 = \rho_2 u_2$; combine with above to obtain

$$\frac{T}{T^*} = \left(\frac{1+\gamma}{1+\gamma M^2}\right)^2 M^2;\ \frac{T_t}{T_t^*} = \frac{2(\gamma+1)M^2\left(1+\frac{\gamma-1}{2}M^2\right)}{\left(1+\gamma M^2\right)^2}$$

$$\frac{P_t}{P_t^*} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}\left(\frac{1+\gamma}{1+\gamma M^2}\right)\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1}$$

Heat addition at constant area



Heat addition at const. area

- Implications
  - Stagnation temperature always increases towards M = 1
  - Stagnation pressure always decreases towards M = 1 (stagnation temperature increasing, more heat addition)
  - Can't cross M = 1 with constant area heat addition!
  - M = 1 corresponds to the maximum possible heat addition
  - …but there's no particular reason we have to keep area (A) constant when we add heat!
- What if neither the initial state (1) nor final state (2) is the choked (*) state? Use $P_2/P_1 = (P_2/P^*)/(P_1/P^*)$ etc.

$$\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}; \frac{T_2}{T_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \frac{M_2^2}{M_1^2};$$

$$\frac{T_{2t}}{T_{1t}} = \frac{M_2^2}{M_1^2}\frac{\left(1+\gamma M_1^2\right)^2}{\left(1+\gamma M_2^2\right)^2}\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}; \frac{P_{2t}}{P_{1t}} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\gamma/\gamma-1}$$

Heat addition at constant pressure

USC Viterbi School of Engineering

- Relevant for hypersonic propulsion if maximum allowable pressure (i.e. structural limitation) is the reason we can't decelerate the ambient air to M = 0)
- Momentum equation: AdP + $\dot{m}$ du = 0 $\Rightarrow$ u = constant
- Reference state ( )*: use M = 1 again but nothing special happens there
- Again energy equation not useful except to calculate q

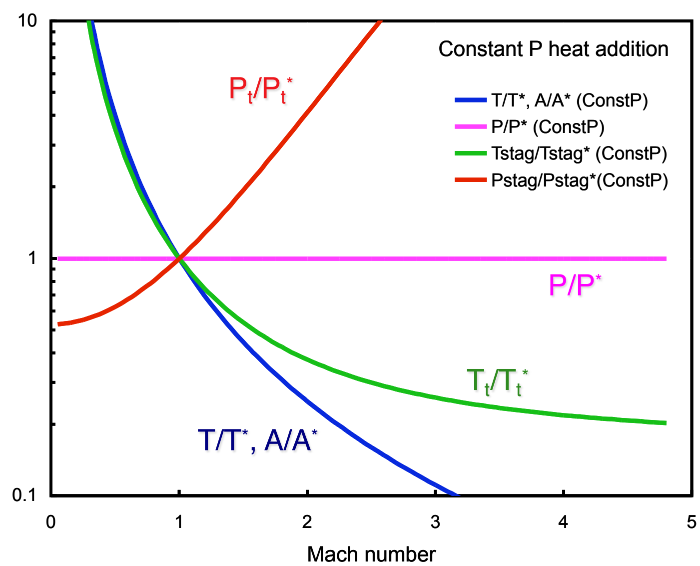
$$\frac{T}{T^*} = \frac{A}{A^*} = \frac{1}{M^2} \qquad \frac{P}{P^*} = 1 \qquad \frac{T_t}{T_t^*} = \frac{2}{(\gamma+1)M^2}\left(1 + \frac{\gamma-1}{2}M^2\right)$$

$$\frac{P_t}{P_t^*} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}\left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1}$$

- Implications
  - Stagnation temperature increases as M decreases, i.e. heat addition corresponds to decreasing M
  - Stagnation pressure decreases as M decreases, i.e. heat addition decreases stagnation P
  - Area increases as M decreases, i.e. as heat is added

Heat addition at constant P

USC Viterbi School of Engineering



Heat addition at constant pressure

USC Viterbi School of Engineering

- What if neither the initial state (1) nor final state (2) is the reference (*) state? Again use $P_2/P_1 = (P_2/P^*)/(P_1/P^*)$ etc.

$$\frac{T_2}{T_1} = \frac{A_2}{A_1} = \frac{M_1^2}{M_2^2} \qquad \frac{T_{2t}}{T_{1t}} = \frac{M_1^2}{M_2^2}\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2} \qquad \frac{P_{2t}}{P_{1t}} = \left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\gamma/\gamma-1}$$

