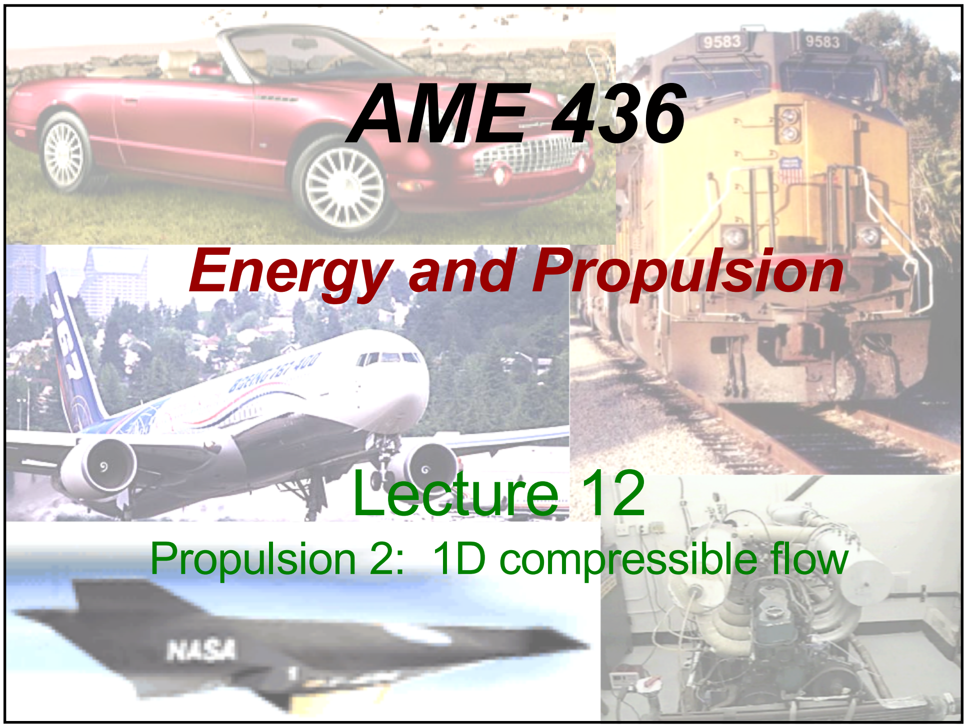

AME 436

Energy and Propulsion

Lecture 12

Propulsion 2: 1D compressible flow

Outline

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- Governing equations
- Analysis of 1D flows
  - Isentropic, variable area
  - Shock
  - Constant area with friction (Fanno flow)
  - Heat addition
    - » Constant area (Rayleigh)
    - » Constant P
    - » Constant T

1D steady flow of ideal gases

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- Assumptions
  - Ideal gas, steady, quasi-1D
  - Constant C_P, C_v, $\gamma \equiv C_P/C_v$
  - Unless otherwise noted: adiabatic, reversible, constant area
  - Note since 2nd Law states $dS \geq \delta Q/T$ (= for reversible, > for irreversible), reversible + adiabatic $\Rightarrow$ isentropic ($dS = 0$)
- Governing equations
  - Equations of state $h_2 - h_1 = C_P(T_2 - T_1)$
    $P = \rho RT$; $S_2 - S_1 = C_p \ln(T_2/T_1) - R\ln(P_2/P_1)$
  - Isentropic ($S_2 = S_1$) (where applicable): $P_2/P_1 = (T_2/T_1)^{\gamma/(\gamma-1)}$
  - Mass conservation: $\dot{m} = \rho_1 u_1 A_1 = \rho_2 u_2 A_2$
  - Momentum conservation, constant area duct (see lecture 11):
    $$AdP + \dot{m}du + C_f(\rho u^2/2)Cdx = 0$$
    - » C_f = friction coefficient; C = circumference of duct
    - » No friction: $P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$
  - Energy conservation: $h_1 + u_1^2/2 + q - w = h_2 + u_2^2/2$
    q = heat input per unit mass = fQ_R if due to combustion
    w = work output per unit mass

