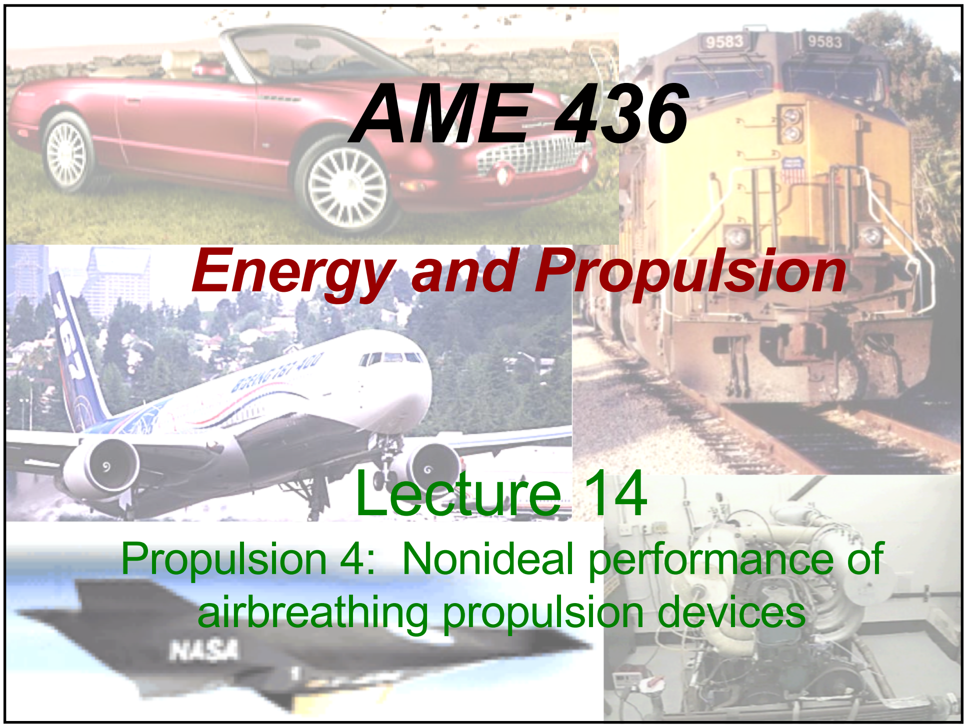

Outline

- Component performance - definitions
- AirCycles4Propulsion spreadsheet
- Non-ideal performance
  - Diffuser
  - Compressor
  - Burner
  - Turbine
  - Nozzle
  - Fan
  - Heat loss

Component performance

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- Diffusers (1 → 2) & nozzles (7 → 9) subject to stagnation pressure losses due to viscosity, shocks, flow separation
- More severe for diffusers since dP/dx > 0 - unfavorable pressure gradient, aggravates tendency for flow separation
- If adiabatic, T_t = constant, thus $\tau_d \equiv T_{2t}/T_{1t} = 1$ & $\tau_n \equiv T_{9t}/T_{7t} = 1$
- If also reversible, $\pi_d \equiv P_{2t}/P_{1t} = 1$ and $\pi_n \equiv P_{9t}/P_{7t} = 1$
- If reversible, $\tau_d = \pi_d^{(\gamma-1)/\gamma} = 1$, $\tau_n = \pi_n^{(\gamma-1)/\gamma} = 1$, but if irreversible $\pi_d^{(\gamma-1)/\gamma} < 1$ or $\pi_n^{(\gamma-1)/\gamma} < 1$ even though $\tau_d = 1$, $\tau_n = 1$
- Define diffuser efficiency
  $\eta_d = \pi_d^{(\gamma-1)/\gamma}$ = 1 if ideal
- Define nozzle efficiency
  $\eta_n = \pi_n^{(\gamma-1)/\gamma}$ = 1 if ideal
- For nozzle - also need to consider what happens if $P_{exit} \neq P_{ambient}$, i.e. $P_9 \neq P_1$

Compressors, fans & turbines

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- Compressors (2 → 3) & fans (2' → 3') subject to losses due to
  - Flow separation around compressor blades (again, unfavorable dP/dx)
  - Gas leakage between blade tips and compressor walls
- How to define compression efficiency η_{comp}?
  - If irreversible dS > δQ/T; if adiabatic (δQ = 0) dS > 0
  - For same P ratio, more work input (more ΔT) during compression, less work output (less ΔT) during expansion through turbine
  - Define η_{comp} similar to piston/cylinder (Lecture 6):

$$\eta_{comp} \equiv \frac{\text{Reversible adiabatic work input for given P ratio}}{\text{Actual work input required for same P ratio}} = \frac{-\dot{m}_a C_P (T_{3t,ideal} - T_{2t})}{-\dot{m}_a C_P (T_{3t} - T_{2t})}$$

$$\eta_{comp} = \frac{-\dot{m}_a C_P [T_{2t}(P_{3t}/P_{2t})^{\gamma-1/\gamma} - T_{2t}]}{-\dot{m}_a C_P (T_{3t} - T_{2t})} = \frac{(\pi_c)^{\gamma-1/\gamma} - 1}{T_{3t}/T_{2t} - 1} \text{ or } \frac{T_{3t}}{T_{2t}} = 1 + \frac{(\pi_{comp})^{\gamma-1/\gamma} - 1}{\eta_{comp}}$$

$$\text{Actual work} = -\dot{m}_a C_P (T_{3t} - T_{2t}) = -\dot{m}_a C_P T_{2t} \left[\frac{(\pi_{comp})^{\gamma-1/\gamma} - 1}{\eta_{comp}} \right]$$

Compressors, fans & turbines

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- Fan efficiency η_{fan} defined similarly but not as problematic since smaller pressure ratios

$$\eta_{fan} \equiv \frac{(P_{3t}'/P_{2t}')^{\frac{\gamma-1}{\gamma}} - 1}{T_{3t}'/T_{2t}' - 1}; \text{ Actual work } = -\alpha \dot{m}_a C_P T_{2t}' \left[\frac{(\pi_c')^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{fan}} \right]$$

- Turbines (4 → 5 and 5 → 6) less problematic since dP/dx < 0, but need to bleed compressor air through turbine blades for cooling
- Define turbine efficiency η_{turb}:

$$\eta_{turb} \equiv \frac{\text{Actual work output for given P ratio}}{\text{Reversible adiabatic work output for same P ratio}} = \frac{-\dot{m}_a C_P (T_{5t} - T_{4t})}{-\dot{m}_a C_P (T_{5t,ideal} - T_{4t})}$$

$$\eta_{turb} = \frac{-\dot{m}_a C_P (T_{5t} - T_{4t})}{-\dot{m}_a C_P \left[T_{4t} (P_{5t}/P_{4t})^{\frac{\gamma-1}{\gamma}} - T_{4t} \right]} = \frac{T_{5t}/T_{4t} - 1}{(\pi_{turb})^{\frac{\gamma-1}{\gamma}} - 1} \text{ or } \frac{T_{5t}}{T_{4t}} = 1 + \eta_{turb} \left[(\pi_{turb})^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$\text{Actual work} = -\dot{m}_a C_P (T_{5t} - T_{4t}) = -\dot{m}_a C_P T_{4t} \eta_{turb} \left[(\pi_{turb})^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