Heat addition at constant temperature

USC Viterbi School of Engineering

- Probably most appropriate case for hypersonic propulsion since temperature (materials) limits is usually the reason we can't decelerate the ambient air to M = 0
- T = constant $\Rightarrow$ c (sound speed) = constant
- Momentum: $AdP + \dot{m}du = 0 \Rightarrow dP/P + \gamma MdM = 0$
- Reference state $(\ )^*$: use M = 1 again

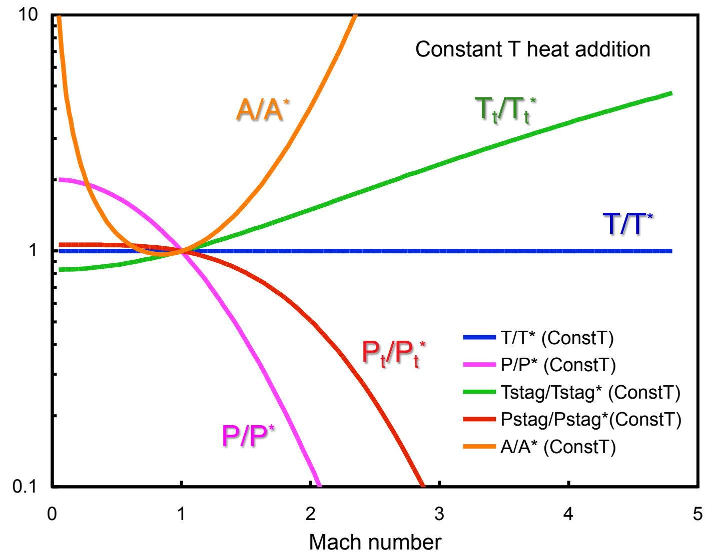
$$\frac{T}{T^*} = 1 \qquad \frac{P}{P^*} = \exp\left[\frac{\gamma}{2}\left(1-M^2\right)\right] \qquad \frac{T_t}{T_t^*} = \frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)$$

$$\frac{A}{A^*} = \frac{1}{M}\exp\left[\frac{-\gamma}{2}\left(1-M^2\right)\right] \quad \frac{P_t}{P_t^*} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}\exp\left[\frac{\gamma}{2}\left(1-M^2\right)\right]\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1}$$

- Implications
  - Stagnation temperature increases as M increases
  - Stagnation pressure decreases as M increases, i.e. heat addition decreases stagnation P
  - Minimum area (i.e. throat) at $M = \gamma^{-1/2}$
  - Large area ratios needed due to exp[ ] term

Heat addition at constant T

USC Viterbi School of Engineering



Heat addition at constant temperature

USC Viterbi School of Engineering

- What if neither the initial state (1) nor final state (2) is the reference (*) state? Again use $P_2/P_1 = (P_2/P^*)/(P_1/P^*)$ etc.

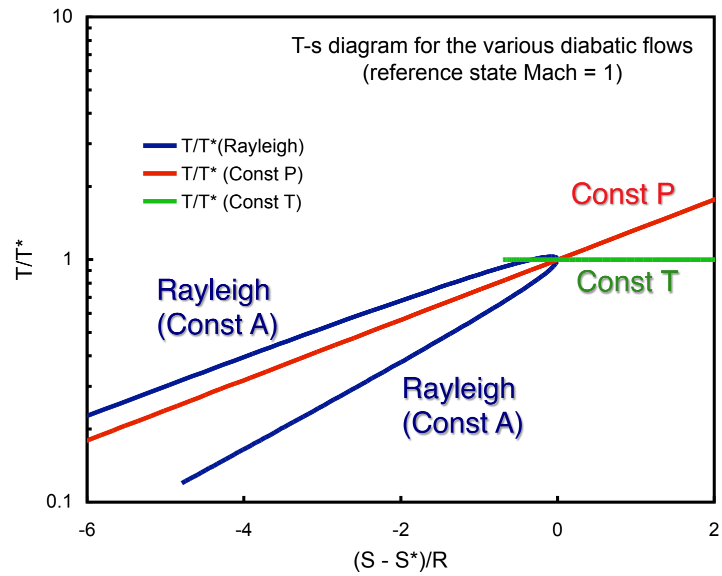
$$\frac{P_2}{P_1} = \frac{\exp\left[\frac{\gamma}{2}\left(1-M_2^2\right)\right]}{\exp\left[\frac{\gamma}{2}\left(1-M_1^2\right)\right]} = \exp\left[\frac{\gamma}{2}\left(M_1^2-M_2^2\right)\right] \qquad \frac{T_{2t}}{T_{1t}} = \frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}$$