1D steady flow of ideal gases

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- Types of analyses: everything constant except…
  - Area (isentropic nozzle flow)
  - Entropy (shock)
  - Momentum (Fanno flow) (constant area with friction)
  - Diabatic ($q \neq 0$) - several possible assumptions
    - » Constant area (Rayleigh flow) (useful if limited by space)
    - » Constant T (useful if limited by materials) (sounds weird, heat addition at constant T…)
    - » Constant P (useful if limited by structure)
    - » Constant M (covered in some texts but really contrived, let's skip it)
- Products of analyses
  - Stagnation temperature (defined later)
  - Stagnation pressure (defined later)
  - Mach number = $u/c = u/(\gamma RT)^{1/2}$ (c = sound speed at local conditions in the flow (NOT at ambient condition!))
  - From this, can get exit velocity u_9, exit pressure P_9 and thus thrust

Isentropic nozzle flow

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- Reversible, adiabatic ⇒ S = constant, A ≠ constant, w = 0
- Energy equation, q = w = 0 (momentum equation not used):

$$h_1 + u_1^2/2 = h_2 + u_2^2/2 \Rightarrow h_2 - h_1 = C_P(T_2 - T_1); M_1 = \frac{u_1}{c_1} = \frac{u_1}{\sqrt{\gamma R T_1}}; M_2 = \frac{u_2}{\sqrt{\gamma R T_2}}$$

$$\Rightarrow C_P T_1 + \frac{M_1^2 \gamma R T_1}{2} = C_P T_2 + \frac{M_2^2 \gamma R T_2}{2} \Rightarrow T_1\left(1 + \frac{\gamma R}{2C_P}M_1^2\right) = T_2\left(1 + \frac{\gamma R}{2C_P}M_2^2\right)$$

$$\frac{R}{C_P} = \frac{\gamma - 1}{\gamma} \Rightarrow T_1\left(1 + \frac{\gamma - 1}{2}M_1^2\right) = T_2\left(1 + \frac{\gamma - 1}{2}M_2^2\right)$$

- Define stagnation temperature T_t = temperature of gas when decelerated adiabatically to M = 0 (doesn't need to be reversible)

$$\Rightarrow T_{1t} = T_1\left(1 + \frac{\gamma - 1}{2}M_1^2\right); T_{2t} = T_2\left(1 + \frac{\gamma - 1}{2}M_2^2\right);$$

- Energy equation is just $T_{1t} = T_{2t}$: sum of thermal energy (the 1 term) and kinetic energy (the $(\gamma-1)M^2/2$ term) =constant

Isentropic nozzle flow

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- Pressure is related to temperature through isentropic (adiabatic + reversible) compression law:

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1} \Rightarrow \frac{P_2}{P_1} = \left(\frac{T_{2t}}{T_{1t}}\right)^{\gamma/\gamma-1}\left(\frac{1 + \frac{\gamma R}{2C_P}M_1^2}{1 + \frac{\gamma R}{2C_P}M_2^2}\right)^{\gamma/\gamma-1}$$

$$\text{but } T_{2t} = T_{1t} \Rightarrow P_2\left(1 + \frac{\gamma R}{2C_P}M_2^2\right)^{\gamma/\gamma-1} = P_1\left(1 + \frac{\gamma R}{2C_P}M_1^2\right)^{\gamma/\gamma-1}$$

- Define stagnation pressure P_t = pressure of gas stream when decelerated adiabatically and reversibly to M = 0

$$\Rightarrow P_{1t} = P_1\left(1 + \frac{\gamma - 1}{2}M_1^2\right)^{\gamma/\gamma-1}; P_{2t} = P_2\left(1 + \frac{\gamma - 1}{2}M_2^2\right)^{\gamma/\gamma-1};$$