- Fan turbine: replace state 4 with 5 and replace state 5 with 6

Component performance - combustors

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- Combustor (4 → 5) and afterburner (6 → 7) - stagnation pressure losses due to turbulence (required for mixing of fuel and air) and heat addition at M > 0
- Define combustor (burner) or afterburner efficiency in terms of stagnation pressure ratio π_b or π_{ab} (= 1 if ideal)
- For heat addition at finite M, stagnation pressure will always decrease; recall (lecture 12)

$$\frac{dP_t}{dT_t} = -\frac{\gamma M^2}{2} \frac{P_t}{T_t} \; or \; \frac{d(\ln P_t)}{d(\ln T_t)} = -\frac{\gamma M^2}{2}$$

AirCycles4Propulsion.xlsx

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- See also Lecture 9 (AirCycles4Recips.xlsx)
- Thermodynamic model is exact, but heat loss, stagnation pressure losses etc. models are qualitative
- Constant γ is not realistic (changes from about 1.4 to 1.25 during cycle) but only affects results quantitatively (not qualitatively)
- Wall heat transfer model
  - $\Delta T \sim h(T_w - T_{gas,t})$, where heat transfer coefficient h and component temperature T_w are specified - physically reasonable

$$T_{i,b,t} = T_{i,a,t} + h(T_w - T_{i,a,t}); P_{i,b,t} = P_{i,a,t}$$

  - Unlike AirCycles4Recips.xlsx, wall temperature T_w is specified separately for each component since each experiences a constant T, not T averaged over the cycle
  - Increments each cell not each time step
  - Doesn't include effects of varying area, varying turbulence, varying time scale through Mach number, etc. on h

AirCycles4Propulsion.xls

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- Work or heat in or out going from step i to step i+1 computed according to $dQ = \dot{m}_a C_P(T_{i+1,t} - T_{i,t})$ and $dW = -\dot{m}_a C_P(T_{i+1,t} - T_{i,t})$ (heat positive if into system, work positive if out of system)
- Diffuser
  - Mach number decrements from flight Mach number (M_1) to $M_2 = 0$ after diffuser in 25 equal steps
  - Stagnation pressure decrements from its value at M_1 (P_{1t}) to $\pi_d P_{1t} = \eta_d^{\gamma/(\gamma-1)} P_{1t}$ in 25 equal steps
  - Static P and T calculated from M and P_t, T_t
  - Sound speed (c) calculated from T, then u calculated from c and M
  - No heat input or work output in diffuser but may have wall heat transfer (page 7)
  - Fan-flow diffuser exactly same as main flow diffuser except mass flow rate α times higher ($\eta_{d,main\ flow} = \eta_{d,fan\ flow}$ assumed)

AirCycles4Propulsion.xls

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- Compressor
  - Mach number = 0, thus $P = P_t$, $T = T_t$
  - Compression in two steps
    (1) Wall heat transfer at constant P according to formula on page 7 (step i,a to step i,b)
    (2) Compression according to the isentropic compression law with efficiency η_{comp} (step i,b to step i+1,a)

$$\eta_{comp} \equiv \frac{(P_{i+1,t}/P_{i,t})^{\gamma-1/\gamma} - 1}{T_{i+1,a,t}/T_{i,b,t} - 1} \Rightarrow T_{i+1,a,t} = T_{i,b,t}\left(1 + \frac{(P_{i+1,t}/P_{i,t})^{\gamma-1/\gamma} - 1}{\eta_{comp}}\right)$$

  - Compression ends at pressure = $\pi_c P_{2t}$ where π_c is the specified pressure ratio
  - Pressure increments in 25 equal steps
  - Work per step = $-\dot{m}_a C_P(T_{i+1,a,t} - T_{i,a,t})$
  - Fan compressor follows exactly same as rules except that mass flow rate is α times higher and η_{fan} may be different from η_{comp}

AirCycles4Propulsion.xls

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- Combustor and afterburner
  - Enthalpy input due to combustion added in 25 equal steps (1/25 of total stagnation temperature change per step)
  - Heat addition ends at $T_{4t} = T_1\tau_\lambda$ (i.e the turbine inlet temperature limit) or $T_{7t} = T_1\tau_{\lambda,ab}$ (afterburner temperature limit)
  - Wall heat transfer at constant P according to formula on page 7
  - Stagnation pressure drops in 25 equal decrements; P after combustion $P_{4t} = \pi_b P_{3t}$ or for afterburner $P_{7t} = \pi_{ab} P_{6t}$
  - Mach number = 0 throughout

AirCycles4Propulsion.xls

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- Turbine
  - Expansion to pay for compressor work done in two steps:
    (1) Wall heat transfer at constant P according to formula on page 7 (step i,a to step i,b)
    (2) Expansion according to isentropic compression law with efficiency η_{turb} (step i,b to step i+1,a)

$$\eta_{turb} \equiv \frac{T_{i+1,at} / T_{i,b,t} - 1}{\left(P_{i+1,t} / P_{i,t}\right)^{\gamma-1/\gamma} - 1} \Rightarrow T_{i+1,a,t} = T_{i,b,t}\left(1 + \eta_{turb}\left[\left(P_{i+1,t} / P_{i,t}\right)^{\gamma-1/\gamma} - 1\right]\right)$$

  - Each of 25 expansion steps provides (after including losses due to irreversibility) 1/25 of the work required to drive turbine (calculated in compressor analysis)
  - Work per step = $-\dot{m}_a C_P(T_{i+1,a,t} - T_{i,a,t})$,
  - Mach number = 0 throughout
  - Expansion through fan turbine to pay for fan work - same as main turbine, except equate fan work to turbine work

AirCycles4Propulsion.xls

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- Nozzle
  - No heat input or work output
  - Static (not stagnation) pressure decrements from value after afterburner to specified exhaust pressure P_9 in 25 equal steps
  - Stagnation pressure decrements from its value after afterburner (P_{7t}) to $\pi_n P_{7t} = \eta_n^{\gamma/(\gamma-1)} P_{7t}$ in 25 equal steps
  - T_t changes due to wall heat transfer (page 7)
  - M calculated from P (static pressure) and P_t (stagnation pressure)
  - T (static temperature) calculated from M and T_t (stagnation temperature)
  - Sound speed (c) calculated from T
  - u calculated from c and M
  - Same rules apply to fan nozzle except
    - » Go directly from state 3' (after fan compressor) through nozzle to state 9' (no combustor or turbine)
    - » Mass flow is α times higher
    - » Assume $P_{exit} = P_{ambient}$ always, i.e. $P_9' = P_1$

Turbojet cycle - diffuser - effect of η_d

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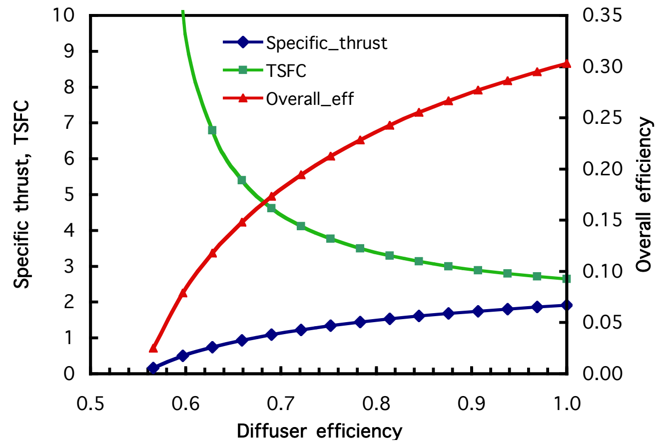
- Obviously thrust & $\eta_{overall}$ decrease, TSFC increases as η_d decreases, but can have pretty bad diffuser ($\eta_d \approx 0.55$) before thrust → 0

Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 30$
$\tau_\lambda = 5$



Turbojet cycle - diffuser - effect of η_d

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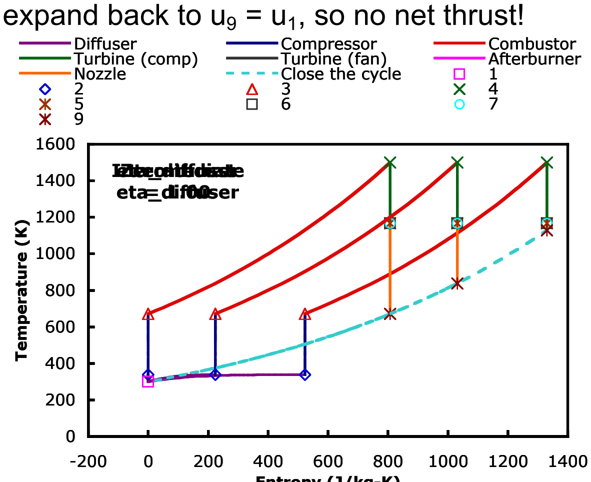
- T_2 same, regardless of η_d, since air decelerated to M = 0
- s_2 increases & P_2 decreases with decreasing η_d
- T_3, T_4, T_5 don't change since π_c, τ_λ & compressor work unchanged
- If P_2 is too low (can be < P_1), after compressor/combustor/turbine, P_5 is too low, can only expand back to $u_9 = u_1$, so no net thrust!

Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 10.97$
$\tau_\lambda = 5$



Turbojet cycle - effect of η_{comp}

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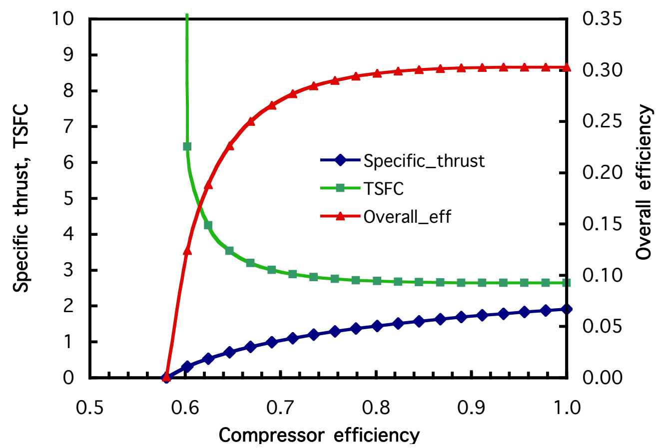
- Obviously thrust & $\eta_{overall}$ decrease, TSFC increases as η_{comp} decreases, but (again) can have pretty bad compressor ($\eta_c \approx 0.58$) before thrust disappears

Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 30$
$\tau_\lambda = 5$



Turbojet cycle - effect of η_{comp}

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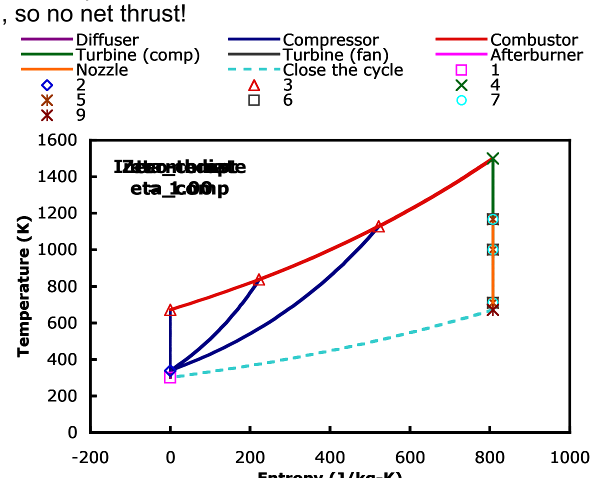
- State 2 always same since air decelerated isentropically to M = 0
- T_3 & s_3 increase as η_{comp} decreases (more work required to drive compressor for same π_{comp}), T_4 same (same τ_λ limit)
- T_5 lower as η_{comp} decreases (more turbine work to drive compressor)
- If η_{comp} too low after paying for compressor work in turbine, P_5 & T_5 are too low, can only expand back to $u_9 = u_1$, so no net thrust!

Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 10.97$
$\tau_\lambda = 5$



Turbojet cycle - burner - effect of π_{burner}

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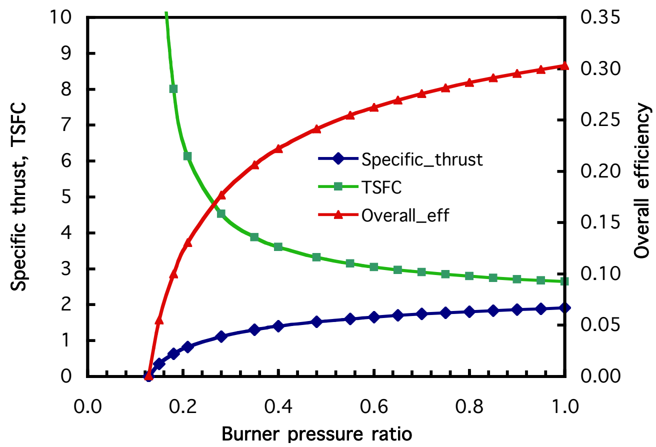
- Obviously thrust & $\eta_{overall}$ decrease, TSFC increases as π_{burner} decreases, but can have really lousy $\pi_{burner} \approx 0.13$ before thrust disappears (though $(\pi_{burner})^{(\gamma-1)/\gamma} \approx 0.56$!)

Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 30$
$\tau_\lambda = 5$



Turbojet cycle - burner - effect of π_{burner}

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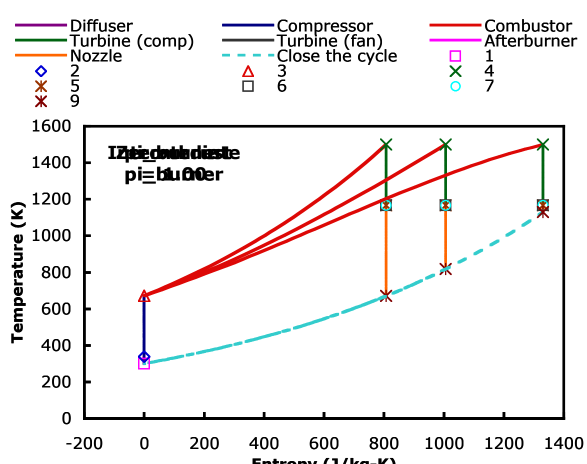
- States 2 & 3 are the same, independent of what burner does
- T_4 unchanged (same τ_λ limit) but as π_{burner} decreases, P_4 decreases, s_4 increases
- T_5 same for all (same turbine work required to drive compressor)
- If π_{burner} too low after paying for compressor work in turbine, P_5 & T_5 are too low, can only expand back to $u_9 = u_1$, so no net thrust!

Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 10.97$
$\tau_\lambda = 5$



Turbojet cycle - turbine - effect of η_{turb}

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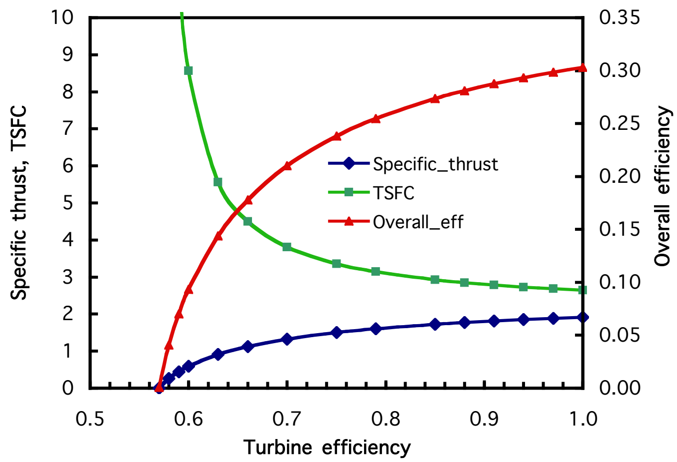
- Obviously thrust & $\eta_{overall}$ decrease, TSFC increases as η_{turb} decreases, but again can have low $\eta_{turb} \approx 0.57$ before thrust disappears

Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 30$
$\tau_\lambda = 5$



Turbojet cycle - turbine - effect of η_{turb}

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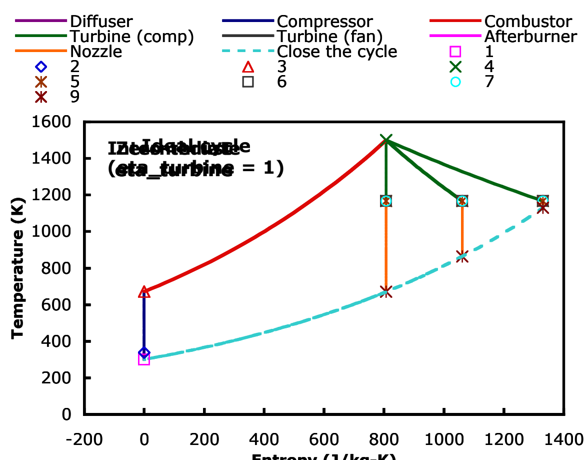
- States 1 - 4 are unchanged, independent of η_{turb}
- T_5 also independent of η_{turb} (same work required to drive compressor) but s_5 increases as $\eta_{turbine}$ decreases
- If η_{turb} too low after paying for compressor work in turbine, P_5 is too low, can only expand back to $u_9 = u_1$, so no net thrust!

Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 10.97$
$\tau_\lambda = 5$



Turbojet cycle - nozzle - effect of η_n

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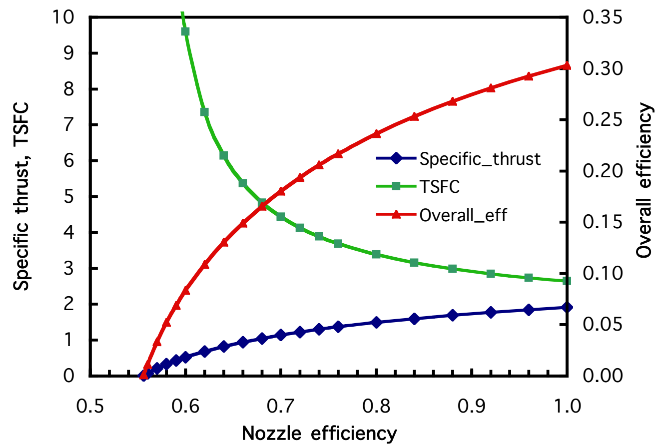
- Obviously thrust & $\eta_{overall}$ decrease, TSFC increases as η_{turb} decreases, but again can have low $\eta_{turb} \approx 0.56$ before thrust → 0
- Interesting that Thrust → 0 for η_d, η_{comp}, η_{turb}, η_{nozzle}, $(\pi_{burner})^{(\gamma-1)/\gamma} \approx 0.56$ (≈ same for all!)

Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 30$
$\tau_\lambda = 5$



Turbojet cycle - nozzle - effect of η_n

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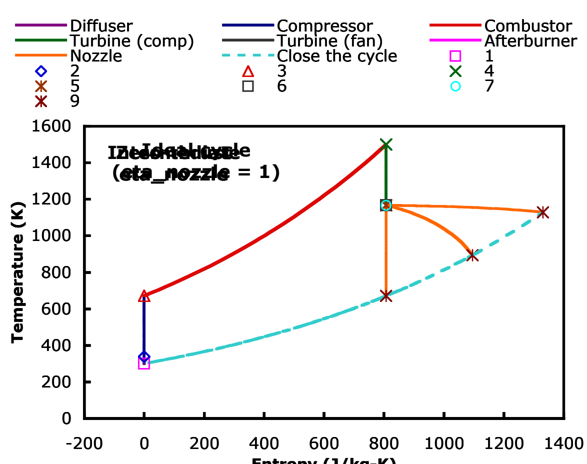
- States 1 - 5 unchanged, independent of η_{turb}
- If η_{nozzle} too low, can only expand back to $u_9 = u_1$, so no net thrust!
- Note that even for zero-thrust cycle, there is still some decrease in T after turbine to create u_9; if $u_9 = 0$ then thrust is negative!)

Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 10.97$
$\tau_\lambda = 5$



Turbojet cycle - nozzle - effect of $P_{exit} \neq P_{amb}$

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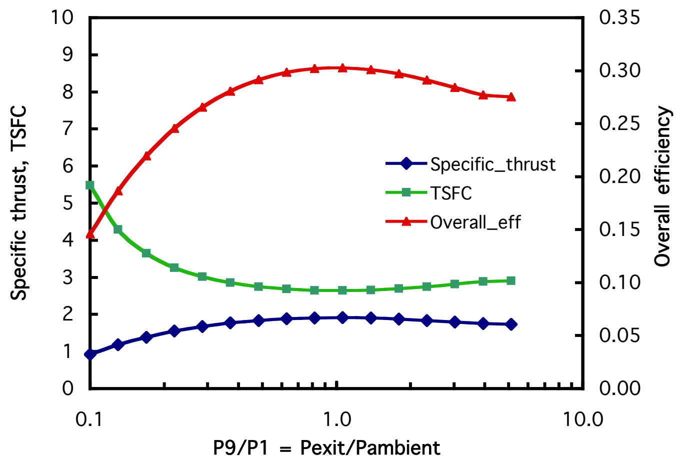
- Thrust & $\eta_{overall}$ decrease, TSFC increases if $P_9 \neq P_1$, but performance very insensitive to P_9/P_1 (note horizontal scale is logarithmic)

Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 30$
$\tau_\lambda = 5$



Turbojet cycle - nozzle - effect of $P_{exit} \neq P_{amb}$

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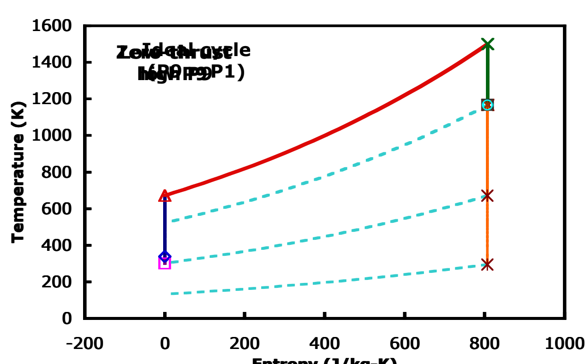
- States 1 - 5 are unchanged, independent of P_9/P_1
- If P_9 too low, negative $(P_9 - P_1)A_9$ term in the thrust equation balances the positive $\dot{m}_a[(1+FAR)u_9 - u_1]$, so no net thrust, even though u_9 increases as P_9 decreases
- If P_9 too high, $u_9 = 0$; analysis fails at higher P_9 although even at $P_9 = P_5$, there is still net thrust (although the $\dot{m}_a[(1+FAR)u_9 - u_1]$ term is zero, the $(P_9 - P_1)A_9$ term generates thrust)