$$\frac{A_2}{A_1} = \frac{M_1}{M_2}\frac{\exp\left[\frac{-\gamma}{2}\left(1-M_2^2\right)\right]}{\exp\left[\frac{-\gamma}{2}\left(1-M_1^2\right)\right]} = \frac{M_1}{M_2}\exp\left[\frac{\gamma}{2}\left(M_2^2-M_1^2\right)\right]$$

$$\frac{P_{2t}}{P_{1t}} = \frac{\exp\left[\frac{\gamma}{2}\left(1-M_2^2\right)\right]}{\exp\left[\frac{\gamma}{2}\left(1-M_1^2\right)\right]}\frac{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1}}{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1}} = \exp\left[\frac{\gamma}{2}\left(M_1^2-M_2^2\right)\right]\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\gamma/\gamma-1}$$

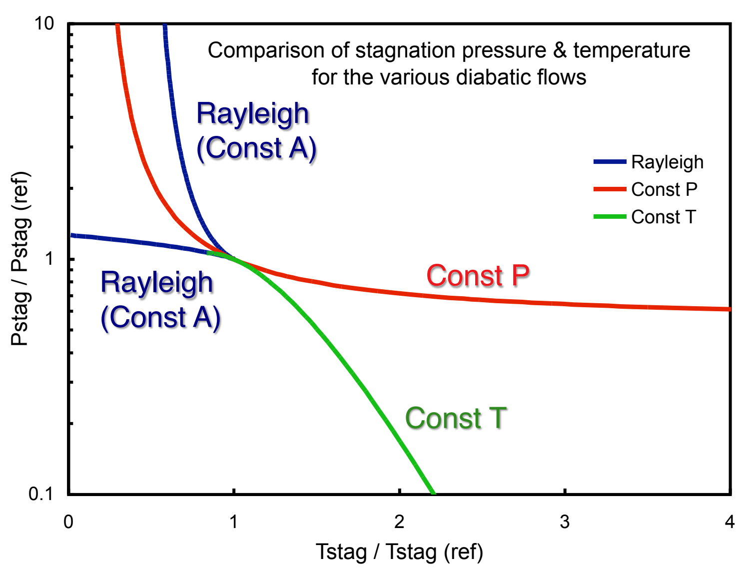
T-s diagram for diabatic flows

USC Viterbi School of Engineering



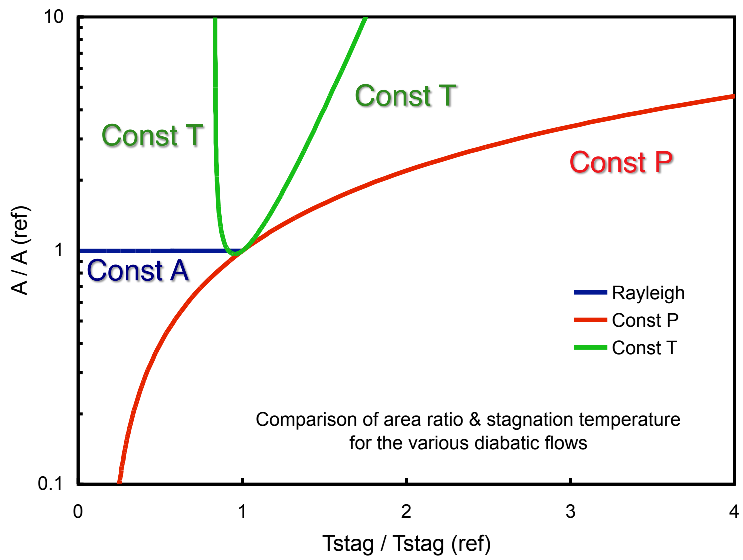
T-s diagram for diabatic flows

USC Viterbi School of Engineering



Area ratios for diabatic flows

USC Viterbi School of Engineering



What is the best way to add heat?

USC Viterbi School of Engineering

- If maximum T or P is limitation, obviously use that case
- What case gives least P_t loss for given increase in T_t?
  - Minimize $d(P_t)/d(T_t)$ subject to mass, momentum, energy conservation, eqn. of state
  - Result (lots of algebra - many trees died to bring you this result)

$$\frac{dP_t}{dT_t} = -\frac{\gamma M^2}{2}\frac{P_t}{T_t} \; or \; \frac{d(\ln P_t)}{d(\ln T_t)} = -\frac{\gamma M^2}{2}$$

- Adding heat (increasing T_t) always decreases P_t
- Least decrease in P_t occurs at lowest possible M – doesn't matter if it's at constant A, P, T, etc.