- Thus the pressure / Mach number relation is simply $P_{1t} = P_{2t}$ assuming reversible flow

Stagnation temperature and pressure

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- Stagnation temperature T_t - measure of total energy (thermal + kinetic) of the flow

$$T_t = T\left(1 + \frac{\gamma - 1}{2} M^2\right)$$

  - T = static temperature - T measured by thermometer moving with flow
  - T_t = temperature of the gas if it is decelerated adiabatically to M = 0
- Stagnation pressure P_t - measure of usefulness of flow (ability to expand flow)

$$P_t = P\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma-1}$$

  - P = static pressure - P measured by pressure gauge moving with flow
  - P_t = P of gas when decelerated reversibly & adiabatically to M = 0
- These relations are definitions of T_t & P_t at a particular state and can be used even if T_t & P_t change during the process
- These relations assume constant γ & R, i.e. constant C_P and M (molecular mass); what if this assumption is invalid? To be discussed in Lecture 15

Isentropic nozzle flow

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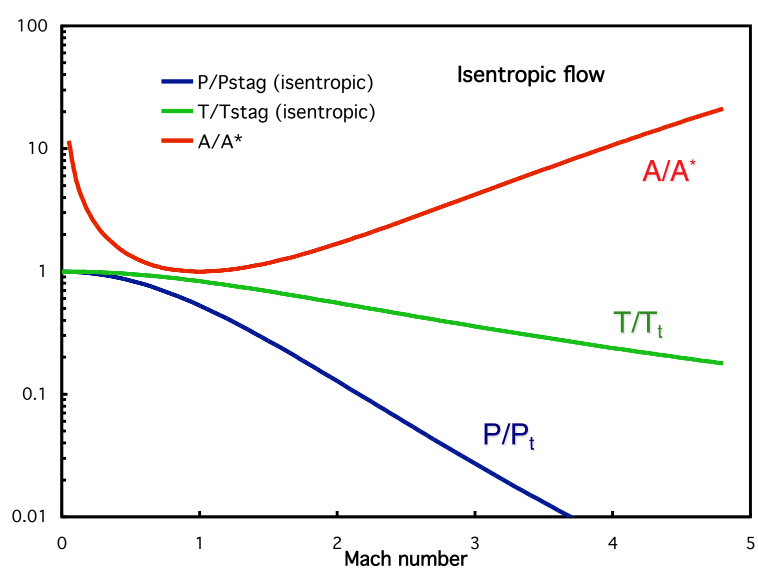

Isentropic nozzle flow

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- Relation of P & T to duct cross-section area A determined through mass conservation

$$\dot{m} = \rho_1 u_1 A_1 = \rho_2 u_2 A_2 \Rightarrow \frac{P_1}{RT_1} u_1 A_1 = \frac{P_2}{RT_2} u_2 A_2 \Rightarrow \frac{P_1}{T_1} M_1 \sqrt{\gamma R T_1} A_1 = \frac{P_2}{T_2} M_2 \sqrt{\gamma R T_2} A_2$$

$$\Rightarrow \frac{P_1}{\sqrt{RT_1}} M_1 A_1 = \frac{P_2}{\sqrt{RT_2}} M_2 A_2$$

$$\Rightarrow \frac{P_{1t} \Big/ \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1}}{\sqrt{RT_{1t}} \Big/ \sqrt{1+\frac{\gamma-1}{2}M_1^2}} M_1 A_1 = \frac{P_{2t} \Big/ \left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1}}{\sqrt{RT_{2t}} \Big/ \sqrt{1+\frac{\gamma-1}{2}M_2^2}} M_2 A_2$$

$$\Rightarrow \frac{P_{1t}}{\sqrt{RT_{1t}}} \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} M_1 A_1 = \frac{P_{2t}}{\sqrt{RT_{2t}}} \left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} M_2 A_2$$

