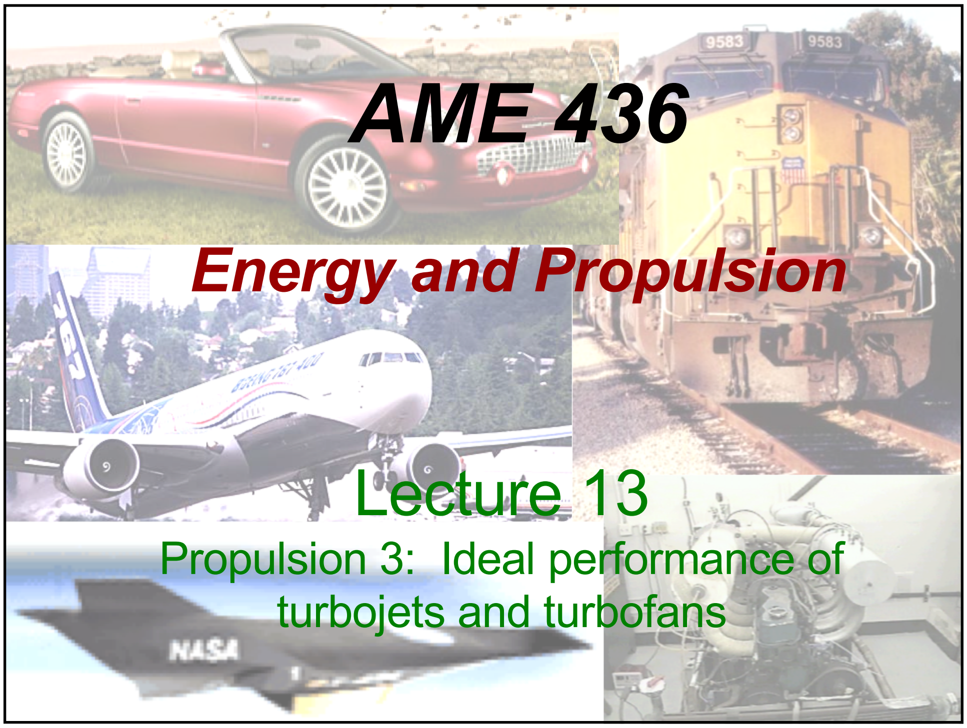

AME 436

Energy and Propulsion

Lecture 13

Propulsion 3: Ideal performance of turbojets and turbofans

Outline

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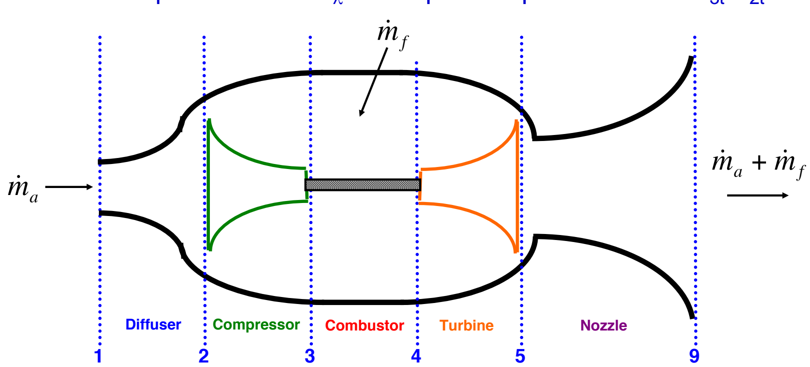
- Turbojet analysis - assumptions and goals
- Process summary
- State-by-state analysis
- Results
  - Thrust
  - Efficiency
  - Fuel consumption
- Effects of
  - Compressor pressure ratio
  - Flight Mach number
  - τ_λ limit
- Sidebar topic: recuperation

Outline (continued)

- Afterburning turbojet
  - Analysis
  - Results - Thrust, Efficiency, Fuel consumption
  - Effects of
    - » Compressor pressure ratio
    - » Flight Mach number
    - » $\tau_{\lambda,AB}$ limit
- Turbofan
  - Analysis
  - Results - Thrust, Efficiency, Fuel consumption
  - Effects of
    - » Fan bypass ratio
    - » Fan pressure ratio
    - » Compressor pressure ratio
    - » Flight Mach number
    - » τ_{λ} limit

Turbojet analysis

- Assumptions
  - Steady, quasi-1D
  - Constant C_P, γ, $P_{exit} = P_{ambient}$ ($P_9 = P_1$ in current notation)
  - Isentropic except for heat addition process
  - Heat addition at M << 1, FAR << 1 up to materials limit temperature T_λ
- Goals: determine Specific Thrust, Thrust Specific Fuel Consumption and thermal & overall efficiency as a function of flight Mach number M_1, turbine inlet temperature limit T_λ & compressor pressure ratio P_{3t}/P_{2t}



Why use Brayton cycle to model gas turbines?

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- Pressure compression ratio (r) = pressure expansion ratio (i.e., $P_9 = P_1$), which yields maximum thrust (Lecture 11)
- Heat input at constant P realistic for steady-flow, M << 1 process (see Rayleigh flow analysis)
- Constant s compression/expansion corresponds to adiabatic and reversible processes - not totally true but not bad
- Notes on Brayton cycle P-v and T-s diagrams
  - v on P-v diagram is specific volume (v) (m^3/kg) which IS a property of the gas (can't use "cylinder volume" V as in unsteady-flow engines since there isn't a fixed mass with changing volume)
  - s is specific entropy (J/kg-K) which IS a property of the gas, heat transfer = $\int Tds$ if mass doesn't change during heat addition
  - P-v diagrams not as useful as with unsteady-flow engines where we can use a cylinder pressure gauge to measure P vs. t and calculate V vs. t from crank angle to get P-V diagram for comparison with ideal cycle (would need a pressure gauge moving with the flow!)

Ideal turbojet cycle - process summary

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Process	Name	Const.	M	P_t	T_t	Comments
$1 \rightarrow 2$	Diffuser	s	↓ to 0	$P_{2t} = P_{1t}$	$T_{2t} = T_{1t}$	Decelerate incoming gas to M = 0; no work
$2 \rightarrow 3$	Compression	s	0	$P_{3t}/P_{2t} = \pi_c$	$T_{3t}/T_{2t} = \pi_c^{(\gamma-1)/\gamma}$	Compressor work input $= C_P(T_{3t}-T_{2t})$
$3 \rightarrow 4$	Combustion (main combustor)	P	0	$P_{4t} = P_{3t}$	$T_{4t} = \tau_\lambda T_1$	Add heat to $\tau_\lambda = T_{4t}/T_1$ - turbine inlet temperature limit
$4 \rightarrow 5$	Expansion through turbine (to pay for compressor work)	s	0	$P_{5t}/P_{4t} = (T_{5t}/T_{4t})^{\gamma/(\gamma-1)}$	$T_{5t} = T_{4t} - (T_{3t} - T_{2t})$	Turbine work output $= C_P(T_{4t}-T_{5t})$ = compressor work input $= C_P(T_{3t}-T_{2t})$
$5 \rightarrow 6$	Expansion through turbine (to pay for fan work)	s	0	$P_{6t}/P_{5t} = (T_{6t}/T_{5t})^{\gamma/(\gamma-1)}$	$T_{6t} = T_{5t} - (T_{3t,f} - T_{2t,f})$	Turbine work output $= C_P(T_{5t}-T_{6t})$ = fan work input $= \alpha C_P(T_{3t}'-T_{2t}')$
$6 \rightarrow 7$	Combustion (afterburner)	P	0	$P_{7t} = P_{6t}$	$T_{7t} = \tau_{\lambda,AB} T_1$	Add heat to $\tau_{\lambda,AB} = T_{7t}/T_1$ - afterburner temperature limit
$7 \rightarrow 9$	Expansion through nozzle	s	↑	$P_{9t} = P_{7t}$	$T_{9t} = T_{7t}$	$P_9 = P_1$

P-V & T-s diagrams for ideal turbojet

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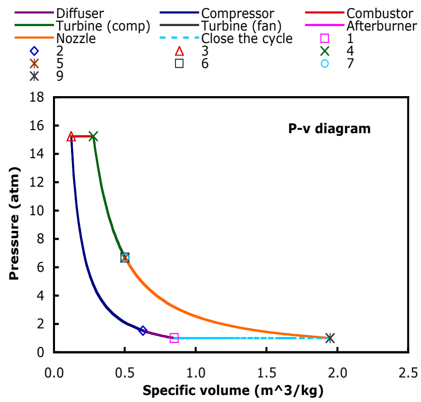
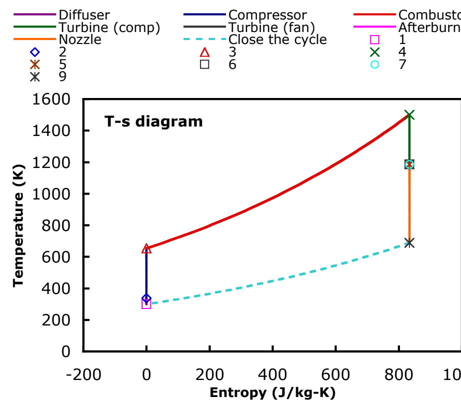
- Model shown is open cycle, where mixture is inhaled, compressed, burned, expanded then thrown away (not recycled)
- In a closed cycle with a fixed (trapped) mass of gas to which heat is transferred to/from, 9 → 1 would be connected (Why don't we do this? Heat transfer is too slow!)





P-V & T-s diagrams for ideal turbojet

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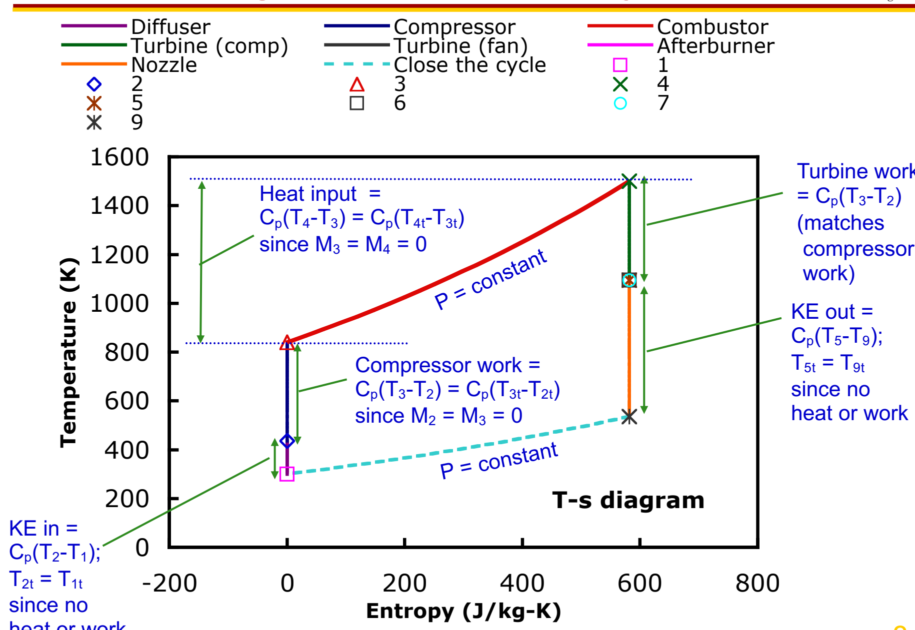

Ideal turbojet cycle - analysis

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- Inlet conditions M_1, T_1, P_1; after diffuser (2), decelerate to $M_2 = 0$

$$M_2 = 0;\ T_2 = T_1\tau_r;\ T_{2t} = T_{1t} = T_1\tau_r;\ P_2 = P_1\left(\tau_r\right)^{\gamma/\gamma-1};\ P_{2t} = P_{1t} = P_1\left(\tau_r\right)^{\gamma/\gamma-1}$$

$$q_{1\to2} = w_{1\to2} = 0;\ \tau_r \equiv 1 + \frac{\gamma-1}{2}M_1^2 = \text{"recovery temperature ratio"}$$

- After compressor (3): isentropic compression by pressure ratio π_c

$$M_3 = 0;\ T_3 = T_{3t} = T_{2t}(\pi_c)^{\gamma-1/\gamma} = T_1\tau_r(\pi_c)^{\gamma-1/\gamma};\ P_3 = P_{3t} = P_{2t}\pi_c = P_1(\tau_r)^{\gamma/\gamma-1}\pi_c$$

$$q_{2\to3} = 0;\ w_{2\to3} = C_P(T_{2t} - T_{3t})$$

- After combustor (4): constant-pressure heat addition to T_λ

$$M_4 = 0;\ T_4 = T_{4t} = T_1\tau_\lambda;\ P_4 = P_{4t} = P_3 = P_1(\tau_r)^{\gamma/\gamma-1}\pi_c$$

$$q_{3\to4} = C_P(T_{4t} - T_{3t});\ w_{3\to4} = 0$$

- After turbine (5): isentropic expansion to pay for compressor work

$$M_5 = 0;\ q_{4\to5} = 0;\ w_{4\to5} = -w_{2\to3} = C_P(T_{4t} - T_{5t}) = -C_P(T_{2t} - T_{3t})$$

$$T_5 = T_{5t} = T_{4t} + (T_{2t} - T_{3t}) = T_1\tau_\lambda + T_1\tau_r - T_1\tau_r\left(\pi_c\right)^{\gamma-1/\gamma} = T_1\left[\tau_\lambda - \tau_r\left(\left(\pi_c\right)^{\gamma-1/\gamma} - 1\right)\right]$$

$$P_5 = P_{5t} = P_{4t}\left(T_{5t}/T_{4t}\right)^{\gamma/\gamma-1} = P_1\left(\tau_r\right)^{\gamma/\gamma-1}\pi_c\left[1 - \left(\tau_r/\tau_\lambda\right)\left(\left(\pi_c\right)^{\gamma-1/\gamma} - 1\right)\right]^{\gamma/\gamma-1}$$