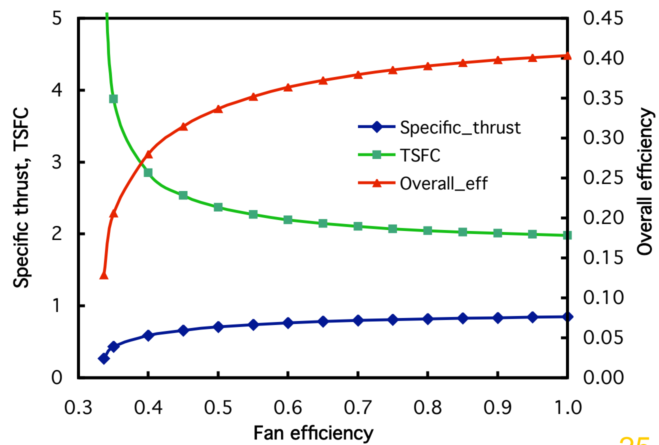
Basic turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 10.97$
$\tau_\lambda = 5$



Turbofan cycle - effect of η_{fan}

- Obviously thrust & $\eta_{overall}$ decrease, TSFC increases as η_{fan} decreases, but can have low $\eta_{fan} \approx 0.33$ before analysis fails
- Actually Thrust ≠ 0 at this point but fan work is so large that turbine exit pressure = ambient pressure; beyond this point analysis fails since turbine work insufficient to supply greedy fan

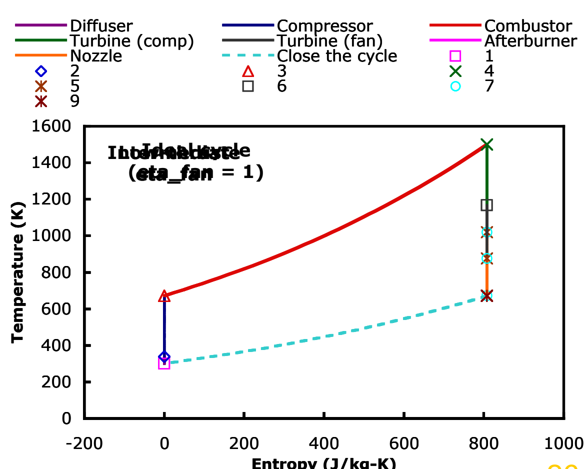
Turbofan
$\alpha = 2$
No afterburner
$M_1 = 0.8$
$\pi_c = 30$
$\pi_c' = 2$
$\tau_\lambda = 5$



Turbofan cycle - effect of η_{fan}

- States 1 - 5 are same, independent of η_{turb}
- If η_{fan} too low, fan work is so large that turbine exit pressure P_6 = ambient pressure P_9; beyond this point analysis fails since turbine work insufficient to supply greedy fan

Turbofan
$\alpha = 2$
No afterburner
$M_1 = 0.8$
$\pi_c = 10.97$
$\pi_c' = 2$
$\tau_\lambda = 5$



Turbojet cycle - effect of heat loss

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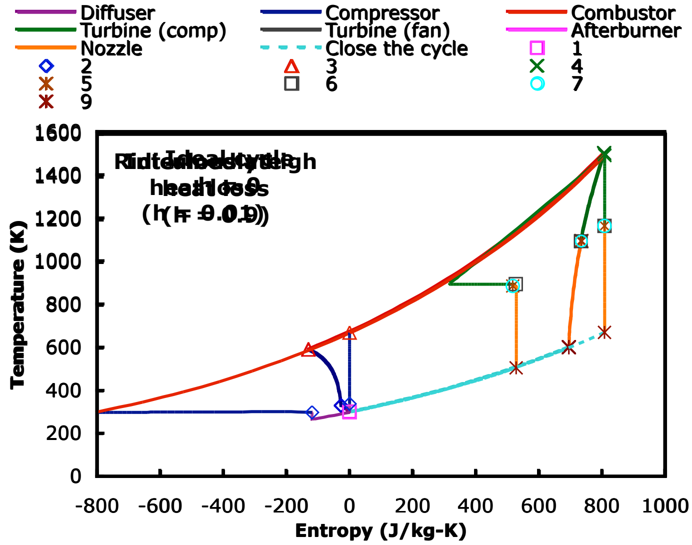
Turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 10.97$
$\tau_\lambda = 5$

T_w = 300K (diffuser, compressor); T_w = 900K (combustor, turbine, nozzle)



Turbojet cycle - effect of heat loss

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- For small heat loss, T_2 & T_3 decrease (since I chose T_w = 300K for diffuser & compressor), and since P_3/P_2 is set by choice of π_c, s_3 decreases
- Heat addition is greater - need to add more heat to get to τ_λ limit limit (starting from lower T, plus wall heat loss during heat addition (combustion)) (I chose T_w = 900K)
- Since heat addition at constant pressure and same maximum temperature, T_4 is the same with heat loss
- Heat loss during turbine & nozzle expansion also result in decreased T (since I chose T_w = 900K for turbine & nozzle); since P_9 is fixed, lower T_9 causes lower s_9
- Note that s decreases in nozzle, even though $T_9 < T_w$ – why?
  - Heat transfer formula uses stagnation temperature, not static T
  - Since heat loss occurs at walls and gas must decelerate to M = 0 at walls, T_t is a better temperature to use in heat transfer estimate
- Actually, whether T_t or T provides a better estimate of heat transfer depends on “temperature recovery factor” – beyond scope of this course; we’ll use T_t

Turbojet cycle - effect of heat loss

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- For ridiculously high heat transfer coefficient h
  - Gas temperature follows T_w during compression (constant-T compression, like Stirling cycle)
  - Heat added at constant P
  - During turbine & nozzle expansion, gas follows ≈ constant-P path until T_t = 900K (= T_w for turbine & nozzle), then remainder of turbine work extracted isothermally (again like Stirling cycle)
  - Gas expands adiabatically through nozzle (no change in T_t during nozzle expansion - adiabatic since T_t = wall T)
- Effect of heat loss on performance (plot next page)
  - Barely affects specific thrust since the pressures (thus velocities, thus propulsive efficiencies) don't change much
  - Much more thermal energy must be added, thus TSFC increases, thermal &overall efficiency decrease
- Special notes
  - T-s diagram shows static temp. T, not stagnation temp. T_t
  - $Q = \int Tds$, $\eta_{Carnot} = 1 - T_L/T_H$, $P = \rho RT$, etc., for static T
  - Unless the formula specifically says T_t or P_t, use T or P