- Really messy but …

Isentropic nozzle flow

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- … for adiabatic reversible flow $T_{1t} = T_{2t}$ and $P_{1t} = P_{2t}$
Also define throat area A^* = area at M = 1 then

$$\left(1+\frac{\gamma-1}{2}(1)^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} (1) A^* = \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} M A \Rightarrow$$

$$\frac{A(M)}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{(\gamma+1)/2(\gamma-1)}$$

- A/A^* shows a minimum at M = 1, thus it is indeed a throat
- How to use A/A^* relations if neither initial state (call it 1) nor final state (call it 2) are at the throat (* condition)?

$$\Rightarrow \frac{A_2}{A_1} = \frac{\frac{A_2}{A^*}}{\frac{A_1}{A^*}} = \frac{M_1}{M_2}\left[\frac{\left(1+\frac{\gamma-1}{2}M_2^2\right)}{\left(1+\frac{\gamma-1}{2}M_1^2\right)}\right]^{(\gamma+1)/2(\gamma-1)}$$

Isentropic nozzle flow

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- Mass flow and velocity can be determined similarly:

$$\frac{\dot{m}}{A} = \rho u = \frac{P}{RT} M\sqrt{\gamma RT} \Rightarrow \frac{\dot{m}}{A} = \frac{P_t}{\sqrt{RT_t}}\sqrt{\gamma} M\left(1+\frac{\gamma-1}{2}M^2\right)^{-(\gamma+1)/2(\gamma-1)}$$

$$\Rightarrow \frac{\dot{m}}{A^*} = \frac{P_t}{\sqrt{RT_t}}\sqrt{\gamma}(1)\left(1+\frac{\gamma-1}{2}(1)^2\right)^{-(\gamma+1)/2(\gamma-1)} \Rightarrow \boxed{\dot{m} = A^*\frac{P_t}{\sqrt{RT_t}}\sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}}$$

$$T_t = T\left(1+\frac{\gamma-1}{2}M^2\right) \Rightarrow M = \frac{u}{\sqrt{\gamma RT}} = \sqrt{\frac{2}{\gamma-1}\left(\frac{T_t}{T}-1\right)}$$

$$\Rightarrow u = \sqrt{\frac{2\gamma}{\gamma-1}RT_t\left(1-\frac{T}{T_t}\right)} \Rightarrow \boxed{u = \sqrt{\frac{2\gamma}{\gamma-1}RT_t\left(1-\left(\frac{P}{P_t}\right)^{\gamma-1/\gamma}\right)}}$$

Isentropic nozzle flow

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- Summary

$$T_t = T\left(1+\frac{\gamma-1}{2}M^2\right) = \text{constant} \qquad P_t = P\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1} = \text{constant}$$

$$\frac{\dot{m}}{A} = \frac{P_t}{\sqrt{RT_t}}\sqrt{\gamma} M\left(1+\frac{\gamma-1}{2}M^2\right)^{(\gamma+1)/2(\gamma-1)} \qquad \dot{m} = A^*\frac{P_t}{\sqrt{RT_t}}\sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{(\gamma+1)/2(\gamma-1)}$$

A^* = area at M = 1

- Recall assumptions: 1D, reversible, adiabatic, ideal gas, const. γ
- Implications
  - P and T decrease monotonically as M increases
  - Area is minimum at M = 1 - need a "throat" to transition from M < 1 to M > 1 or vice versa
  - $\dot{m}/A$ is maximum at M = 1 - flow is "choked" at throat - any change in downstream conditions cannot affect
  - Note for supersonic flow, M (and u) INCREASE as area increases - this is exactly opposite subsonic flow as well as intuition (e.g. garden hose - velocity increases as area decreases)