Ideal turbojet cycle - analysis

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- After nozzle (9): isentropic expansion to $P_9 = P_1$ ($P_e = P_a$)

$$q_{4\to5} = w_{4\to5} = 0;\ P_9 = P_1;\ P_{9t} = P_{5t} = P_1\left(\tau_r\right)^{\gamma/\gamma-1}\pi_c\left[1 - \left(\tau_r/\tau_\lambda\right)\left(\left(\pi_c\right)^{\gamma-1/\gamma} - 1\right)\right]^{\gamma/\gamma-1}$$

$$P_{9t}/P_9 = P_1\left(\tau_r\right)^{\gamma/\gamma-1}\pi_c\left[1 - \left(\tau_r/\tau_\lambda\right)\left(\left(\pi_c\right)^{\gamma-1/\gamma} - 1\right)\right]^{\gamma/\gamma-1}\Big/P_1 = \left(1 + \frac{\gamma-1}{2}M_9^2\right)^{\gamma/\gamma-1}$$

$$\Rightarrow M_9 = \sqrt{\frac{2}{\gamma-1}\left\{\tau_r\left(\pi_c\right)^{\gamma-1/\gamma}\left[1 - \left(\tau_r/\tau_\lambda\right)\left(\left(\pi_c\right)^{\gamma-1/\gamma} - 1\right)\right] - 1\right\}}$$

$$T_9 = T_5\left(P_9/P_5\right)^{\gamma-1/\gamma} = T_1\left[\tau_\lambda - \tau_r\left(\left(\pi_c\right)^{\gamma-1/\gamma} - 1\right)\right]\Big/\left\{\tau_r\left(\pi_c\right)^{\gamma-1/\gamma}\left[1 - \left(\tau_r/\tau_\lambda\right)\left(\left(\pi_c\right)^{\gamma-1/\gamma} - 1\right)\right]\right\}$$

$$T_9 = T_1\tau_\lambda\Big/\tau_r\left(\pi_c\right)^{\gamma-1/\gamma}$$

$$T_{9t} = T_{5t} = T_1\left[\tau_\lambda - \tau_r\left(\left(\pi_c\right)^{\gamma-1/\gamma} - 1\right)\right]$$

Ideal turbojet cycle - Thrust

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- Recall (Lecture 11) $Thrust = \dot{m}_a[(1+FAR)u_9 - u_1] + (P_9 - P_1)A_9$

  For $FAR << 1,\ P_9 = P_1:\ Thrust \approx \dot{m}_a[u_9 - u_1]$

  $$\text{Specific Thrust (ST)} \equiv \frac{Thrust}{\dot{m}_a c_1} = \frac{u_9}{c_9}\frac{c_9}{c_1} - \frac{u_1}{c_1} = M_9\frac{\sqrt{\gamma R T_9}}{\sqrt{\gamma R T_1}} - M_1 = M_9\sqrt{\frac{T_9}{T_1}} - M_1$$

  $$\text{ST} = \sqrt{\frac{2}{\gamma-1}\left\{\tau_r(\pi_c)^{\gamma-1/\gamma}\left[1-(\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]-1\right\}}\sqrt{\frac{\tau_\lambda}{\tau_r(\pi_c)^{\gamma-1/\gamma}}} - M_1$$

  $$\text{ST} = \sqrt{\frac{2}{\gamma-1}\left[\tau_\lambda\left(1-1/\tau_r(\pi_c)^{\gamma-1/\gamma}\right)-\tau_r\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]} - M_1$$

- This gives the thrust in terms of
  - Air flow ($\dot{m}_a$)
  - Sound speed at ambient conditions (c_1)
  - Flight Mach number M_1 and $\tau_r \equiv 1 + [(\gamma-1)/2]M_1^2$
  - Compressor pressure ratio π_c
  - Materials-limited temperature at turbine inlet = $\tau_\lambda T_1$

Ideal turbojet cycle - comments

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- Recall the assumptions required to get these results
  - 1D steady flow
  - Ideal gas, constant specific heats
  - FAR << 1, $P_9 = P_1$
  - Isentropic compression & expansion
  - Constant-P combustion at $M_3 = M_4 = 0$, add heat to τ_λ limit
- Special case: $\pi_c = 1$ (ramjet, no compressor)

  $$\frac{Thrust}{\dot{m}_a c_1} = \sqrt{\frac{2}{\gamma-1}\left[\tau_\lambda(1-1/\tau_r)\right]} - M_1 = \left(\sqrt{\frac{\tau_\lambda}{\tau_r}}-1\right)M_1 \quad \text{(no thrust at } M_1 = 0!\text{)}$$

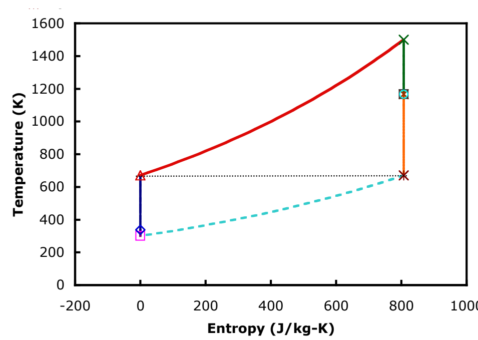
- If $\tau_\lambda = \tau_r(\pi_c)^{(\gamma-1)/\gamma}$ then Thrust = 0
  - τ_λ materials limited temperature reached just by decelerating the gas to M = 0 and compressing it in the compressor
  - No "head room" in terms of temperature to enable heat addition
  - Could happen at unrealistically high π_c or (more realistically) at very high M_1 (thus high τ_r)

Ideal turbojet cycle - notes on thrust

- Since either too low or too high π_c leads to Thrust = 0, there is a value of π_c that maximizes Thrust (but not any type of efficiency):

$$\frac{\partial(ST)}{\partial(\pi_c)} = 0 \Rightarrow \pi_c = \left(\frac{\sqrt{\tau_\lambda}}{\tau_r}\right)^{\gamma/\gamma-1}$$

- At this condition, $T_3 = T_9 = (\tau_\lambda)^{1/2}T_1$, i.e. temperature at end of compression = temperature at end of expansion
- For typical $\tau_\lambda = 5$, $\tau_r = 1.128$ (M = 0.8), $\gamma = 1.4$, optimal $\pi_c = 10.97$



Ideal turbojet cycle - thermal efficiency

$$\text{Note } T_t - T = T\left(1+\frac{\gamma-1}{2}M^2\right) - T = \frac{\gamma-1}{2}M^2T = \frac{\gamma-1}{2}\frac{u^2}{c^2}T$$

$$= \frac{\gamma-1}{\gamma RT}\frac{u^2}{2}T = \left(\frac{\gamma}{\gamma-1}R\right)^{-1}\frac{u^2}{2} = \frac{1}{C_P}\frac{u^2}{2} \Rightarrow \text{KE per unit mass} = \frac{u^2}{2} = C_P\left(T_t - T\right);\text{ then}$$

$$\eta_{th} = \frac{\text{what you get}}{\text{what you pay for}} = \frac{\text{KE}_{\text{out}} - \text{KE}_{\text{in}}}{\text{heat in}} = \frac{u_9^2/2 - u_1^2/2}{C_P(T_{4t} - T_{3t})}$$

$$= \frac{C_P(T_{9t}-T_9) - C_P(T_{1t}-T_1)}{C_P(T_{4t}-T_{3t})} = \frac{T_1\left[\tau_\lambda - \tau_r\left(\left(\pi_c\right)^{\gamma-1/\gamma} - 1\right)\right] - T_1\tau_\lambda\Big/\tau_r\left(\pi_c\right)^{\gamma-1/\gamma} - T_1\tau_r + T_1}{T_1\tau_\lambda - T_1\tau_r\left(\pi_c\right)^{\gamma-1/\gamma}}$$

$$= \frac{\tau_\lambda - \tau_r\left(\pi_c\right)^{\gamma-1/\gamma} - \tau_\lambda\Big/\tau_r\left(\pi_c\right)^{\gamma-1/\gamma} + 1}{\tau_\lambda - \tau_r\left(\pi_c\right)^{\gamma-1/\gamma}} = 1 - \frac{\tau_\lambda\Big/\tau_r\left(\pi_c\right)^{\gamma-1/\gamma} - 1}{\tau_\lambda - \tau_r\left(\pi_c\right)^{\gamma-1/\gamma}} = 1 - \frac{1}{\tau_r\left(\pi_c\right)^{\gamma-1/\gamma}}$$

$$\Rightarrow \boxed{\eta_{th} = 1 - \frac{1}{\left(\pi_r\pi_c\right)^{\gamma-1/\gamma}} = 1 - \frac{1}{\left(r\right)^{\gamma-1/\gamma}}};\quad r = \pi_r\pi_c;\ \pi_r = \text{"recovery pressure"} = \left(\tau_r\right)^{\gamma/\gamma-1}$$

Ideal turbojet cycle - thermal efficiency

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- Note thermal efficiency $\eta_{th} = 1 - 1/r^{(\gamma-1)/\gamma}$, where $r = \pi_r\pi_c$, i.e. the combined pressure rise due to "ram effect" compression (decelerating the gas from fM$_1$ to M = 0) AND the mechanical compression - each has the same effect on thermal efficiency
- Very similar to the Otto cycle ($\eta_{th} = 1 - 1/r^{(\gamma-1)}$), where r is the volume (not pressure) ratio)
  - Why the difference? Otto is constant V heat addition and expansion back to the initial V, whereas Brayton is constant P heat addition and expansion back to the initial P
- In either case $\eta_{th} = 1 - T_L/T_H$ and T_L/T_H is the same for each Carnot strip in this cycle; $T_L/T_H = (P_L/P_H)^{(\gamma-1)/\gamma} = (V_H/V_L)^{(\gamma-1)}$, thus $\eta_{th} = 1 - (P_L/P_H)^{(\gamma-1)/\gamma} = 1 - (V_H/V_L)^{(\gamma-1)}$; the only difference is that Otto cycles are specified in terms of volume compression ratio whereas Brayton cycles are specified in terms of pressure compression ratio

Ideal turbojet cycle - fuel consumption

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- Recall (Lecture 11) Thrust Specific Fuel Consumption (TSFC):

$$TSFC \equiv \frac{\dot{m}_f}{Thrust}\frac{Q_R}{c_1} = \left(\frac{\dot{m}_a c_1}{Thrust}\right)\frac{FAR \cdot Q_R}{c_1^2} = \frac{1}{ST}\frac{FAR \cdot Q_R}{\gamma R T_1}$$

- To compute TSFC is to compute FAR; energy balance on combustor (heat input = change in total enthalpy):

$$\dot{m}_f Q_R = (\dot{m}_a + \dot{m}_f)C_P T_{4t} - (\dot{m}_a)C_P T_{3t}; (FAR)Q_R = (1 + FAR)C_P T_{4t} - (1)C_P T_{3t}$$

$$\text{but } FAR << 1,\ T_{4t} = T_1\tau_\lambda,\ T_{3t} = T_1\tau_r(\pi_c)^{\gamma-1/\gamma} \Rightarrow \frac{(FAR)Q_R}{C_P T_1} = \tau_\lambda - \tau_r(\pi_c)^{\gamma-1/\gamma}$$

$$\Rightarrow TSFC \equiv \frac{1}{ST}\frac{(FAR)Q_R}{\gamma R T_1} = \frac{1}{ST}\frac{(FAR)Q_R}{C_P T_1}\frac{C_P T_1}{\gamma R T_1} = \frac{1}{ST}\left(\tau_\lambda - \tau_r(\pi_c)^{\gamma-1/\gamma}\right)\frac{\frac{\gamma}{\gamma-1}RT_1}{\gamma R T_1}$$

$$\Rightarrow TSFC = \frac{1}{ST}\frac{1}{\gamma-1}\left(\tau_\lambda - \tau_r(\pi_c)^{\gamma-1/\gamma}\right); \eta_o = \frac{M_1}{TSFC} = \frac{(\gamma-1)M_1 ST}{\tau_\lambda - \tau_r(\pi_c)^{\gamma-1/\gamma}}$$