Turbojet cycle - effect of heat loss

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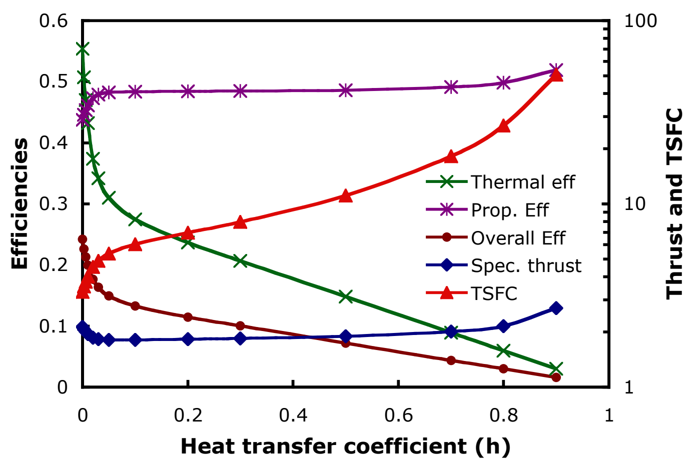
Turbojet
No fan
No afterburner
$M_1 = 0.8$
$\pi_c = 10.97$
$\tau_\lambda = 5$



Combination of all non-ideal effects

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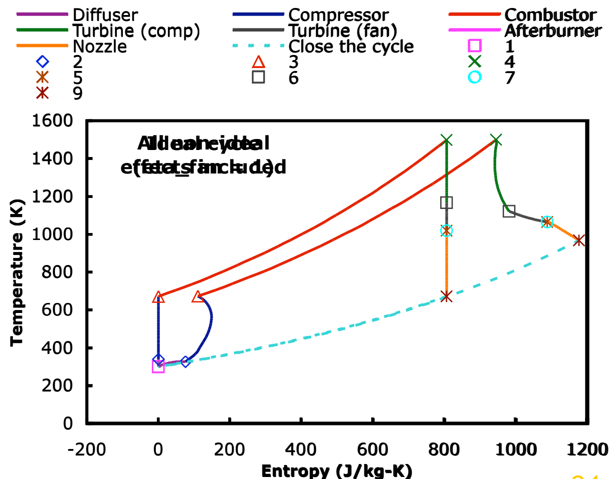
	Spec. Thrust	TSFC	Thermal Eff.	Prop. Eff.	Overall Eff.	Specific impulse (sec)
Ideal	0.9314	2.5193	0.5504	0.5770	0.3176	5247.4
Non-ideal	0.3800	6.2803	0.1976	0.6445	0.1274	2104.9

Turbofan, no afterburner
$M_1 = 0.8$, $\pi_c = 10.97$, $\tau_\lambda = 5$
$\pi_c' = 2$, $\alpha = 2$
$\eta_{diffuser} = 1$ or 0.9
$\eta_{comp} = 1$ or 0.9
$\pi_{burner} = 1$ or 0.9
$\eta_{turb} = 1$ or 0.9
$\eta_{nozzle} = 1$ or 0.9
$\eta_{fan} = 1$ or 0.9
h = 0 or 0.02 (all wall temperatures = average gas temp for that component)
Drag coefficient = 0 or 0.2



How accurate is aircycles4recips.xls?

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- Real engine: GE90-85B turbofan, no afterburner
- Static sea level ($M_1 = 0$, $T_1 = 300K$, $P_1 = 1$ atm) or cruise ($M_1 = 0.83$, $T_1 = 219K$, $P_1 = 0.235$ atm)
- Reported properties: $\pi_c = 39.3$; $\pi_c' = 1.65$; $\alpha = 8.4$; Inlet area = 7.66 m^2; $C_D = 0$ (drag not included in reported performance)
- Guessed properties
  - Turbine inlet temp. = 1700K $\Rightarrow \tau_\lambda = 5.67$ (static) or 7.76 (cruise)
  - $\eta_{diffuser} = 1$ (static) or 0.97 (cruise); $\eta_{comp} = 0.9$; $\pi_{burner} = 0.98$; $\eta_{turb} = 0.9$; $\eta_{nozzle} = 0.98$; $\eta_{fan} = 0.9$
  - h = 0.01 (all wall temperatures = average gas T for that component)
- Results (very sensitive to $\eta_{diffuser}$ because of huge fan air flow!)

	Spec. Thrust (static)	TSFC (static)	Spec. Thrust (cruise)	TSFC (cruise)
Spreadsheet	0.988	1.095	**0.774**	**2.355**
Actual	0.788	1.183	**??? (air flow not reported)**	**2.213**

Example - numerical

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For a nonideal turbofan with bypass ratio (α) = 8, γ = 1.35, compressor pressure ratio (π_c) = 30, fan pressure ratio (π_c') = 1.8, flight Mach number (M_1) = 0.8, turbine inlet temperature = 1800K, ambient pressure (P_1) = 0.25 atm and ambient temperature (T_1) = 225 K, with $P_9 = P_1$ and FAR << 1, and component efficiencies as follows: diffuser (η_d) = 0.97, compressor (η_c) = 0.90, combustor (π_b) = 0.98, turbine (η_t) = 0.90, nozzle (η_d) = 0.98, fan (η_f) = 0.90, determine the following (this is the same as the example in lecture 13 except non-ideal components are presumed here):

(a) T, P and M after the diffuser (station 2)

$$M_2 = 0 \text{ by assumption; } T_2 = T_{2t} = T_{1t} = T_1\left(1+\frac{\gamma-1}{2}M_1^2\right) = (225K)\left(1+\frac{1.35-1}{2}0.8^2\right) = 250.2K$$

$$P_2 = P_{2t} = (\eta_d)^{\gamma/\gamma-1}P_{1t} = (\eta_d)^{\gamma/\gamma-1}P_1\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1} = (0.97)^{1.35/1.35-1}(0.25atm)\left(1+\frac{1.35-1}{2}0.8^2\right)^{1.35/1.35-1} = 0.335\ atm$$

(b) T, P and M after the compressor (station 3)

$$M_3 = 0 \text{ by assumption; } \pi_c = 30 = P_{3t}/P_{2t} \Rightarrow P_{3t} = P_3 = 30\times 0.335atm = 10.04\,atm$$

$$\frac{T_{3t}}{T_{2t}} = 1+\frac{(\pi_{comp})^{\gamma-1/\gamma}-1}{\eta_{comp}} \Rightarrow T_3 = T_{3t} = 250.2K\left(1+\frac{(30)^{\gamma-1/\gamma}-1}{0.9}\right) = 643.6K$$

(c) T, P and M after the combustor (station 4)

$M_4 = 0$; $P_{4t} = P_4 = (\pi_b)P_{3t} = 0.98(10.04\text{ atm}) = 9.84$ atm ; $T_{4t} = T_4 = 1800$K

Example - numerical (continued)

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(d) T, P and M after the compressor turbine (station 5). Balance turbine & compressor work:

$$C_p(T_{3t}-T_{2t}) = C_p(T_{4t}-T_{5t}) \Rightarrow 643.6K - 250.2K = 1800K - T_{5t} \Rightarrow T_{5t} = 1406.6\ K$$

$$\frac{T_{5t}}{T_{4t}} = 1+\eta_{turb}\left[(\pi_{turb})^{\gamma-1/\gamma}-1\right] \Rightarrow \pi_{turb} = \left[1+\frac{1}{\eta_{turb}}\left(\frac{T_{5t}}{T_{4t}}-1\right)\right]^{\gamma/\gamma-1} = \left[1+\frac{1}{0.9}\left(\frac{1406.6}{1800}-1\right)\right]^{1.35/1.35-1} = 0.342$$

$$\pi_{turb} = P_{5t}/P_{4t} \Rightarrow P_{5t} = 9.84(0.342) = 3.37\,atm;\ M_5 = 0 \text{ by assumption}$$