Summary of heat addition processes

	Const. A	Const. P	Const. T
M	Goes to M = 1	Decreases	Increases
Area	Constant	Increases	Min. at $M = \gamma^{-1/2}$
P	Decr. M < 1 Incr. M > 1	Constant	Decreases
P_t	Decreases	Decreases	Decreases
T	Incr. except for a small region at M < 1	Increases	Constant
T_t	Increases	Increases	Increases

AirCycles4Hypersonics.xls

- Diffuser – same as AirCycles4Propulsion.xls except that M_2 is specified rather than $M_2 = 0$
- Compressor – NONE ☺
- Combustor
  - Heat addition may be at constant area (Rayleigh flow), P or T
  - Mach number after diffuser is a specified quantity (not necessarily zero) - Mach number after diffuser sets compression ratio since there is no mechanical compressor
  - Rayleigh curves starting at states 1 and 2 included to show constant area / no friction on T-s
- Turbine – NONE ☺
- Nozzle – same as AirCycles4Propulsion.xls

AirCycles4Hypersonics.xls

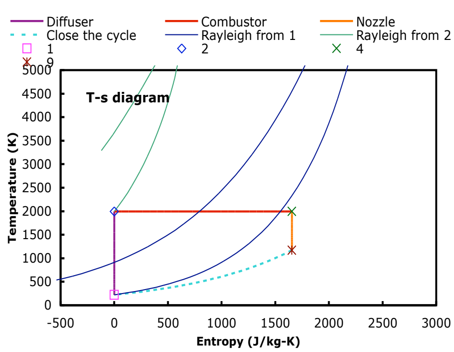
USC Viterbi School of Engineering

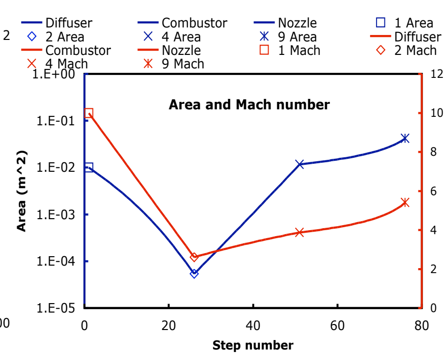
- Combustion parameter $\tau_\lambda = T_{4t}/T_1$ (specifies stagnation temperature, not static temperature, after combustion)
- Caution on choosing τ_λ
  - If $\tau_\lambda T_1 < \tau_r T_1$ ($\tau_r = 1 + (\gamma-1)/2\ M_1^2$) (maximum allowable temperature after heat addition > temperature after deceleration) then no heat can be added (actually, spreadsheet will try to refrigerate the gas…)
  - For constant-area heat addition, if $\tau_\lambda T_1$ is too large, you can't add that much heat (beyond thermal choking point) & spreadsheet "chokes"
  - For constant-T heat addition, if $\tau_\lambda T_1$ is too large, pressure after heat addition < ambient pressure - overexpanded jet - still works but performance suffers
  - For constant-P heat addition, no limits! ☺ But temperatures go sky-high ☹
  - All cases: f (fuel mass fraction) needed to obtain specified τ_λ is calculated - make sure this doesn't exceed $f_{stoichiometric}$!

Hypersonic propulsion (const. T) - T-s diagrams

USC Viterbi School of Engineering

- With Const. T combustion, maximum temperature within sane limits, but as more heat is added, P decreases, eventually $P_4 < P_9$
- Also, latter part of cycle has low Carnot-strip efficiency since constant T and P lines will converge



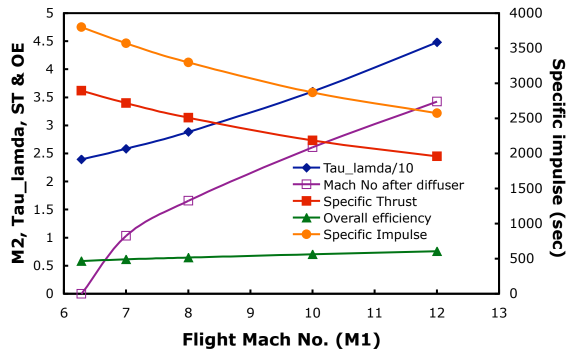


Const. T combustion, $M_1 = 10$; $M_2 = 2.61$ ($T_2 = 2000K$)
Stoich. H_2-air ($f = 0.0283$, $Q_R = 1.2 \times 10^8$ J/kg $\Rightarrow \tau_\lambda = 35.6$)

Hypersonic propulsion (const. T) - effect of M_1

USC Viterbi School of Engineering

- Minimum M_1 = 6.28 - below that T_2 < 2000 even if M_2 = 0
- No maximum M_2
- $\eta_{overall}$ improves slightly at high M_1 due to higher $\eta_{thermal}$ (lower T_9)