Ideal turbojet; use ST from page 11

Ideal turbojet cycle - fuel consumption

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- Note on FAR: how do we know that we can add enough fuel to reach the τ_λ limit before we run out of O_2 in the air? From the previous page, energy balance on combustor:

$$(FAR)Q_R / C_P T_1 = \tau_\lambda - \tau_r (\pi_c)^{\gamma-1/\gamma}$$

Using realistic numbers FAR_{stoich} = 0.068, Q_R = 4.3 x 10^7 J/kg, C_P = 1400 J/kgK, T_1 = 300K, $(FAR_{stoich}Q_R)/(C_PT_1)$ = 7.2, and we require 0 < FAR < FAR_{stoich}, thus at stoichiometric

$$\tau_\lambda - \tau_r (\pi_c)^{\gamma-1/\gamma} \approx 7.3$$

But typically the maximum allowable turbine inlet temperature T_{4t} is at most 1800K; with T_1 = 300K, τ_λ = 6 < 7.2, so we can never add the stoichiometric amount of fuel - we reach the materials limit first

Ideal turbojet cycle - fuel consumption

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- Also note that FAR can be calculated via

$$(FAR)Q_R / C_P T_1 = \tau_\lambda - \tau_r (\pi_c)^{\gamma-1/\gamma} \Rightarrow FAR = (C_P T_1 / Q_R)\left(\tau_\lambda - \tau_r (\pi_c)^{\gamma-1/\gamma}\right)$$

- For typical values T_1 = 300K, π_c = 30, γ = 1.4, τ_λ = 5, τ_r = 1 (M_1 = 0), Q_R = 4.3 x 10^7 J/kg, C_P = 1400 J/kgK, FAR_{stoich} = 0.068

  FAR ≈ 0.023; ϕ = FAR/FAR_{stoich} = 0.34 << 1

- Wait - isn't ϕ = 0.34 too lean to burn? For premixed hydrocarbon-air flame, yes, for non-premixed flame (e.g. spray flame, like diesel but continuous, not injected at discrete times), not a problem

Ideal turbojet cycle - results - effect of π_c

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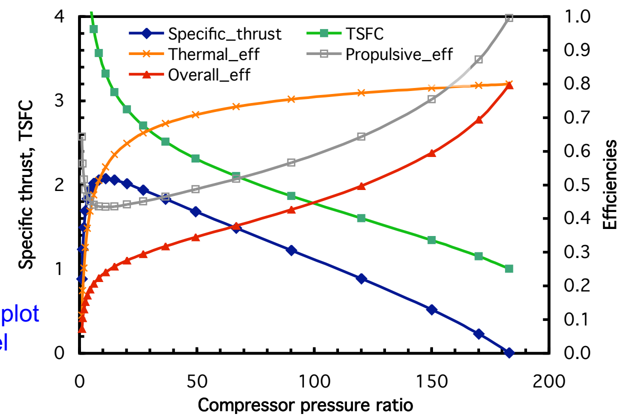
- Next few slides: baseline values M_1 = 0.8, γ = 1.4, τ_λ = 5, π_c = 30
- One parameter changed, others fixed
- For very low π_c, η_{th} is low, so both thrust and TSFC are low
- At very (unrealistically) high π_c, very little fuel can be added, thus Thrust decreases, but TSFC is great!

M_1 = 0.8
γ = 1.4
τ_λ = 5
π_c = varies



Double-click plot to open Excel spreadsheet

Ideal turbojet cycle - results - effect of π_c

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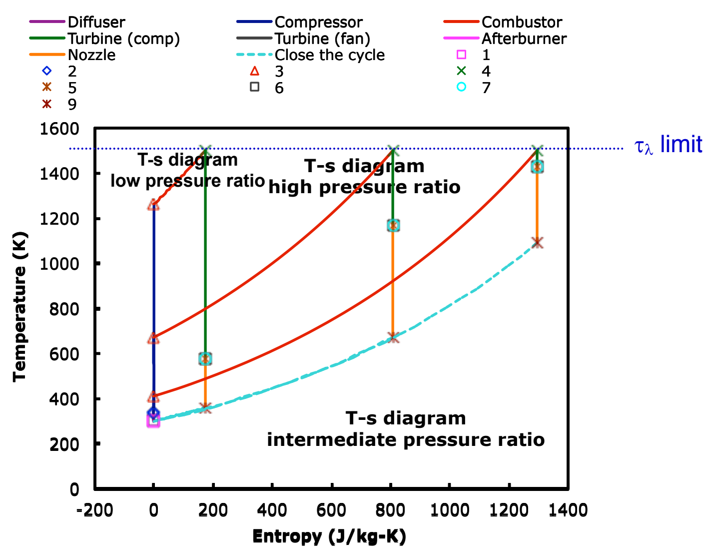
- T-s diagrams show tall skinny T-s diagrams for high π_c, "banana" shaped cycles for low π_c, and "fat" cycles for intermediate π_c



Ideal turbojet cycle - results - effect of M_1

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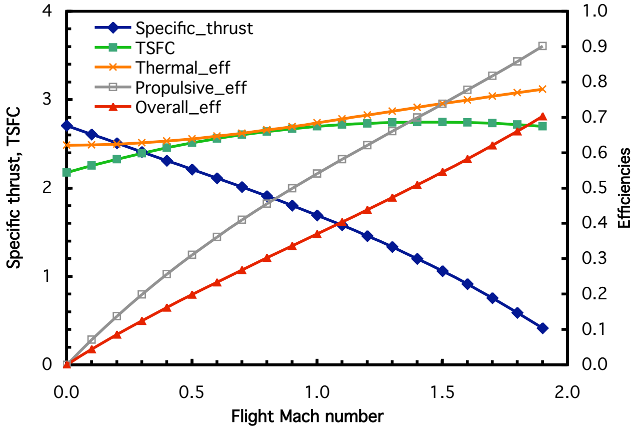
- ST decreases as M_1 increases: less fuel can be added (τ_λ limit)
- η_{th} increases as M_1 increases: total P ratio = $\pi_r\pi_c$ increases
- $\eta_{prop} = 2(u_1/u_9)/(1+ u_1/u_9)$ increases as M_1 increases: $u_1/u_9 \rightarrow 1$
- As a result, $\eta_{overall} = \eta_{th}\eta_{prop}$ increases as M_1 increases
- TSFC increases as M_1 increases since TSFC = $M_1/\eta_{overall}$

M_1 = varies
$\gamma = 1.4$
$\tau_\lambda = 5$
$\pi_c = 30$



Ideal turbojet cycle - results - effect of M_1

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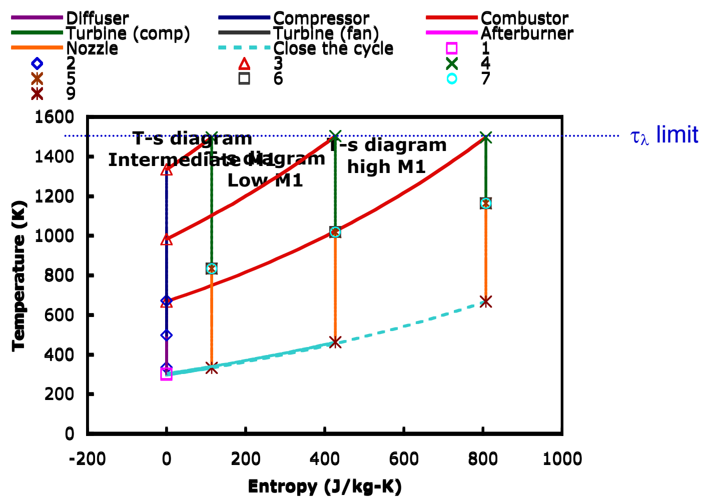
- T-s diagrams similar to π_c effect - tall skinny diagrams for high M_1, "banana" shaped cycles for low M_1, and "fat" cycles for intermediate M_1



Ideal turbojet cycle - results - effect of τ_λ

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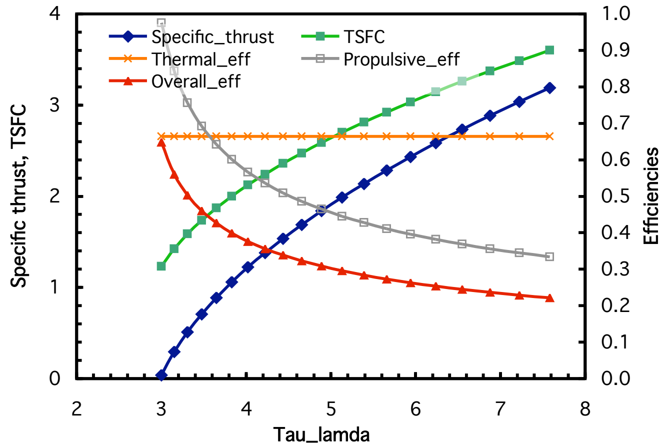
- As τ_λ increases, specific thrust increases but so does TSFC due to lower propulsive efficiency (η_{prop} = 2(u_1/u_9)/(1+ u_1/u_9); for fixed u_1, u_9 increases with increasing τ_λ)
- At very low τ_λ, no heat addition is possible, thus no thrust

M_1 = 0.8
γ = 1.4
τ_λ = varies
π_c = 30



Ideal turbojet cycle - results - effect of τ_λ

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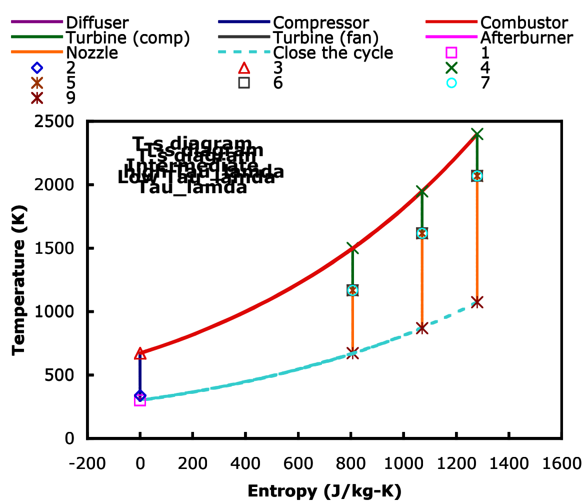
- T-s diagrams just show increasing heat addition as τ_λ increases, no change in η_{th} (which can be seen on T-s) but decreases in η_{prop} (which can't be seen on T-s or P-v)



Sidebar topic: Recuperation

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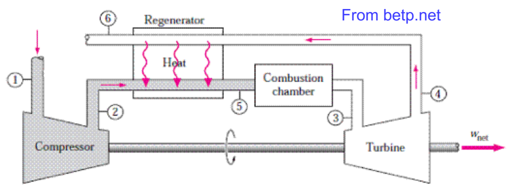
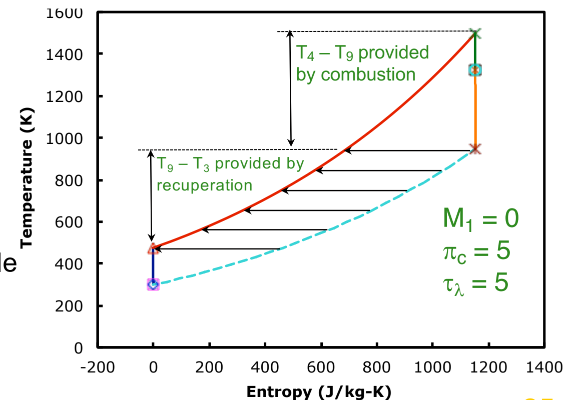
- If $T_9 > T_3$, some heat from exhaust can be transferred to incoming air after compression but before combustion to reduce fuel consumption – recuperation
- Only practical for stationary power generation ($M_1 = 0$) because heat exchangers are too large and heavy for aircraft engines
- For $M_1 = 0$, $T_9 > T_3$ requires

$$\pi_c < \tau_\lambda^{\gamma/2(\gamma-1)}$$

else no recuperation possible
- Analysis same as baseline Brayton except for calculation of heat addition

From betp.net





Recuperation - analysis

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$$M = 0:\ T_2 = T_1;\ T_3 = T_2\pi_c^{\gamma-1/\gamma} = T_1R;\ T_4 = \tau_\lambda T_1;\ T_9 = T_4/R = \tau_\lambda T_1/R;\ R \equiv \pi_c^{\gamma-1/\gamma}$$

$$\eta_{th} = \frac{\text{Net work}}{\text{Net heat input}} = \frac{C_P(T_4 - T_9) - C_P(T_3 - T_2)}{C_P(T_4 - T_3) - \varepsilon C_P(T_9 - T_3)}$$

$$\varepsilon \equiv \text{ Heat exchanger effectiveness} = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transger}}$$

$$\eta_{th} = \frac{(\tau_\lambda T_1 - \tau_\lambda T_1/R) - (T_1R - T_1)}{(\tau_\lambda T_1 - T_1R) - \varepsilon(\tau_\lambda T_1/R - T_1R)} = \frac{(\tau_\lambda - R)(R-1)}{\tau_\lambda(R-\varepsilon) - R^2(1-\varepsilon)};R \equiv \pi_c^{\gamma-1/\gamma}$$

$$\text{Checks}: \varepsilon = 0 \text{ (no recuperation)}, \eta_{th} = 1 - 1/R = 1 - 1/r^{\gamma-1/\gamma} \text{ (standard Brayton cycle)}$$

$$\varepsilon = 1, R \to 1 \text{ (perfect recuperation, no T rise due to compression):}$$