Isentropic nozzle flow

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- When can choking occur? If M ≥ 1 or

$$\frac{P_t}{P^*} \geq \left(1+\frac{\gamma-1}{2}(1)^2\right)^{\gamma/\gamma-1} = \left(\frac{\gamma+1}{2}\right)^{\gamma/\gamma-1} = 1.89 \text{ for } \gamma = 1.4$$

  so need pressure ratio > 1.89 for choking (if all assumptions satisfied…)
- Where did P_t come from? Mechanical compressor (turbojet) or vehicle speed (high flight Mach number M_1)
- Where did T_t come from? Combustion! (Even if at high M thus high T_t, no thrust unless T_t increased; otherwise just reversible compression & expansion)

Stagnation temperature and pressure

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- Why are T_t and P_t so important? Recall isentropic expansion with stagnation conditions T_t and P_t to exit pressure P_9 yields

$$u = \sqrt{\frac{2\gamma}{\gamma-1}RT_t\left(1-\left(\frac{P_9}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

- For exit pressure P_9 = ambient pressure P_1 and FAR << 1,

$$Thrust = \dot{m}_a(u_9 - u_1) = \dot{m}_a\sqrt{\frac{2\gamma RT_{1t}}{\gamma-1}}\left[\sqrt{\frac{T_{9t}}{T_{1t}}\left(1-\left(\frac{P_9}{P_{9t}}\right)^{\frac{\gamma-1}{\gamma}}\right)} - \sqrt{\left(1-\left(\frac{P_1}{P_{1t}}\right)^{\frac{\gamma-1}{\gamma}}\right)}\right]$$

- Thrust increases as T_t and P_t increase, but everything is inside square root, plus P_t is raised to small exponent - hard to make big improvements with better designs having larger T_t or P_t

Stagnation temperature and pressure

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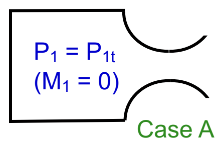
- From previous page

$$Thrust = \dot{m}_a(u_9 - u_1) = \dot{m}_a\sqrt{\frac{2\gamma RT_{1t}}{\gamma-1}}\left[\sqrt{\frac{T_{9t}}{T_{1t}}\left(1-\left(\frac{P_9}{P_{9t}}\right)^{\frac{\gamma-1}{\gamma}}\right)} - \sqrt{\left(1-\left(\frac{P_1}{P_{1t}}\right)^{\frac{\gamma-1}{\gamma}}\right)}\right]$$

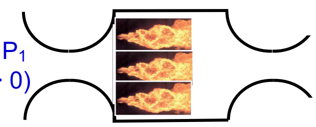
- No thrust if $P_{1t} = P_{9t}, P_9 = P_1$ & $T_{1t} = T_{9t}$; to get thrust we need either

A. $T_{9t} = T_{1t}$, $P_{9t} = P_{1t} = P_1$; $P_9 < P_1$
(e.g. tank of high-P, ambient-T gas)



Case A

B. $T_{9t} > T_{1t}$, $P_{9t} = P_{1t} > P_1 = P_9$
(e.g. high-M ramjet/scramjet, no P_t losses)



Case B

Stagnation temperature and pressure

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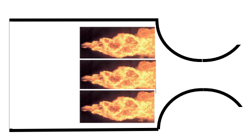
C. $T_{9t} > T_{1t}$, $P_{9t} > P_{1t} = P_1 = P_9$
(e.g. low-M turbojet or fan)
  - Fan: $T_{9t}/T_{1t} = (P_{9t}/P_{1t})^{(\gamma-1)/\gamma}$ due to adiabatic compression
  - Turbojet: $T_{9t}/T_{1t} > (P_{9t}/P_{1t})^{(\gamma-1)/\gamma}$ due to adiabatic compression plus heat addition
  - Could get thrust even with $T_{9t} = T_{1t}$, but how to pay for fan or compressor work without heat addition???