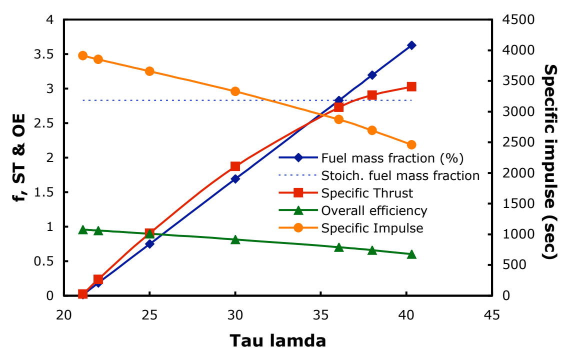


Const. T combustion, M_1 = varies; M_2 adjusted so that T_2 = 2000K;
H_2-air (Q_R = 1.2 x 10^8 J/kg), τ_λ adjusted so that f = $f_{stoichiometric}$

Hypersonic propulsion (const. T) - effect of τ_λ

USC Viterbi School of Engineering

- M_1 = 10, T_2 = 2000K specified $\Rightarrow$ M_2 = 2.61
- At τ_λ = 21.1 no heat can be added
- At τ_λ = 35.6, f = 0.0283 (stoichiometric H_2-air)
- At τ_λ = 40.3 (assuming one had a fuel with higher heating value than H_2), P_4 = P_9
- f & Specific Thrust increase as more fuel is added (τ_λ increasing), $\eta_{overall}$ & I_{SP} decrease due to low $\eta_{thermal}$ at high heat addition (see T-s diagram)

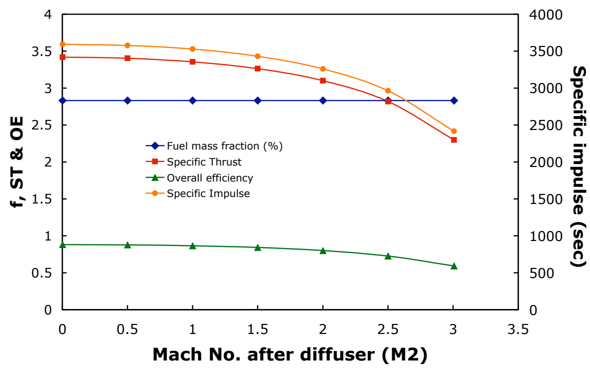


Const. T combustion, M_1 = 10; M_2 = 2.61 (T_2 = 2000K)
H_2-air (Q_R = 1.2 x 10^8 J/kg), τ_λ (thus f) varies

Hypersonic propulsion (const. T) - effect of M_2

USC Viterbi School of Engineering

- Maximum M_2 = 3.01 - above that $P_4 < P_9$ after combustion (you could go have higher M_2 but why would you want to - heat addition past $P_4 = P_9$ would reduce thrust!)
- No minimum M_2, but lower M_2 means higher T_2 - maybe beyond materials limits (after all, high T_{1t} is the whole reason we're looking at alternative ways to burn at hypersonic Mach numbers)
- $\eta_{overall}$ decreases at higher M_2 due to lower $\eta_{thermal}$ (lower T_2)

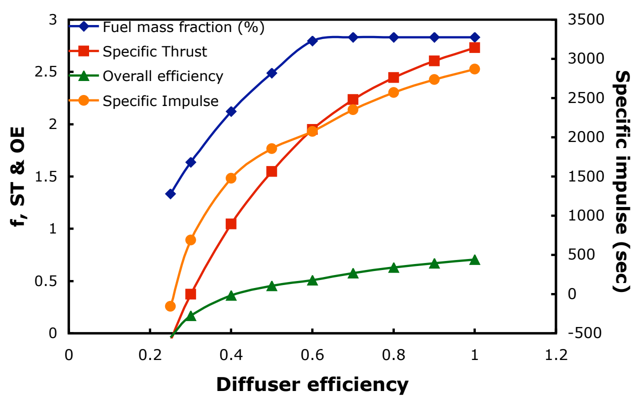


Const. T combustion, M_1 = 10; M_2 varies;
H_2-air ($Q_R = 1.2 \times 10^8$ J/kg), τ_λ adjusted so that $f = f_{stoichiometric}$

Hypersonic propulsion (const. T) - effect of η_d

USC Viterbi School of Engineering

- Obviously as η_d decreases, all performance parameters decrease
- If η_d too low, pressure after stoichiometric heat addition < P_1, so need to decrease heat addition (thus τ_λ)
- Diffuser can be pretty bad ($\eta_d \approx 0.25$) before no thrust