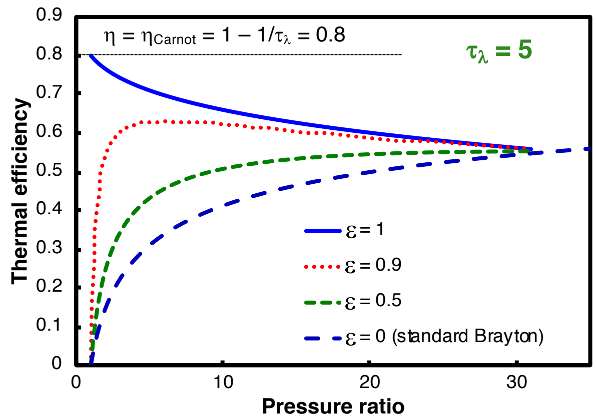
$$\eta_{th} \to 1 - 1/\tau_\lambda = 1 - 1/(T_4/T_1) = 1 - (T_1/T_4) \text{ (Same as Carnot since}$$

$$\text{heat addition only at T} = \text{T}_4 \text{ and heat rejection only at T} = \text{T}_1\text{)}$$

$$R \to 1 \text{ (no compression)}: \eta_{th} \to 0 \text{ (unless } \varepsilon = 1\text{)}$$

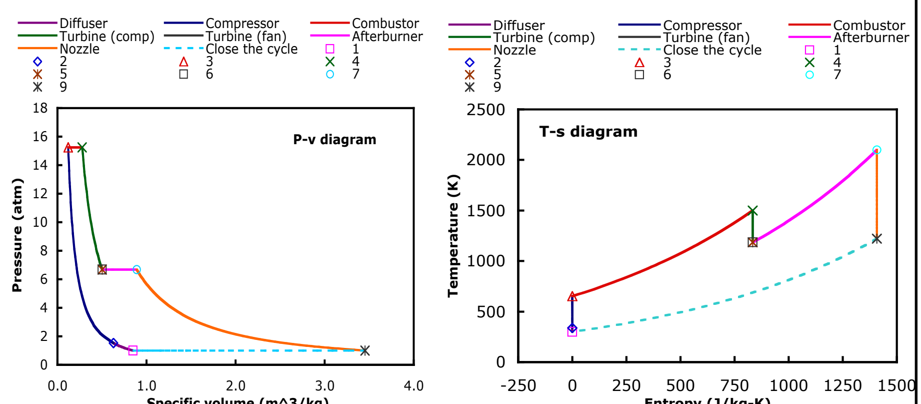
Recuperation - results

- Substantially increases efficiency when π_c is low
- No recuperation possible at high π_c (≈ 32 for case shown) because then $T_9 < T_3$ (exhaust T < post-compression T)
- High exchanger efficiency required for best results
- At sufficiently high ε (≈ 0.52 for case shown), there is an optimum π_c that maximizes efficiency (optimum $\pi_c \rightarrow 1$ as $\varepsilon \rightarrow 1$)



Afterburning turbojet cycle

- In turbojet cycle FAR << FAR_{stoich} since τ_λ limit well below T_{ad} for stoichiometric HC-air, thus much O_2 never mixes with fuel & burns - lost opportunity to generate power/thrust
- Solution: afterburner - add more fuel AFTER turbine
  - No moving parts after turbine, easier to cool $\Rightarrow \tau_{\lambda,AB} > \tau_\lambda$



P-V & T-s - afterburning turbojet

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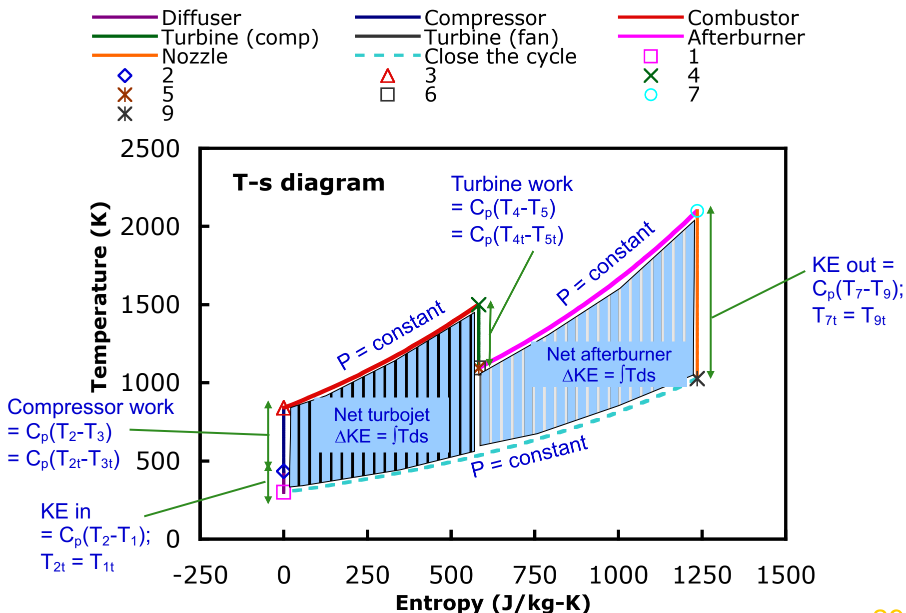

Afterburning turbojet cycle - analysis

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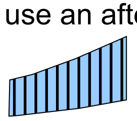
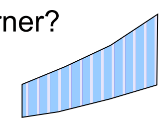
- So why doesn't everyone use an afterburner?
- Thermal efficiency is low!  vs. 
- Also - not useful to generate shaft work - can't put 2nd turbine after afterburner - too hot!
- Analysis of afterburning turbojet same as non-afterburning up to state 5 (after turbine); from page 9

$$M_5 = 0;\ q_{4\to5} = 0;\ w_{4\to5} = C_P(T_{4t} - T_{5t}) = -C_P(T_{2t} - T_{3t})$$

$$T_5 = T_{5t} = T_{4t} + (T_{2t} - T_{3t}) = T_1\tau_\lambda + T_1\tau_r - T_1\tau_r(\pi_c)^{\gamma-1/\gamma} = T_1\left[\tau_\lambda - \tau_r\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right]$$

$$P_5 = P_{5t} = P_{4t}(T_{5t}/T_{4t})^{\gamma/\gamma-1} = P_1(\tau_r)^{\gamma/\gamma-1}\pi_c\left[1 - (\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right]^{\gamma/\gamma-1}$$

Afterburning turbojet cycle - analysis

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- After afterburner (7): constant-P heat addition to $T_7 = T_1\tau_{\lambda,AB}$

$$M_7 = 0;\ T_7 = T_{7t} = T_1\tau_{\lambda,AB};\ q_{5\to7} = C_P(T_{7t} - T_{5t});\ w_{5\to7} = 0$$

$$P_7 = P_{7t} = P_5 = P_1(\tau_r)^{\gamma/\gamma-1}\pi_c\left[1-(\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]^{\gamma/\gamma-1}$$

- After nozzle (9): isentropic expansion to $P_9 = P_1$

$$q_{7\to9} = w_{7\to9} = 0;\ P_9 = P_1;\ P_{9t} = P_{7t} = P_{5t} = P_1(\tau_r)^{\gamma/\gamma-1}\pi_c\left[1-(\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]^{\gamma/\gamma-1}$$

$$P_{9t}/P_9 = P_1(\tau_r)^{\gamma/\gamma-1}\pi_c\left[1-(\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]^{\gamma/\gamma-1} \Big/ P_1 = \left(1+\frac{\gamma-1}{2}M_9^2\right)^{\gamma/\gamma-1}$$

$$\Rightarrow M_9 = \sqrt{\frac{2}{\gamma-1}\left\{\tau_r(\pi_c)^{\gamma-1/\gamma}\left[1-(\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]-1\right\}}$$

- Since all pressure ratios are same, M_9 is same as without afterburner!

Afterburning turbojet cycle - analysis

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- … but more heat has been added, so T_9 is larger, so $c_9 = \sqrt{\gamma R T_9}$ and thus u_9 is larger

$$T_{9t} = T_{7t} = T_1\tau_{\lambda,AB};\ T_9 = T_7(P_9/P_7)^{\gamma-1/\gamma} = T_1\tau_{\lambda,AB}(P_1/P_5)^{\gamma-1/\gamma}$$

$$T_9 = T_1\tau_{\lambda,AB}\left\{\tau_r(\pi_c)^{\gamma-1/\gamma}\left[1-(\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]\right\}^{-1}$$

$$\text{Specific Thrust (ST)} \equiv \frac{Thrust}{\dot m_a c_1} = \frac{\dot m_a(u_9-u_1)}{\dot m_a c_1} = M_9\sqrt{\frac{T_9}{T_1}} - M_1$$

For FAR << 1, $P_9 = P_1$ only !!!

$$ST = \sqrt{\frac{\frac{2}{\gamma-1}\tau_{\lambda,AB}\left\{\tau_r(\pi_c)^{\gamma-1/\gamma}\left[1-(\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]-1\right\}}{\tau_r(\pi_c)^{\gamma-1/\gamma}\left[1-(\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]}} - M_1$$

$$ST = \sqrt{\frac{2}{\gamma-1}\tau_{\lambda,AB}\left[1-\frac{1}{\tau_r(\pi_c)^{\gamma-1/\gamma}\left[1-(\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]}\right]} - M_1$$

Afterburning turbojet cycle - thrust

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- π_c = 1 (ramjet, no compressor)

$$ST \equiv \frac{Thrust}{\dot{m}_a c_1} = \sqrt{\frac{2}{\gamma-1}\left[\tau_{\lambda,AB}\left(1-1/\tau_r\right)\right]} - M_1 = \left(\sqrt{\frac{\tau_{\lambda,AB}}{\tau_r}}-1\right)M_1 \quad \text{(same as turbojet)}$$

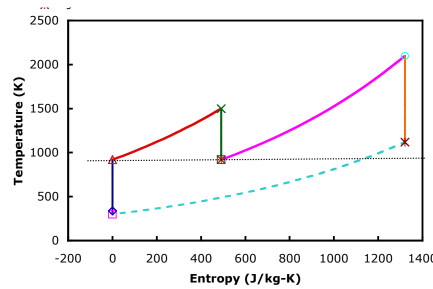
(i.e., an afterburning ramjet is just a ramjet since no compressor)

- Too high π_c leads to net work = 0 for main part, thus pressure ratio = 1 for afterburner part, thus no work for either part
- Too low π_c leads to no pressure ratio for main or afterburner part, thus no work for either part
- Value of π_c at maximum Thrust:

$$\frac{\partial(ST)}{\partial(\pi_c)} = 0 \Rightarrow \pi_c = \left(\frac{\tau_\lambda + \tau_r}{2\tau_r}\right)^{\gamma/\gamma-1}$$

for which $T_{3t} = T_{5t}$!

(Doesn't depend on $\tau_{\lambda,AB}$ at all!)