(e) T, P and M after the fan turbine (station 6)

For the fan stream, $T_2' = T_2 = 250.2$K and $P_2' = P_2 = 0.335$ atm (same as main stream)
Then we need to compute the fan stream work and equate it to the turbine work:

$$\frac{T_{3t}'}{T_{2t}'} = 1+\frac{(\pi_c')^{\gamma-1/\gamma}-1}{\eta_{fan}} \Rightarrow T_3' = T_{3t}' = 250.2K\left(1+\frac{(1.8)^{\gamma-1/\gamma}-1}{0.9}\right) = 296.0K;\ \frac{P_{3t}'}{P_{2t}'} = \pi_c' \Rightarrow P_{3t}' = (1.8)(0.335atm) = 0.603atm$$

$$W_{2-3,fan} = \dot{m}_a' C_p(T_{2t}'-T_{3t}') = -W_{5-6,turbine} = -\dot{m}_a C_p(T_{5t}-T_{6t}) \Rightarrow T_{6t} = T_{5t} + (\dot{m}_a'/\dot{m}_a)(T_{2t}'-T_{3t}')$$

$$\Rightarrow T_{6t} = T_6 = T_{5t} + \alpha(T_{2t}'-T_{3t}') = 1406.6K + 8(250.2K - 296.0K) = 1040.5K$$

$$\frac{T_{6t}}{T_{5t}} = 1+\eta_{turb}\left[(\pi_{turb})^{\gamma-1/\gamma}-1\right] \Rightarrow \pi_{turb} = \left[1+\frac{1}{\eta_{turb}}\left(\frac{T_{6t}}{T_{5t}}-1\right)\right]^{\gamma/\gamma-1} = \left[1+\frac{1}{0.9}\left(\frac{1040.5K}{1406.6K}-1\right)\right]^{1.35/1.35-1} = 0.268$$

$$\pi_{turb} = \frac{P_{6t}}{P_{5t}} \Rightarrow P_{6t} = P_6 = 3.37atm(0.268) = 0.902\,atm;\ M_6 = 0 \text{ by assumption}$$

Example - numerical (continued)

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(f) T, P and M after the nozzle (station 9, and 9' for the fan stream) (note that for the fan stream, nothing happens between stations 3 and 6)

$$P_9 = 0.25atm \text{ by assumption; } P_{9t} = (\eta_{nozzle})^{\gamma/\gamma-1} P_{6t} = (0.98)^{1.35/1.35-1}(0.902atm) = 0.834atm$$

$$\frac{P_{9t}}{P_9} = \left(1+\frac{\gamma-1}{2}M_9^2\right)^{\gamma/\gamma-1} \Rightarrow \left(\frac{0.834atm}{0.25atm}\right)^{1.35-1/1.35} = \left(1+\frac{1.35-1}{2}M_9^2\right) \Rightarrow M_9 = 1.448$$

$$T_{9t} = T_{6t} = T_9\left(1+\frac{\gamma-1}{2}M_9^2\right) \Rightarrow T_9 = T_{9t}\Big/\left(1+\frac{\gamma-1}{2}M_9^2\right) = 1040.5K\Big/\left(1+\frac{1.35-1}{2}1.448^2\right) = 761.2K$$

$$P_9' = 0.25atm \text{ by assumption; } P_{9t}' = (\eta'_{nozzle})^{\gamma/\gamma-1} P_{6t}' = (\eta'_{nozzle})^{\gamma/\gamma-1} P_{3t}' = (0.98)^{1.35/1.35-1}(0.603atm) = 0.558atm$$

$$\frac{P_{9t}'}{P_9'} = \left(1+\frac{\gamma-1}{2}M_9^2\right)^{\gamma/\gamma-1} \Rightarrow \left(\frac{0.558atm}{0.25atm}\right)^{1.35-1/1.35} = \left(1+\frac{1.35-1}{2}M'^2_9\right) \Rightarrow M_9' = 1.15$$

$$T_{9t}' = T_{6t}' = T_9'\left(1+\frac{\gamma-1}{2}M'^2_9\right) \Rightarrow T_9' = T_{9t}'\Big/\left(1+\frac{\gamma-1}{2}M'^2_9\right) = 296.0K\Big/\left(1+\frac{1.35-1}{2}1.15^2\right) = 240.4K$$

(g) Specific thrust (ST) (recall FAR << 1 and $P_9 = P_1$ by assumption)

$$Thrust \approx \dot{m}_a[u_9 - u_1] + \dot{m}_a'[u_9' - u_1]; ST = \frac{Thrust}{(\dot{m}_a + \dot{m}_a')c_1} = \frac{\dot{m}_a[u_9 - u_1]}{(\dot{m}_a + \dot{m}_a')c_1} + \frac{\dot{m}_a'[u_9 - u_1]}{(\dot{m}_a + \dot{m}_a')c_1}$$

$$ST = \frac{1}{1+\alpha}\left\{\left[M_9\sqrt{T_9/T_1} - M_1\right] + \alpha\left[M_9'\sqrt{T_9'/T_1} - M_1\right]\right\}$$

$$ST = \frac{1}{1+8}\left\{\left[1.448\sqrt{761.2/225} - 0.8\right] + 8\left[1.149\sqrt{240.4K/225K} - 0.8\right]\right\} = 0.552$$

Example - numerical (continued)

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(h) TSFC and overall efficiency

Recalling that only $1/(1+\alpha)$ of the total air flow is burned,

$$TSFC \equiv \frac{\dot{m}_f Q_R}{Thrust \cdot c_1} = \frac{\dot{m}_a C_P(T_{4t} - T_{3t})}{Thrust \cdot c_1} = \frac{(\dot{m}_a + \dot{m}_a')c_1}{Thrust}\frac{\dot{m}_a}{\dot{m}_a + \dot{m}_a'}\frac{C_P}{c_1^2}(T_{4t} - T_{3t}) = \frac{1}{ST}\frac{1}{1+\alpha}\frac{\frac{\gamma}{\gamma-1}R}{\gamma R T_1}(T_{4t} - T_{3t})$$

$$TSFC = \frac{1}{ST}\frac{1}{1+\alpha}\frac{T_{4t} - T_{3t}}{(\gamma-1)T_1} = \frac{1}{0.552}\frac{1}{1+8}\frac{1800K - 643.6K}{(1.35-1)225K} = 2.96; \quad \eta_o = \frac{M_1}{TSFC} = \frac{0.8}{2.96} = 0.271$$

note that compared to the ideal cycle, TSFC suffers slightly more (2.96 vs. 2.32 = 28% increase) than ST (0.552 vs. 0.728 = 24% decrease)