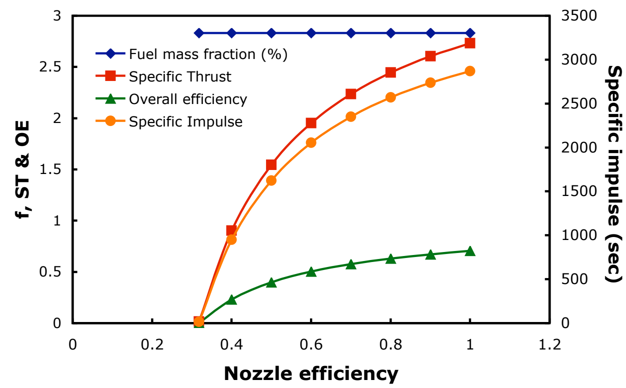


Const. T combustion, M_1 = 10; M_2 = 2.611; H_2-air ($Q_R = 1.2 \times 10^8$ J/kg),
τ_λ adjusted so that $f = f_{stoichiometric}$ or $P_5 = P_1$, whichever is smaller

Hypersonic propulsion (const. T) - effect of η_n

USC Viterbi School of Engineering

- Obviously as η_n decreases, all performance parameters decrease
- Nozzle can be pretty bad ($\eta_n \approx 0.32$) before no thrust, but not as bad as diffuser

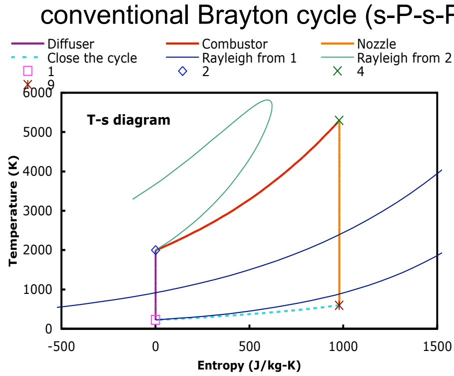


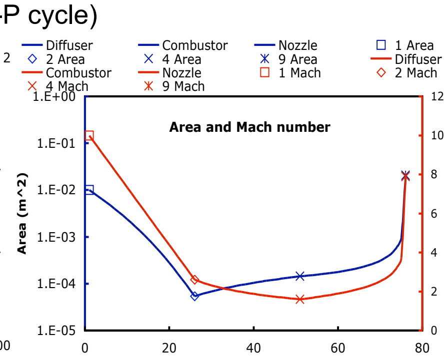
Const. T combustion, $M_1 = 10$; $M_2 = 2.611$; H_2-air ($Q_R = 1.2 \times 10^8$ J/kg), τ_λ adjusted so that $f = f_{stoichiometric}$

Hypersonic propulsion (const. P) - T-s diagrams

USC Viterbi School of Engineering

- With Const. P combustion, no limitations on heat input, but maximum temperature becomes insane (actually dissociation & heat losses would decrease this T substantially)
- Carnot-strip (thermal) efficiency independent of heat input; same as conventional Brayton cycle (s-P-s-P cycle)



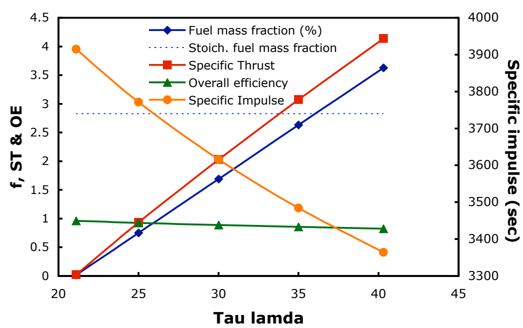


Const. P combustion, $M_1 = 10$; $M_2 = 2.61$ ($T_2 = 2000$K)
Stoich. H_2-air ($f = 0.0283$, $Q_R = 1.2 \times 10^8$ J/kg $\Rightarrow \tau_\lambda = 35.6$)

Hypersonic propulsion (const. P) - performance

USC Viterbi School of Engineering

- $M_1 = 10$, $T_2 = 2000K$ specified $\Rightarrow M_2 = 2.61$
- Still, at $\tau_\lambda = 21.1$ no heat can be added
- At $\tau_\lambda = 35.6$, f = 0.0283 (stoichiometric H_2-air)
- No upper limit on τ_λ (assuming one has a fuel with high enough Q_R)
- f & Specific Thrust increase as more fuel is added (τ_λ increasing), $\eta_{overall}$ & I_{SP} decrease only slightly at high heat addition due to lower $\eta_{propulsive}$