Afterburner - thermal efficiency

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$$\eta_{th} = \frac{\mathrm{KE}_{out} - \mathrm{KE}_{in}}{\text{heat in}} = \frac{u_9^2/2 - u_1^2/2}{C_P(T_{4t}-T_{3t}) + C_P(T_{7t}-T_{5t})} = \frac{C_P(T_{9t}-T_9) - C_P(T_{1t}-T_1)}{C_P(T_{4t}-T_{3t}) + C_P(T_{7t}-T_{5t})}$$

$$= \frac{T_1\tau_{\lambda,AB} - T_1\tau_{\lambda,AB}\left\{\tau_r(\pi_c)^{\gamma-1/\gamma}\left[1-(\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]\right\}^{-1} - T_1\tau_r + T_1}{\left(T_1\tau_\lambda - T_1\tau_r(\pi_c)^{\gamma-1/\gamma}\right) + \left(T_1\tau_{\lambda,AB} - T_1\left[\tau_\lambda - \tau_r\left((\pi_c)^{\gamma-1/\gamma}-1\right)\right]\right)}$$

$$= \boxed{1 - \frac{1}{(\pi_r\pi_c)^{\gamma-1/\gamma}}}\boxed{\left[\frac{1}{\tau_{\lambda,AB}-\tau_r}\left(\tau_r(\pi_c)^{\gamma-1/\gamma} + \frac{\tau_{\lambda,AB}\tau_\lambda}{\tau_\lambda - \tau_r\left((\pi_c)^{\gamma-1/\gamma}-1\right)}\right)\right]}$$

Turbojet η_{th} New term > 1

- As expected, η_{th} is lower; similar to Diesel {heat addition at lower P (thus lower T) than "baseline" turbojet (or Otto) cycle}

Afterburning turbojet - fuel consumption

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- To compute TSFC we need FAR; now 2 fuel streams, again do energy balance on combustors (now 2 instead of just 1):

$$\dot{m}_f Q_R = \dot{m}_a C_P (T_{4t} - T_{3t}) + \dot{m}_a C_P (T_{7t} - T_{5t})$$

$$(FAR)Q_R = C_P (T_{4t} - T_{3t}) + C_P (T_{7t} - T_{5t})$$

but $T_{4t} = T_1\tau_\lambda,\ T_{3t} = T_1\tau_r(\pi_c)^{\gamma-1/\gamma},\ T_{7t} = T_1\tau_{\lambda,AB},\ T_{3t} = T_1\tau_r(\pi_c)^{\gamma-1/\gamma}$,

$$T_{5t} = T_1\left[\tau_\lambda - \tau_r\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right]$$

$$\Rightarrow \frac{(FAR)Q_R}{C_P T_1} = \tau_{\lambda,AB} - \tau_r$$

$$TSFC = \left(\frac{Thrust}{\dot{m}_a c_1}\right)^{-1} \frac{FAR \cdot Q_R}{C_P T_1} \frac{1}{\gamma - 1} \text{ as with turbojet}$$

$$\Rightarrow TSFC = \frac{1}{ST}\frac{1}{\gamma-1}\left(\tau_{\lambda,AB} - \tau_r\right)$$

ST from page 32

Afterburning turbojet - effect of π_c

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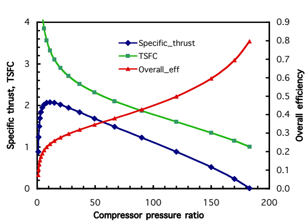
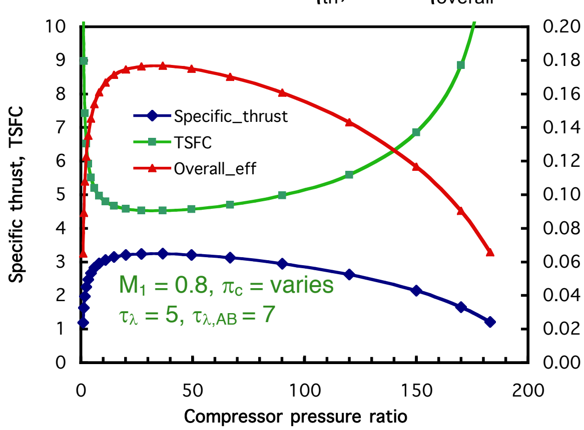
- For parameters used, maximum ST increases 1.5x with afterburner but $\eta_{overall}$ decreases substantially
- For low π_c, η_{th} is low, so both thrust and TSFC are low
- At high π_c, "check mark" cycle - main cycle has high η_{th} but low work, afterburning cycle is banana-like - low η_{th}, thus $\eta_{overall}$ is low



Standard turbojet from page 19



Afterburning turbojet - effect of π_c

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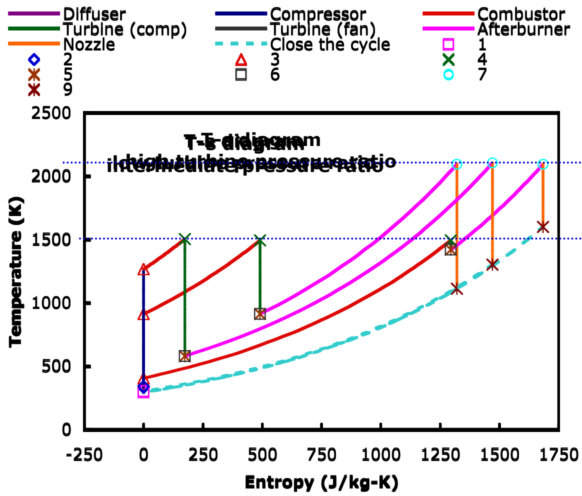
- High π_c: T-s diagrams show "check mark" cycle - tall skinny main T-s diagrams & "banana" shaped afterburner cycles
- Low π_c: cycle is banana-shaped; fattest cycles for intermediate π_c

$M_1 = 0.8$
π_c = varies
$\tau_\lambda = 5$
$\tau_{\lambda,AB} = 7$



Afterburning turbojet - effect of M_1

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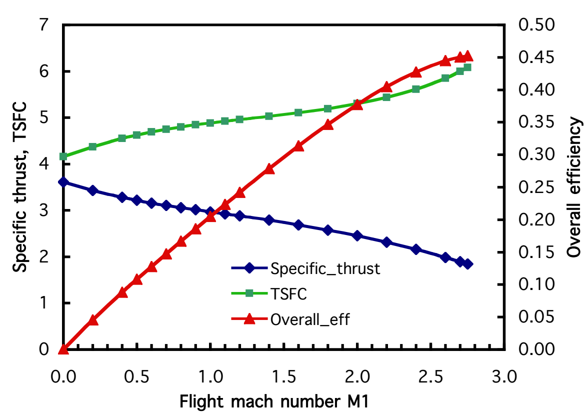
- High M_1: ST decreases since less fuel can be added (τ_λ limit)
- η_{th} increases with M_1 since total pressure ratio = $\pi_r\pi_c$ increases
- $\eta_{prop} = 2(u_1/u_9)/(1+ u_1/u_9)$ increases with M_1 since $u_1/u_9 \rightarrow 1$
- TSFC increases even though $\eta_{overall} = \eta_{th}\eta_{prop}$ increases since PDR's definition of TSFC has M_1 in it – biased against high M_1

M_1 = varies
$\pi_c = 10.97$
$\tau_\lambda = 5$
$\tau_{\lambda,AB} = 7$



Afterburning turbojet cycle - results - effect of M_1

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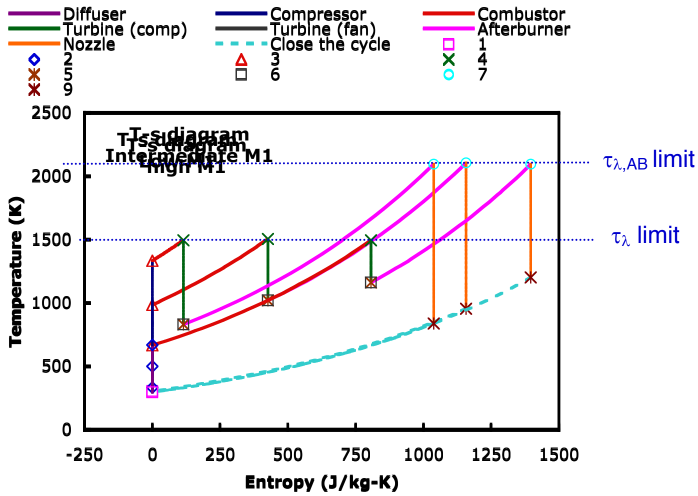
- T-s diagrams similar to π_c effect - "banana" cycles for low π_c, "fat" cycles for intermediate π_c



M_1 = varies
π_c = 10.97
τ_λ = 5
$\tau_{\lambda,AB}$ = 7

Afterburning turbojet - effect of $\tau_{\lambda,AB}$

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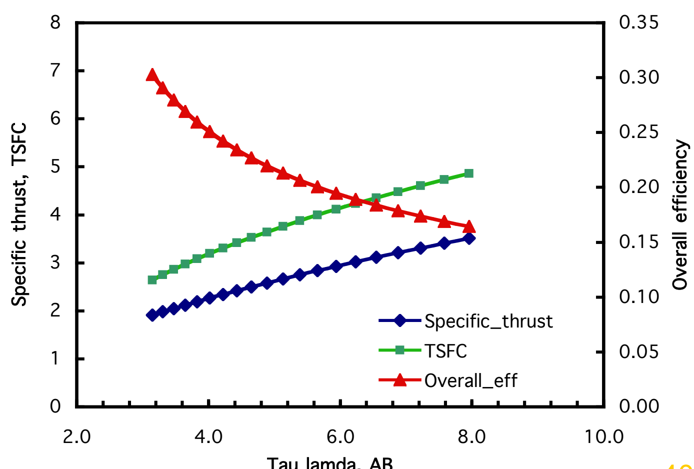
- As $\tau_{\lambda,AB}$ increases, Thrust increases but so does TSFC due to lower $\eta_{prop} = 2(u_1/u_9)/(1+ u_1/u_9)$; u_9 increases for fixed u_1
- At very low $\tau_{\lambda,AB}$, no afterburning heat addition is possible, thus cycle reverts to plain turbojet

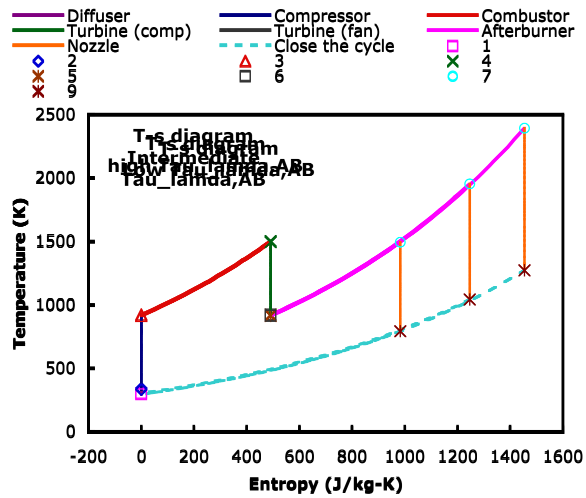
M_1 = 0.8
π_c = 33.03
τ_λ = 5
$\tau_{\lambda,AB}$ = varies



Afterburning turbojet - effect of $\tau_{\lambda,AB}$

- No change in η_{th} of afterburning part as $\tau_{\lambda,AB}$ increases (which can be seen on T-s) but decrease in η_{prop} (which can't be seen on T-s or P-v) due to higher u_9 for fixed u_1

$M_1 = 0.8$
$\pi_c = 33.03$
$\tau_\lambda = 5$
$\tau_{\lambda,AB}$ = varies



Turbofan

- Turbojets and afterburning turbojets 'suffer' from large exhaust velocity u_9 - provides large thrust per unit mass flow (thus large specific thrust, ST) but poor propulsive efficiency, thus poor TSFC and overall efficiency
- Lecture 11: what you really want to do is accelerate ∞ mass of air by $1/\infty$ amount, thus $u_9 = u_1(1 + 1/\infty)$, thus propulsive efficiency $\rightarrow 1$
- How to improve propulsive efficiency? Turbofan
  - Extract more power from turbine than needed to drive compressor
  - Use extra power to drive "fan" that accelerates large mass of air by 1/large amount
  - No free lunch, combusted stream produces less thrust

Turbofan - P-v & T-s diagrams

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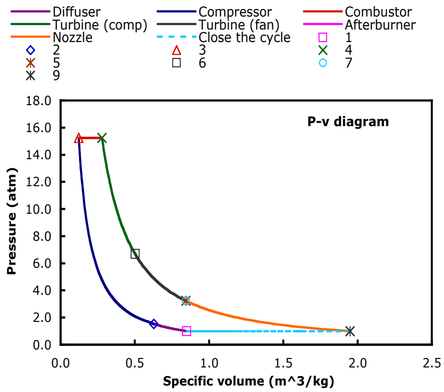
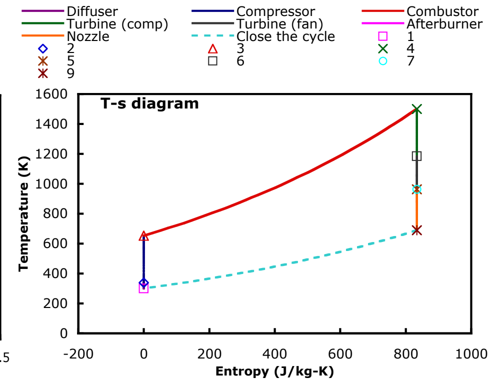
- P-v and T-s diagrams look exactly the same as turbojet since an isentropic expansion through a nozzle or turbine look the same





P-V & T-s diagrams for turbofan

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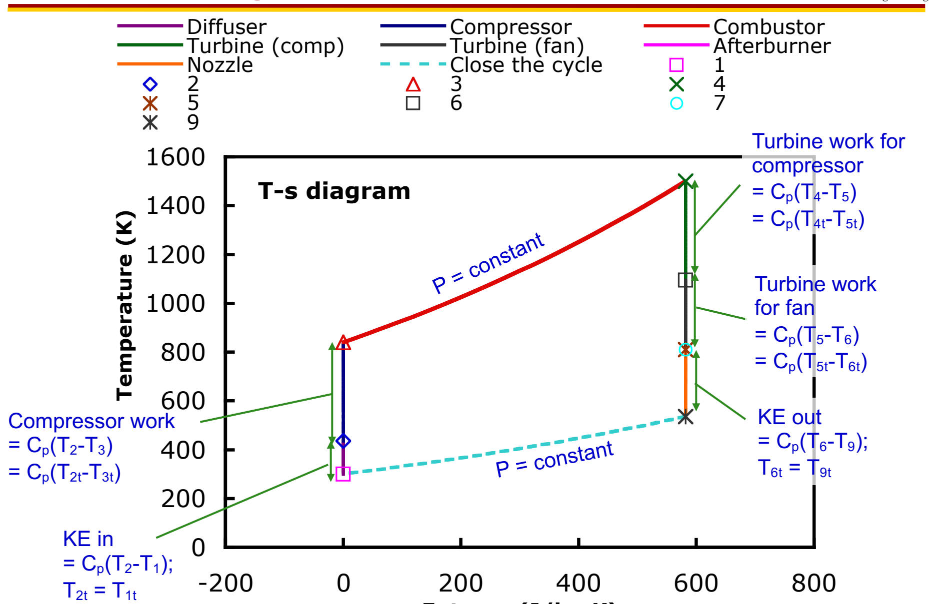