(i) Propulsive efficiency

$$\eta_p \equiv \frac{\text{Thrust power}}{\Delta(\text{Kinetic energy})} = \frac{Thrust \cdot u_1}{\dot{m}_a(u_9^2 - u_1^2)/2 + \dot{m}_a'(u_9^2{}' - u_1^2{}')/2} = \frac{Thrust}{(\dot{m}_a + \dot{m}_a')c_1} \cdot \frac{2(\dot{m}_a + \dot{m}_a')c_1 u_1}{\dot{m}_a(u_9^2 - u_1^2) + \dot{m}_a'(u_9'^2 - u_1'^2)}$$

$$\eta_p = ST\frac{2(1+\alpha)c_1^2 M_1}{(M_9^2 c_9^2 - M_1^2 c_1^2) + \alpha(M_9'^2 c_9'^2 - M_1'^2 c_1'^2)} = ST\frac{2(1+\alpha)(\gamma R T_1)M_1}{(M_9^2(\gamma R T_9) - M_1^2(\gamma R T_1)) + \alpha(M_e'^2(\gamma R T_9') - M_1^2(\gamma R T_1))}$$

$$= ST\frac{2(1+\alpha)M_1 T_1}{(M_9^2 T_9 - M_1^2 T_1) + \alpha(M_9'^2 T_9' - M_1^2 T_1)} = 0.552\frac{(2)(1+8)(0.8)(225K)}{(1.448^2 761.2K - 0.8^2 225K) + 8(1.149^2 240.4K - 0.8^2 225K)}$$

$$= 0.630$$

Example - numerical (continued)

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(j) Thermal efficiency

$$\eta_{th} \equiv \frac{\Delta(\text{Kinetic energy})}{\dot{m}_f Q_R} = \frac{\dot{m}_a\left(u_9^2 - u_1^2\right)/2 + \dot{m}_a'\left(u_9^2{}' - u_1^2{}'\right)/2}{\dot{m}_a C_P (T_{4t} - T_{3t})} = \frac{1}{2}\frac{\left(u_9^2 - u_1^2\right) + \alpha\left(u_9^2{}' - u_1^2{}'\right)}{C_P(T_{4t} - T_{3t})}$$

$$\eta_{th} = \frac{1}{2}\frac{\left(M_9^2 c_9^2 - M_1^2 c_1^2\right) + \alpha\left(M'^2_9 c_9^2 - M'^2_1 c_1^2\right)}{C_P(T_{4t} - T_{3t})} = \frac{1}{2}\frac{\left(M_9^2 \gamma R T_9 - M_1^2 \gamma R T_1\right) + \alpha\left(M'^2_9 \gamma R T_9' - M'^2_1 \gamma R T_1\right)}{\frac{\gamma}{\gamma - 1} R (T_{4t} - T_{3t})}$$

$$\eta_{th} = \frac{\gamma - 1}{2}\frac{\left(M_9^2 T_9 - M_1^2 T_1\right) + \alpha\left(M'^2_9 T_9' - M'^2_1 T_1\right)}{T_{4t} - T_{3t}}$$

$$\eta_{th} = \frac{1.35 - 1}{2}\frac{\left(1.448^2 761.2K - 0.8^2 225K\right) + 8\left(1.149^2 240.4K - 0.8^2 225K\right)}{1800K - 643.6K} = 0.430$$

Note that $\eta_{th}\eta_p$ = (0.430)(0.630) = 0.271 = η_o as advertised. Note that compared to the ideal cycle where η_{th} = 0.629, η_p = 0.549 and η_o = 0.345, for the non-ideal case the thermal and overall efficiencies decreased (as expected) but propulsive efficiency actually increased slightly due to lower exit velocities.

(k) Specific impulse

$$I_{SP} = \frac{1}{TSFC}\frac{Q_R}{g_{earth} c_1} = \frac{1}{2.96}\frac{4.3 \times 10^7 J/kg}{(9.81 m/\sec^2)\sqrt{1.35(8.314 J/moleK)(mole/0.02897kg)(225K)}} = 5010\sec$$

Example - graphical

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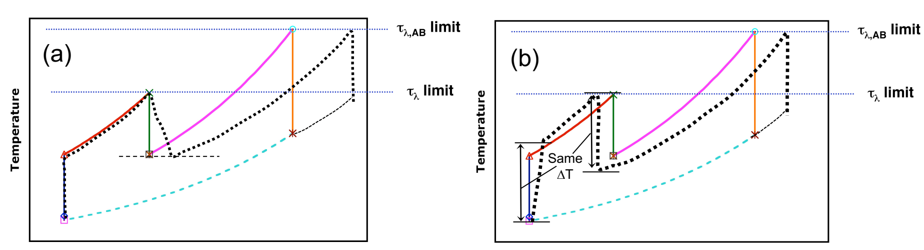
In an ideal τ_λ-limited afterburning turbojet, how would the T-s diagrams be affected if

(a) The turbine is irreversible, but all other components are still ideal

The cycles are the same up to the end of the first heat addition process. The turbine work (thus ΔT) does not change, but the irreversibility during expansion increases the entropy, so that afterburning heat addition occurs along a lower constant-pressure curve up to the same temperature limit. Although not shown, since the afterburner ΔT is the same for the modified cycle, the heat addition, thus area under the T-s curve, is the same for the modified afterburner.

(b) A new compressor is used with a higher pressure ratio, but this compressor is irreversible (all other components are still ideal)

The compression process ends at a higher pressure and (since it is irreversible) a higher entropy than the base cycle. Heat addition then occurs along this constant-pressure curve up to the turbine temperature limit. More work, thus more ΔT, is required from the turbine, so the afterburning heat addition occurs along a lower constant pressure curve.



Example - graphical

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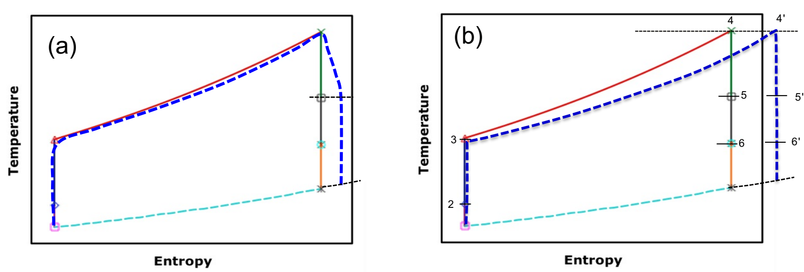
In an ideal turbofan, how would the T-s diagrams be affected if

(a) The fan is removed and the redesigned turbine that drives the compressor is irreversible

The cycle is the same up to the end of heat addition. The ΔT and thus the T after the compressor turbine is the same but the entropy is higher. There is no fan turbine. Expansion through the nozzle continues to ambient pressure.

(b) There are substantial pressure losses in the combustor

The cycle is the same up to the end of the compression process. Heat addition occurs at decreasing pressure but up to the same temperature limit as the base cycle. The compressor and fan work requirements are unchanged, so the temperatures at states 5 and 6 are unchanged.



Summary - non-ideal effects

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- Irreversible diffusion, compression, expansion (due to viscosity, shocks, flow separation, etc.) all reduce thrust & η_o
  - Stagnation pressure loss in diffuser - less expansion possible at end of cycle
  - More ΔT (thus more work) to drive compressor
  - More ΔP needed to extract turbine work required to drive compressor
  - Surprisingly, component efficiency at thrust = 0 nearly same for diffuser, compressor, turbine, nozzle! (0.55 - 0.6 for cases shown)
- Effect of combustor pressure loss relatively small
- $P_{exit} \neq P_{ambient}$ ($P_9 \neq P_1$) reduces thrust & η_o but effect is small
- Heat transfer to/from gas during cycle
  - Modeling of heat losses different for steady flow device
    - » Each component has its own T (unlike unsteady flow engines - use 1 wall temperature)
    - » Stagnation vs. static temperature
  - As with unsteady flow engines, more work required to compress hot gas than cold gas, lowers η_o - best performance at h = 0