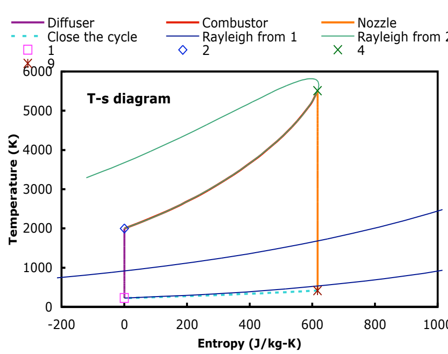


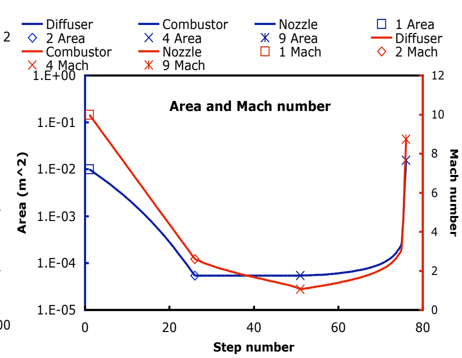
Const. P combustion, $M_1 = 10$; $M_2 = 2.61$ ($T_2 = 2000K$)
H_2-air ($Q_R = 1.2 \times 10^8$ J/kg), τ_λ (thus f) varies

Hypersonic propulsion (const. A) - T-s diagrams

USC Viterbi School of Engineering

- With Const. A combustion, heat input limited by thermal choking, maximum temperature even more insane than constant P
- … but Carnot-strip efficiency is awesome!

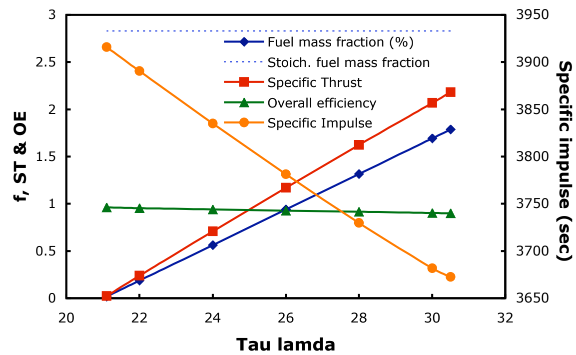




Const. A combustion, $M_1 = 10$; $M_2 = 2.61$ ($T_2 = 2000K$)
H_2-air (f = 0.0171, $Q_R = 1.2 \times 10^8$ J/kg $\Rightarrow \tau_\lambda = 30.1$; (can't add stoichiometric amount of fuel at constant area for this M_1 and M_2))

Hypersonic propulsion (const. A) - performance

- $M_1 = 10$, $T_2 = 2000K$ specified $\Rightarrow M_2 = 2.61$
- Still, at $\tau_\lambda = 21.1$ no heat can be added
- At $\tau_\lambda = 30.5$, thermal choking at $f = 0.0193 < 0.0283$
- f & Specific Thrust increase as more fuel is added (τ_λ increasing), $\eta_{overall}$ & I_{SP} decrease slightly at high heat addition due to lower $\eta_{propulsive}$



Const. A combustion, $M_1 = 10$; $M_2 = 2.61$ ($T_2 = 2000K$)
H_2-air ($Q_R = 1.2 \times 10^8$ J/kg), τ_λ (thus f) varies

Example

Consider a very simple propulsion system in a standard atmosphere at 100,000 feet (227K and 0.0107 atm, with $\gamma = 1.4$) in which

(1) Incoming air is decelerated isentropically from $M_1 = 15$ until T = 3000K
(2) Heat is added at constant T until ambient pressure is reached (not a good way to operate, but this represents a sort of maximum heat addition)

(a) To what Mach number could the air be decelerated if the maximum allowable gas temperature is 3000K? What is the corresponding pressure?

$$T_1\left(1+\frac{\gamma-1}{2}M_1^2\right) = T_2\left(1+\frac{\gamma-1}{2}M_2^2\right)$$

$$(227K)\left(1+\frac{1.4-1}{2}15^2\right) = (3000K)\left(1+\frac{1.4-1}{2}M_2^2\right)$$

$$M_2^2 = \frac{2}{1.4-1}\left[\frac{227K}{3000K}\left(1+\frac{1.4-1}{2}15^2\right)-1\right] = 12.403 \Rightarrow M_2 = 3.522$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1} = \left(\frac{3000K}{227K}\right)^{1.4/1.4-1} = 8391; P_2 = 8391P_1 = 8391(.0107) = 89.79atm$$

Example - continued

(b) What is the exit Mach (M_9) number? (Note that heat is added until $P_3 = P_1$, so there is no separate nozzle (3 → 9) in this case.) What is the area ratio?