Turbofan - analysis

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- So why doesn't everyone use a fan? (They do now)
- But fan is large, so specific thrust is low if you include ALL air flow
- Analysis of turbofan same as non-afterburning up to state 5 (after turbine for compressor work); from page 9:

$$M_5 = 0;\ q_{4\to5} = 0;\ w_{4\to5} = C_P(T_{4t} - T_{5t}) = -C_P(T_{2t} - T_{3t})$$

$$T_5 = T_{5t} = T_{4t} + (T_{2t} - T_{3t}) = T_1\tau_\lambda + T_1\tau_r - T_1\tau_r(\pi_c)^{\gamma-1/\gamma} = T_1\left[\tau_\lambda - \tau_r\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right]$$

$$P_5 = P_{5t} = P_{4t}(T_{5t}/T_{4t})^{\gamma/\gamma-1} = P_1(\tau_r)^{\gamma/\gamma-1}\pi_c\left[1 - (\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right]^{\gamma/\gamma-1}$$

- Fan: use prime (') superscript for (non-combusting) fan stream:

$$M_2' = 0;\ T_2' = T_1\tau_r;\ T_{2t}' = T_{1t}' = T_1\tau_r;\ P_2' = P_1(\tau_r)^{\gamma/\gamma-1};\ q_{1\to2} = w_{1\to2} = 0$$

$$M_3' = 0;\ T_3' = T_{3t}' = T_{2t}'(\pi_c')^{\gamma-1/\gamma} = T_1\tau_r(\pi_c')^{\gamma-1/\gamma}\ ;$$

$$P_3' = P_{3t}' = P_{2t}'\pi_c' = P_1(\tau_r)^{\gamma/\gamma-1}\pi_c'$$

$$q_{2\to3} = 0;\ w_{2\to3} = C_P(T_{2t}' - T_{3t}')$$

Turbofan - analysis

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- Fan stream continued…

$$q_{3'\to9'} = w_{3'\to9'} = 0;\ P_9' = P_1;\ P_{9t}' = P_{3t}' = P_1(\tau_r)^{\gamma/\gamma-1}\pi_c'$$

$$P_{9t}'/P_9' = P_1(\tau_r)^{\gamma/\gamma-1}\pi_c'/P_1 = \left(1 + \frac{\gamma-1}{2}(M_9')^2\right)^{\gamma/\gamma-1}$$

$$\Rightarrow M_9' = \sqrt{\frac{2}{\gamma-1}\left[\tau_r(\pi_c')^{\gamma-1/\gamma} - 1\right]}$$

$$T_9' = T_3'(P_9'/P_3')^{\gamma-1/\gamma} = T_1\tau_r(\pi_c')^{\gamma-1/\gamma}\left(P_1 \big/ P_1(\tau_r)^{\gamma/\gamma-1}\pi_c'\right)^{\gamma-1/\gamma}$$

$$T_9' = T_1\tau_r(\pi_c')^{\gamma-1/\gamma}\left(P_1 \big/ P_1(\tau_r)^{\gamma/\gamma-1}\pi_c'\right)^{\gamma-1/\gamma} = T_1$$

Note fan stream exit temperature T_9' is same as ambient temperature T_1 - gas undergoes reversible adiabatic compression then expansion back to initial pressure (but fan work input comes out as KE of fan stream)

Turbofan - analysis

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- Need to complete energy balance on the main stream (fan work = fan turbine work) to determine properties after fan turbine 5→ 6
- Define Bypass Ratio (α) = ratio of mass flow through fan to mass flow through compressor $(\alpha \equiv \dot{m}_a'/\dot{m}_a)$

$$M_6 = 0;\ q_{5\to 6} = 0;\ w_{5\to 6} = \dot{m}_a C_P (T_{5t} - T_{6t}) = -\dot{m}_a' C_P (T_{2t}' - T_{3t}')$$

$$\Rightarrow T_{6t} = T_{5t} - \alpha(T_{3t}' - T_{2t}') = T_1\left[\tau_\lambda - \tau_r\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right] - \alpha(T_{3t}' - T_{2t}')$$

$$T_{6t} = T_6 = T_1\left[\tau_\lambda - \tau_r\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right] - \alpha\left[T_1\tau_r(\pi_c')^{\gamma-1/\gamma} - T_1\tau_r\right]$$

$$P_{6t} = P_6 = P_{5t}(T_{6t}/T_{5t})^{\gamma/\gamma-1}$$

$$= P_1(\tau_r)^{\gamma/\gamma-1}\pi_c\left[1 - (\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right]^{\gamma/\gamma-1}\left\{\frac{T_1\left[\tau_\lambda - \tau_r\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right] - \alpha\left[T_1\tau_r(\pi_c')^{\gamma-1/\gamma} - T_1\tau_r\right]}{T_1\left[\tau_\lambda - \tau_r\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right]}\right\}^{\gamma/\gamma-1}$$

$$= P_1(\tau_r)^{\gamma/\gamma-1}\pi_c\left\{\left[1 - (\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right] - \alpha\left[(\tau_r/\tau_\lambda)\left((\pi_c')^{\gamma-1/\gamma} - 1\right)\right]\right\}^{\gamma/\gamma-1}$$

Turbofan - analysis

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- Expand isentropically back to $P_9 = P_1$

$$P_{9t} = P_{6t} = P_1(\tau_r)^{\gamma/\gamma-1}\pi_c\left\{\left[1 - (\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right] - \alpha(\tau_r/\tau_\lambda)\left((\pi_c')^{\gamma-1/\gamma} - 1\right)\right\}^{\gamma/\gamma-1}$$

$$P_9 = P_1; P_{9t}/P_9 = \left(1 + \frac{\gamma-1}{2}M_9^2\right)^{\gamma/\gamma-1}$$

$$\Rightarrow M_9 = \sqrt{\frac{2}{\gamma-1}\left(\tau_r\pi_c^{\gamma-1/\gamma}\left\{\left[1 - (\tau_r/\tau_\lambda)\left((\pi_c)^{\gamma-1/\gamma} - 1\right)\right] - \alpha\left[(\tau_r/\tau_\lambda)\left((\pi_c')^{\gamma-1/\gamma} - 1\right)\right]\right\} - 1\right)}$$

- T_9 is the same with or without fan since in either case you're expanding isentropically back to $P_9 = P_1$ (but of course M_9 and thus T_{9t} are different; exit velocity u_9 is less with fan due to the energy extracted from the main stream to pay for the fan work)

$$T_9 = T_1\tau_\lambda / \tau_r(\pi_c)^{\gamma-1/\gamma}$$

Turbofan - analysis - Thrust

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- Then finally compute the Thrust (assuming $P_9 = P_1$, FAR << 1)

$$Thrust = \dot{m}_a\left[\left(1+FAR\right)u_9 - u_1\right] + \left(P_9 - P_1\right)A_9 + \dot{m}_a'\left[u_9' - u_1\right]$$

$$\approx \dot{m}_a\left[u_9 - u_1\right] + \dot{m}_a'\left[u_9' - u_1\right]$$

$$ST \equiv \frac{Thrust}{(\dot{m}_a + \dot{m}_a')c_1} = \frac{\dot{m}_a\left[u_9 - u_1\right]}{(\dot{m}_a + \dot{m}_a')c_1} + \frac{\dot{m}_a'\left[u_9 - u_1\right]}{(\dot{m}_a + \dot{m}_a')c_1}$$

$$= \frac{1}{1+\alpha}\left\{\left[M_9\sqrt{\frac{T_9}{T_1}} - M_1\right] + \alpha\left[M_9'\sqrt{\frac{T_9'}{T_1}} - M_1\right]\right\}$$

$$= \frac{1}{1+\alpha}\left\{\left[M_9\sqrt{\frac{T_9}{T_1}} - M_1\right] + \alpha\left[\sqrt{\frac{2}{\gamma-1}\left(\tau_r\left(\pi_c'\right)^{\gamma-1/\gamma} - 1\right)}\sqrt{\frac{T_1}{T_1}} - M_1\right]\right\}$$

$$ST = \frac{1}{1+\alpha}\left\{M_9\sqrt{\frac{\tau_\lambda}{\tau_r\left(\pi_c\right)^{\gamma-1/\gamma}}} + \alpha\sqrt{\frac{2}{\gamma-1}\left(\tau_r\left(\pi_c'\right)^{\gamma-1/\gamma} - 1\right)}\right\} - M_1$$

where M_9 (for main stream) is given on the previous page

Turbofan - analysis - TSFC

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- Now 3 parameters: α, π_c, π_c'; not just π_c as with the turbojet
- TSFC calculation is almost same as with turbojet, since same amount of fuel is used, but thrust is different of course:

$$TSFC = \frac{\dot{m}_f Q_R}{Thrust \cdot c_1} = \frac{\dot{m}_a}{\dot{m}_a + \dot{m}_a'}\frac{(\dot{m}_a + \dot{m}_a')c_1 \cdot FAR \cdot Q_R}{Thrust \cdot c_1^2}$$

$$= \frac{1}{1+\alpha}\left(\frac{Thrust}{(\dot{m}_a + \dot{m}_a')c_1}\right)^{-1}\frac{C_P(T_{4t} - T_{3t})}{\gamma R T_1} = \frac{1}{1+\alpha}\frac{1}{\gamma-1}\frac{1}{ST}\left(\tau_\lambda - \tau_r\left(\pi_c\right)^{\gamma-1/\gamma}\right)$$

- "It can be shown" (with A LOT of algebra) that a value of α minimizes TSFC or maximumizes $\eta_{overall} = M_1/TSFC$

$$\frac{\partial\left(\eta_{overall}\right)}{\partial(\alpha)} = 0 \Rightarrow 2\left(u_9 - u_1\right) = \left(u_9' - u_1\right)$$

At this optimum, thrust per unit mass of the main stream ($u_9 - u_1$) is half that of the fan stream ($u_9' - u_1$), i.e., the main stream is bled almost completely out of KE to feed the greedy fan (by drawing more power out of the turbine)

Turbofan - analysis – TSFC & η_{th}

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- The optimal value of α (at $\partial(\eta_{overall})/\partial(\alpha) = 0$) is

$$\alpha_{optimal} = \frac{1}{\left(\pi_c'\right)^{\gamma-1/\gamma} - 1}\left\{\frac{\tau_\lambda}{\tau_r}\left[1 - \frac{1}{\tau_r\left(\pi_c\right)^{\gamma-1/\gamma}} - \frac{1}{4\tau_\lambda}\left(\sqrt{\tau_r\left(\pi_c'\right)^{\gamma-1/\gamma} - 1} + \sqrt{\tau_r - 1}\right)^2\right] - \left(\left(\pi_c\right)^{\gamma-1/\gamma} - 1\right)\right\}$$

- Thermal efficiency η_{th} of turbofan is exactly same as turbojet - same T-s cycle, just different "stopover" points 5 & 6 along the T-s path - the advantage of the turbofan is the propulsive efficiency η_{prop} (which you can't see on T-s diagram), not the thermal efficiency (which does show on T-s)

Turbofan - effect of fan bypass ratio α

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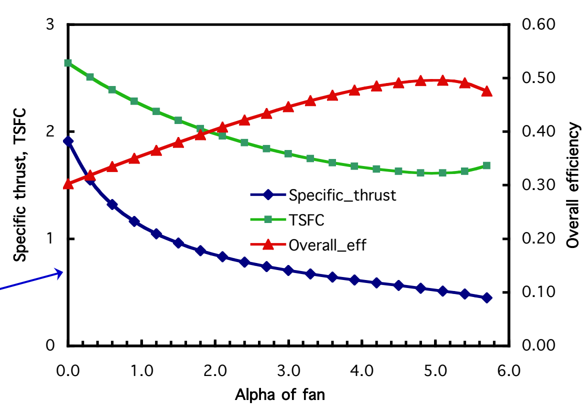
- Specific thrust decreases considerably as α increases (more mass of air being accelerated by a small amount)
- TSFC decreases, η_o increases as α increases, but little benefit beyond a point (η_{prop} already good; recall $\eta_o = \eta_{th}\eta_{prop}$; fan provides NO BENEFIT to η_{th}
- As promised, optimum α (≈ 5 in this case); improves η_o ≈ 50%

$M_1 = 0.8$
$\pi_c = 30$
$\pi_{c'} = 2$
$\tau_\lambda = 5$
α = varies

α = 0 is standard turbojet without fan (see p. 19)



Turbofan - effect of fan bypass ratio α

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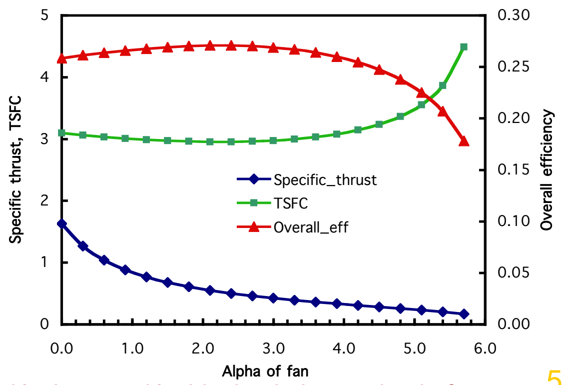
- There is a limit to the value of the fan – for large α, fan area is large, increases the drag (D) = $C_D \rho_1 u_1^2 A_1/2$, (C_D = drag coefficient; A_1 = area of inlet (main + fan streams))
- D must be subtracted from Thrust

$$Drag = C_D \rho_1 u_1^2 A_1 / 2 = C_D (\rho_1 u_1 A_1) u_1 / 2 = C_D (1+\alpha)\dot{m}_a u_1 / 2 = C_D (1+\alpha)\dot{m}_a c_1 M_1 / 2$$

- Optimal $\alpha \approx 2$ with large drag (C_D = 0.7) compared to optimal $\alpha \approx$ 5 with no drag

$M_1 = 0.8$
$\pi_c = 30$
$\pi_{c'} = 2$
$\tau_\lambda = 5$
α = varies
$C_D = 0.7$



Turbofan - effect of fan pressure ratio π_c'

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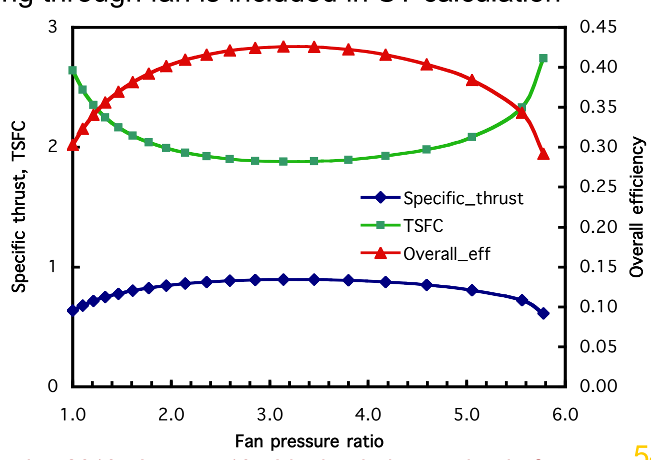
- Little effect of fan pressure ratio π_c' except at low ratios, where we revert to turbojet, and high ratios where we bleed too much energy from main stream (recall optimum is $u_9' - u_1 = 2(u_9 - u_1)$)
- Specific thrust small because α fixed, so even at $\pi_c' = 1$ (no fan), mass of air going through fan is included in ST calculation

$M_1 = 0.8$
$\pi_c = 30$
$\pi_{c'}$ = varies
$\tau_\lambda = 5$
$\alpha = 2$
$C_D = 0$



Turbofan - effect of compressor π_c

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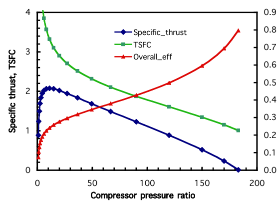
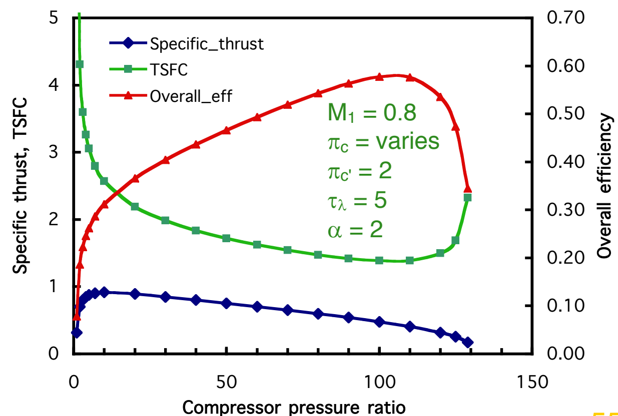
- As with turbojet, for very low compressor pressure ratio π_c, thermal efficiency is low, so both thrust and η_o are low
- At unrealistically high π_c, very little fuel can be added; after paying for compressor & fan work and expanding main stream, exit velocity $u_9 < u_1$ - negative thrust from main stream!



Standard turbojet ($\alpha = 0$) from page 19



Turbofan - results - effect of π_c

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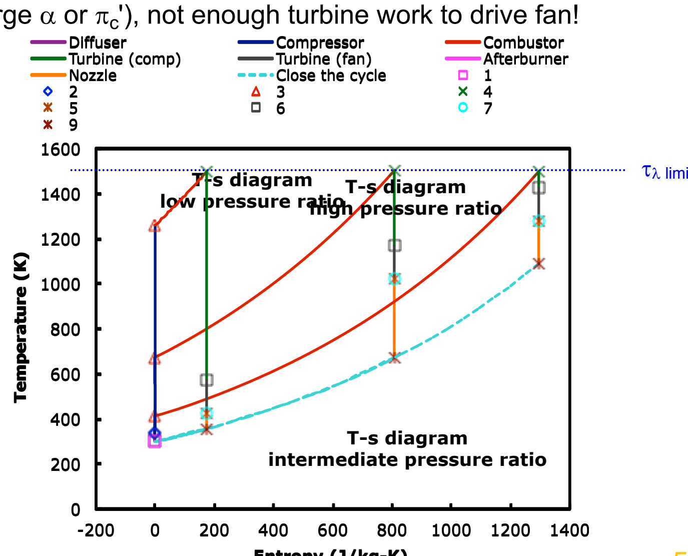
- T-s diagrams same as turbojet except for tradeoff between KE of main stream and work going into fan
- If low π_c (or large α or $\pi_{c'}$), not enough turbine work to drive fan!

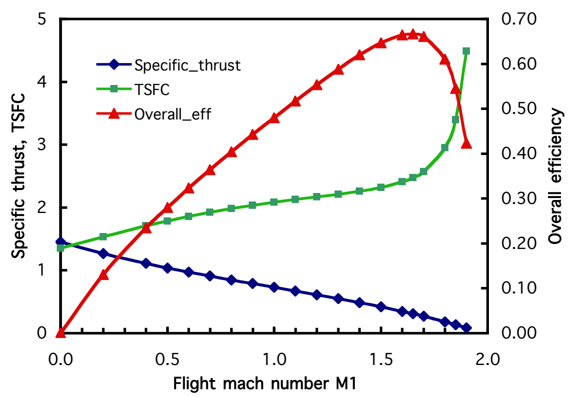
$M_1 = 0.8$
π_c = varies
$\pi_{c'} = 2$
$\tau_\lambda = 5$
$\alpha = 2$



Turbofan - results - effect of M_1

- For high M_1, ST decreases since less fuel can be added (τ_λ limit)
- η_{th} increases with M_1 since total pressure ratio = $\pi_r\pi_c$ increases
- $\eta_{prop} = 2(u_1/u_9)/(1+ u_1/u_9)$ increases with M_1 since $u_1/u_9 \to 1$
- At very high M_1, high η_{prop} anyway, no benefit of fan
- At very high M_1, again after paying for compressor & fan work and expanding main stream, $u_9 < u_1$ - negative thrust from main stream!

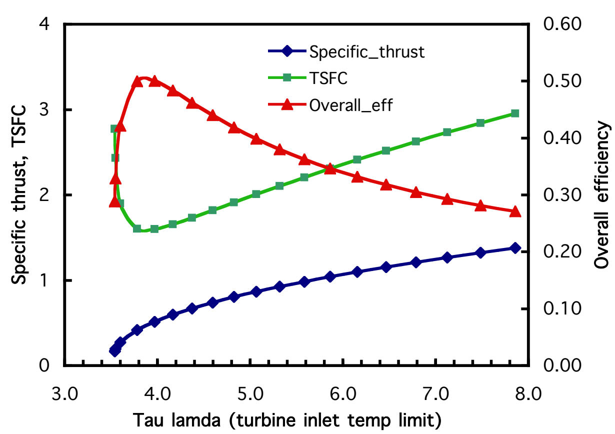
M_1 = varies
$\pi_c = 30$
$\pi_{c'} = 2$
$\tau_\lambda = 5$
$\alpha = 2$



Turbofan - results - effect of τ_λ

- As τ_λ increases, ST increases but η_o decreases due to lower η_{prop}
- For high τ_λ, a better approach would be to extract more turbine work to drive fan and thus get higher η_{prop} - use higher α and/or π_c'
- At very low τ_λ, no heat addition is possible, thus no thrust

$M_1 = 0.8$
$\pi_c = 30$
$\pi_{c'} = 2$
τ_λ = varies
$\alpha = 2$



Example - numerical

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For an ideal turbofan with bypass ratio (α) = 8, γ = 1.35, compressor pressure ratio (π_c) = 30, fan pressure ratio (π_c') = 1.8, flight Mach number (M_1) = 0.8, turbine inlet temperature = 1800K, ambient pressure (P_1) = 0.25 atm and ambient temperature (T_1) = 225 K, with $P_9 = P_1$ and FAR << 1 determine:

(Note that most of these answers could be obtained from the formulas given earlier in this lecture without re-derivations, but this is needed for consistency with the example in the next lecture where non-ideal effects are considered and the simple formulas cannot be used.)

(a) T, P and M after the diffuser (station 2)

$$M_2 = 0 \text{ by assumption; } T_2 = T_{2t} = T_{1t} = T_1\left(1+\frac{\gamma-1}{2}M_1^2\right) = (225)\left(1+\frac{1.35-1}{2}0.8^2\right) = 250.2K$$

$$P_2 = P_{2t} = P_{1t} = P_1\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1} = (0.25)\left(1+\frac{1.35-1}{2}0.8\right)^{1.35/1.35-1} = 0.377\ atm$$

(b) T, P and M after the compressor (station 3)

$$M_3 = 0 \text{ by assumption; } \pi_c = 30 = P_{3t}/P_{2t} \Rightarrow P_{3t} = P_3 = 30\times 0.377atm = 11.3atm$$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\gamma-1/\gamma} \Rightarrow T_3 = T_{3t} = 250.2K(30)^{0.35/1.35} = 604.3K$$

(c) T, P and M after the combustor (station 4)

$M_4 = 0$ by assumption; $P_{4t} = P_4 = P_3 = 11.3$ atm; $T_{4t} = T_4 = 1800$K by assumption

Example - numerical (continued)

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(d) T, P and M after the compressor turbine (station 5)

To balance turbine work with compressor work:

$$C_p(T_{3t}-T_{2t}) = C_p(T_{4t}-T_{5t}) \Rightarrow 604.3K - 250.2K = 1800K - T_{5t} \Rightarrow T_{5t} = 1445.9\ K$$

$$\left(\frac{P_5}{P_4}\right) = \left(\frac{T_5}{T_4}\right)^{\gamma/\gamma-1} \Rightarrow P_5 = P_{5t} = 11.3atm\left(\frac{1445.9K}{1800K}\right)^{1.35/0.35} = 4.85atm;\ M_5 = 0 \text{ by assumption}$$

(e) T, P and M after the fan turbine (station 6)

For the fan stream, $T_2' = T_2 = 250.2$K and $P_2' = P_2 = 0.377$ atm.
Then we need to compute the fan stream work and equate it to the turbine work:

$$T_3' = T_{3t}' = T_{2t}'\left(\pi_c'\right)^{\gamma-1/\gamma} = 250.2K(1.8)^{1.35-1/1.35} = 291.4\ K; P_3' = P_{3t}' = P_{2t}'\pi_c' = 0.377atm\times 1.8 = 0.678atm$$

$$W_{2-3,fan} = \dot{m}_a' C_p\left(T_{2t}' - T_{3t}'\right) = -W_{5-6,turbine} = -\dot{m}_a C_p\left(T_{5t} - T_{6t}\right) \Rightarrow T_{6t} = T_{5t} + \left(\dot{m}_a'/\dot{m}_a\right)\left(T_{2t}' - T_{3t}'\right)$$

$$\Rightarrow T_{6t} = T_6 = T_{5t} + \alpha\left(T_{2t}' - T_{3t}'\right) = 1445.9K + 8(250.2K - 291.4K) = 1116.3K$$

$$\left(\frac{P_6}{P_5}\right) = \left(\frac{T_6}{T_5}\right)^{\gamma/\gamma-1} \Rightarrow P_6 = 4.85atm\left(\frac{1116.3K}{1445.9K}\right)^{1.35/0.35} = 1.79\ atm$$