$$P_3/P_2 = \exp\left[\frac{\gamma}{2}\left(M_2^2 - M_3^2\right)\right]; \ln\left(P_3/P_2\right) = \frac{\gamma}{2}\left(M_2^2 - M_3^2\right)$$

$$M_3^2 = M_2^2 - \frac{2}{\gamma}\ln\left(P_3/P_2\right) = 3.522^2 - \frac{2}{1.4}\ln\left(1/8391\right) = 25.31 \Rightarrow M_3 = 5.031$$

$$A_3/A_2 = \frac{M_2}{M_3}\exp\left[\frac{\gamma}{2}\left(M_3^2 - M_2^2\right)\right] = \frac{3.522}{5.031}\exp\left[\frac{1.4}{2}\left(5.031^2 - 3.522_2^2\right)\right] = 5869$$

(c) What is the specific thrust?

$ST = \text{Thrust}/\dot{m}_a c_1 = \dot{m}_a(u_9 - u_1)/\dot{m}_a c_1 = (M_9 c_9 - M_1 c_1)/c_1 = M_e(T_e/T_1)^{1/2} - M_1,$

$ST = M_9(T_9/T_1)^{1/2} - M_1 = 5.031(3000K/227K)^{1/2} - 15 = 3.289$

Example - continued

(d) What is the thrust specific fuel consumption and overall efficiency?

$TSFC = (\text{Heat input})/\text{Thrust}*c_1 = [\dot{m}_a(C_P(T_{3t} - T_{2t})c_1]/[\text{Thrust}*c_1^2]$
$= [(\dot{m}_a c_1)/\text{Thrust}]\ [(\gamma/(\gamma-1))R(T_{3t}-T_{2t})/(\gamma R T_1)] = [1/(ST)]\ [1/(\gamma-1)]\ [(T_{3t}-T_{2t})/T_1]$

$$T_{2t} = T_2\left(1 + \frac{\gamma-1}{2}M_2^2\right); T_{3t} = T_3\left(1 + \frac{\gamma-1}{2}M_e^2\right); T_2 = T_3 = T_e$$

$$\frac{T_{3t} - T_{2t}}{T_1} = \frac{T_3}{T_1}\left(1 + \frac{\gamma-1}{2}M_e^2\right) - \frac{T_2}{T_1}\left(1 + \frac{\gamma-1}{2}M_2^2\right) = \frac{T_2}{T_1}\left(1 + \frac{\gamma-1}{2}M_e^2 - 1 + \frac{\gamma-1}{2}M_2^2\right)$$

$$\frac{T_{3t} - T_{2t}}{T_1} = \frac{T_2}{T_1}\frac{\gamma-1}{2}\left(M_e^2 - M_2^2\right) = \frac{3000}{227}\frac{1.4-1}{2}\left(5.031^2 - 3.522^2\right) = 34.11$$

$$TSFC = \frac{1}{ST}\frac{1}{\gamma-1}\frac{T_{3t} - T_{2t}}{T_1} = \frac{1}{3.289}\frac{1}{1.4-1}\left(34.11\right) = 26.01; \eta_o = \frac{M_1}{TSFC} = \frac{15}{26.01} = 0.577$$

(e) Can stoichiometric hydrogen-air mixtures generate enough heat to accomplish this? Determine if the heat release per unit mass = $f_{stoich}Q_R$ is equal to or greater than the heat input needed = $C_P(T_{3t} - T_{2t})$.

$$C_P(T_{3t} - T_{2t}) = \frac{\gamma R}{\gamma - 1}\frac{T_{3t} - T_{2t}}{T_1}T_1 = \frac{1.4\left[(8.314 J/moleK)/(0.02897 kg/mole)\right]}{1.4-1}(34.11)(227K)$$

$$f_{stoich}Q_r = (0.0283)\left(1.20\times10^8 J/kg\right) = 3.396\times10^6 J/kg; C_P(T_{3t} - T_{2t}) = 7.777\times10^6 J/kg$$

Requirement is higher by a factor of ≈ 2.3, so H_2-air cannot provide this much heat release

Summary

- Propulsion at high Mach numbers is very different from conventional propulsion because
  - The optimal thermodynamic cycle (decelerate to M = 0) yields impractically high T & P
  - Deceleration from high M to low M without major P_t losses is difficult
  - Propulsive efficiency ≈ $2u_1/(u_1+u_9)$ is always high
- 3 ways of adding heat discussed
  - Constant T
    - » Probably most practical case
    - » Low efficiency with large heat addition
    - » Large area ratios
  - Constant P - best performance but very high T
  - Constant A - thermal choking limits heat input