Example - numerical (continued)

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(e) T, P and M after the nozzle (station 9, and 9' for the fan stream) (note that for the fan stream, nothing happens between stations 3 and 6)

$$P_9 = 0.25atm \text{ by assumption; } \frac{P_{9t}}{P_9} = \left(1+\frac{\gamma-1}{2}M_9^2\right)^{\gamma/\gamma-1} \Rightarrow \left(\frac{1.789atm}{0.25atm}\right)^{1.35-1/1.35} = \left(1+\frac{1.35-1}{2}M_9^2\right) \Rightarrow M_9 = 1.95$$

$$T_{9t} = T_{6t} = T_9\left(1+\frac{\gamma-1}{2}M_9^2\right) \Rightarrow T_9 = T_{9t}\Big/\left(1+\frac{\gamma-1}{2}M_9^2\right) = 1116.3K\Big/\left(1+\frac{1.35-1}{2}1.95^2\right) = 670.2K$$

$$P_9' = 0.25atm \text{ by assumption; } \frac{P_{9t}'}{P_9'} = \left(1+\frac{\gamma-1}{2}M_9^2\right)^{\gamma/\gamma-1} \Rightarrow \left(\frac{0.678atm}{0.25atm}\right)^{1.35-1/1.35} = \left(1+\frac{1.35-1}{2}M'^2_9\right) \Rightarrow M_9' = 1.30$$

$$T_{9t}' = T_{6t}' = T_9'\left(1+\frac{\gamma-1}{2}M'^2_9\right) \Rightarrow T_9' = T_{9t}'\Big/\left(1+\frac{\gamma-1}{2}M'^2_9\right) = 291.4K\Big/\left(1+\frac{1.35-1}{2}1.30^2\right) = 225K$$

(f) Specific thrust (ST) (recall FAR << 1 and $P_9 = P_1$ by assumption)

$$Thrust \approx \dot{m}_a\left[u_9 - u_1\right] + \dot{m}_a'\left[u_9' - u_1\right]; ST = \frac{Thrust}{(\dot{m}_a + \dot{m}_a')c_1} = \frac{\dot{m}_a\left[u_9 - u_1\right]}{(\dot{m}_a + \dot{m}_a')c_1} + \frac{\dot{m}_a'\left[u_9 - u_1\right]}{(\dot{m}_a + \dot{m}_a')c_1}$$

$$ST = \frac{1}{1+\alpha}\left\{\left[M_9\sqrt{T_9/T_1} - M_1\right] + \alpha\left[M_9'\sqrt{T_9'/T_1} - M_1\right]\right\}$$

$$ST = \frac{1}{1+8}\left\{\left[1.95\sqrt{670.2/225} - 0.8\right] + 8\left[1.30\sqrt{225K/225K} - 0.8\right]\right\} = 0.728$$

Example - numerical (continued)

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(g) TSFC and overall efficiency

Recalling that only $1/(1+\alpha)$ of the total air flow is burned,

$$TSFC \equiv \frac{\dot{m}_f Q_R}{Thrust \cdot c_1} = \frac{\dot{m}_a C_P (T_{4t} - T_{3t})}{Thrust \cdot c_1} = \frac{(\dot{m}_a + \dot{m}_a')c_1}{Thrust}\frac{\dot{m}_a}{\dot{m}_a + \dot{m}_a'}\frac{C_P}{c_1^2}(T_{4t} - T_{3t}) = \frac{1}{ST}\frac{1}{1+\alpha}\frac{\frac{\gamma}{\gamma-1}R}{\gamma R T_1}(T_{4t} - T_{3t})$$

$$TSFC = \frac{1}{ST}\frac{1}{1+\alpha}\frac{T_{4t} - T_{3t}}{(\gamma-1)T_1} = \frac{1}{0.728}\frac{1}{1+8}\frac{1800K - 604.3K}{(1.35-1)225K} = 2.32; \quad \eta_o = \frac{M_1}{TSFC} = \frac{0.8}{2.32} = 0.345$$

(h) Propulsive efficiency

$$\eta_p \equiv \frac{\text{Thrust power}}{\Delta(\text{Kinetic energy})} = \frac{Thrust \cdot u_1}{\dot{m}_a\left(u_9^2 - u_1^2\right)/2 + \dot{m}_a'\left(u_9^2{}' - u_1^2{}'\right)/2} = \frac{Thrust}{(\dot{m}_a + \dot{m}_a')c_1} \cdot \frac{2(\dot{m}_a + \dot{m}_a')c_1 u_1}{\dot{m}_a\left(u_9^2 - u_1^2\right) + \dot{m}_a'\left(u_9'^2 - u_1'^2\right)}$$

$$\eta_p = ST\frac{2(1+\alpha)c_1^2 M_1}{\left(M_9^2 c_9^2 - M_1^2 c_1^2\right) + \alpha\left(M_9'^2 c_9'^2 - M_1'^2 c_1'^2\right)} = ST\frac{2(1+\alpha)(\gamma R T_1)M_1}{\left(M_9^2(\gamma R T_9) - M_1^2(\gamma R T_1)\right) + \alpha\left(M_e'^2(\gamma R T_9') - M_1^2(\gamma R T_1)\right)}$$

$$= ST\frac{2(1+\alpha)M_1 T_1}{\left(M_9^2 T_9 - M_1^2 T_1\right) + \alpha\left(M_9'^2 T_9' - M_1^2 T_1\right)} = 0.728\frac{(2)(1+8)(0.8)(225)}{\left(1.95^2 670.2K - 0.8^2 225K\right) + 8\left(1.30^2 225K - 0.8^2 225K\right)}$$

$$= 0.549$$

Example - numerical (continued)

(i) Thermal efficiency

$$\eta_{th} \equiv \frac{\Delta(\text{Kinetic energy})}{\dot{m}_f Q_R} = \frac{\dot{m}_a\left(u_9^2 - u_1^2\right)/2 + \dot{m}_a'\left(u_9^{2}{}' - u_1^{2}{}'\right)/2}{\dot{m}_a C_P (T_{4t} - T_{3t})} = \frac{1}{2}\frac{\left(u_9^2 - u_1^2\right) + \alpha\left(u_9^{2}{}' - u_1^{2}{}'\right)}{C_P (T_{4t} - T_{3t})}$$

$$\eta_{th} = \frac{1}{2}\frac{\left(M_9^2 c_9^2 - M_1^2 c_1^2\right) + \alpha\left(M'^2_9 c_9^2 - M'^2_1 c_1^2\right)}{C_P (T_{4t} - T_{3t})} = \frac{1}{2}\frac{\left(M_9^2 \gamma R T_9 - M_1^2 \gamma R T_1\right) + \alpha\left(M'^2_9 \gamma R T_9' - M'^2_1 \gamma R T_1\right)}{\frac{\gamma}{\gamma - 1} R (T_{4t} - T_{3t})}$$

$$\eta_{th} = \frac{\gamma - 1}{2}\frac{\left(M_9^2 T_9 - M_1^2 T_1\right) + \alpha\left(M'^2_9 T_9' - M'^2_1 T_1\right)}{T_{4t} - T_{3t}}$$

$$\eta_{th} = \frac{1.35 - 1}{2}\frac{\left(1.95^2 670.2K - 0.8^2 225K\right) + 8\left(1.30^2 225K - 0.8^2 225K\right)}{1800K - 604.3K} = 0.629$$

Note that $\eta_{th}\eta_p = (0.629)(0.549) = 0.345 = \eta_o$ as advertised

(j) Specific impulse

$$I_{SP} = \frac{1}{TSFC}\frac{Q_R}{g_{earth} c_1} = \frac{1}{2.32}\frac{4.3\times 10^7 J/kg}{(9.81 m/\sec^2)\sqrt{1.35(8.314 J/moleK)(mole/0.02897kg)(225K)}} = 6400\sec$$

Example - graphical

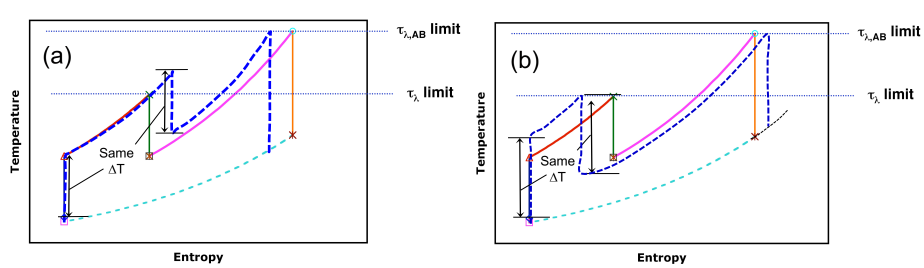
In an ideal τ_λ-limited afterburning turbojet, how would the T-s diagrams be affected if

(a) A new turbine with higher τ_λ is used but $\tau_{\lambda,AB}$ is unchanged

The cycles are the same up to the end of compression process. The temperature after combustion increases. The compressor work and thus turbine work (thus ΔT) does not change. Since ΔT across the turbine does not change but the temperature at the start of the turbine expansion is higher than the base cycle, the pressure after expansion will be higher. Then more heat is added up to the afterburner limit.

(b) A new compressor is used with a higher pressure ratio

The compression process ends at a higher pressure and but the same entropy than the base cycle. Heat addition then occurs along this constant-pressure curve up to the turbine temperature limit, ending at lower entropy. More work, thus more ΔT, is required from the turbine, so the afterburning heat addition occurs along a lower constant pressure curve.



Example - graphical

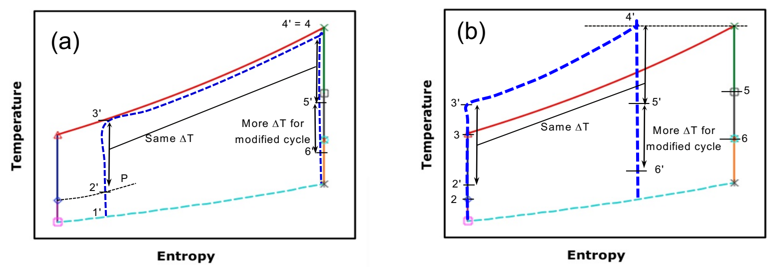
In an ideal turbofan, how would the T-s diagrams be affected if

(a) Ambient temperature is increased (ambient pressure not changed).

T_1 increases and the starting point (1) moves up along the constant-pressure curve. Since the pressure ratios $P_2/P_1 = (T_2/T_1)^{\gamma/\gamma-}$ and $P_3/P_2 = (T_3/T_2)^{\gamma/\gamma-1}$ are the same, T_2 and T_3 increase but P_2 and P_3 do not. The turbine inlet temperature limit, T_4 does not change so state 4' is the same as 4. The increased compressor work must be paid for in the turbine. The fan work increases for the same reason as the compressor work assuming that the fan pressure ratio didn't change.

(b) The flight Mach number is increased

T_2 increases and since $P_3/P_2 = (T_3/T_2)^{\gamma/\gamma-1}$ is the same, T_3 increases. Then with the same turbine inlet temperature limit, T_4 does not change. The increased compressor work must be paid for in the turbine. The fan work increases for the same reason.



Summary

- Turbojet
  - Ideal cycle - compress isentropically, burn isobarically (constant P), expand isentropically back to ambient pressure
  - Matching conditions: compressor work = turbine work & $P_1 = P_9$ (i.e. $P_{exit} = P_{ambient}$)
  - Performance is determined by compressor pressure ratio π_c, turbine inlet temperature limit (τ_λ) and flight Mach number (M_1)
    - » Low π_c: thermal efficiency low, thrust low
    - » High π_c: thermal efficiency high but not much fuel can be added before τ_λ limit reached so again thrust low (also propulsive efficiency low because of high exit M)
    - » Intermediate π_c: thermal efficiency intermediate, thrust highest
    - » Increasing τ_λ always increases thrust but propulsive efficiency suffers
    - » Increasing M_1 increases thermal efficiency (more π_r compression) and propulsive efficiency (higher u_1)

Summary - continued

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- Afterburner
  - Obtain more thrust by adding additional fuel after the turbine
  - This is possible since τ_λ is too low for stoichiometric combustion, thus exhaust of standard turbojet has unburned O_2
  - Additional parameter $\tau_{\lambda,AB}$
  - Since there are no moving parts in the afterburner temperature limitations are not as severe, thus $\tau_{\lambda,AB} > \tau_\lambda$ and substantial additional fuel can be added
  - T-s Carnot cycle strips are shorter for afterburning part of cycle, thus thermal efficiency is greatly reduced
- Turbofan
  - Used to increase propulsive efficiency by accelerating large mass of air by a smaller Δu (no effect on thermal efficiency)
  - Two streams - main (combusted) and fan
  - Additional parameters α, π_c'
  - Fan work is "paid for" with additional turbine stages - lower u_9 for main stream
  - Combined thrust maximized when most of thrust is obtained from fan stream