Case C

Stagination temperature and pressure

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- Note also $\Delta(T_t)$ ~ heat or work transfer

$$\text{Energy equation: } q_{1\to 2} - w_{1\to 2} = \left(h_2 + u_2^2/2\right) - \left(h_1 + u_1^2/2\right)$$

$$q_{1\to 2} - w_{1\to 2} = \left(C_P T_2 + (c_2 M_2)^2/2\right) - \left(C_P T_1 + (c_1 M_1)^2/2\right)$$

$$q_{1\to 2} - w_{1\to 2} = \left(C_P T_2 + \gamma R T_2 M_2^2/2\right) - \left(C_P T_1 + \gamma R T_1 M_1^2/2\right), \text{ but } R = C_P\frac{\gamma-1}{\gamma}$$

$$q_{1\to 2} - w_{1\to 2} = C_P T_2\left(1+\frac{\gamma-1}{2}M_2^2\right) - C_P T_1\left(1+\frac{\gamma-1}{2}M_1^2\right) = C_P(T_{2t} - T_{1t})$$

- Recall (lecture 6) for control volume, steady flow

  $q_{1\to 2} - w_{1\to 2} = C_P(T_2 - T_1)$

- Why T not T_t in that case? KE not included in lecture 6 since KE almost always small (more specifically, M << 1) in reciprocating engines!

Constant everything except S (shock)

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- Q: what if A = constant but S ≠ constant? Can anything happen while still satisfying mass, momentum, energy & equation of state?
- A: YES! (shock)
- Energy equation: no heat or work transfer thus

$$T_{1t} = T_1\left(1+\frac{\gamma-1}{2}M_1^2\right) = T_{2t} = T_2\left(1+\frac{\gamma-1}{2}M_2^2\right) = \text{constant}$$

- Mass conservation

$$\dot{m} = \rho u A; \dot{m} = \text{constant}, A = \text{constant} \Rightarrow \rho u = \text{constant} \Rightarrow \rho_1 u_1 = \rho_2 u_2$$

$$\Rightarrow \frac{P_1}{RT_1}M_1\sqrt{\gamma R T_1} = \frac{P_2}{RT_2}M_2\sqrt{\gamma R T_2} \Rightarrow \frac{P_1 M_1}{\sqrt{T_1}} = \frac{P_2 M_2}{\sqrt{T_2}}$$

$$\Rightarrow \frac{P_1 M_1}{\sqrt{T_1}} = \frac{P_2 M_2}{\sqrt{T_1\dfrac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}}} \Rightarrow P_2 = P_1\frac{M_1}{M_2}\sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}}$$

Constant everything except S (shock)

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- Momentum conservation (constant area, dx = 0)

$$P_1+\rho_1 u_1^2=P_2+\rho_2 u_2^2=P_2+\frac{\rho_1 u_1}{u_2}u_2^2=P_2+\rho_1 u_1 u_2$$

$$\Rightarrow P_1+\left(\frac{P_1}{RT_1}\right)\left(M_1^2\gamma RT_1\right)=P_2+\left(\frac{P_1}{RT_1}\right)\left(M_1 M_2\gamma R\sqrt{T_1T_2}\right)=P_2+\left(\frac{P_1M_1}{\sqrt{T_1}}\right)\left(M_2\gamma\sqrt{T_2}\right)$$

$$\Rightarrow P_1\left(1+\gamma M_1^2\right)=P_2+\left(\frac{P_2M_2}{\sqrt{T_2}}\right)\left(M_2\gamma\sqrt{T_2}\right)\Rightarrow P_1\left(1+\gamma M_1^2\right)=P_2\left(1+\gamma M_2^2\right)$$

$$\Rightarrow P_1\left(1+\gamma M_1^2\right)=P_1\frac{M_1}{M_2}\sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}}\left(1+\gamma M_2^2\right)\Rightarrow\frac{\left(1+\gamma M_1^2\right)^2}{M_1^2\left(1+\frac{\gamma-1}{2}M_1^2\right)}=\frac{\left(1+\gamma M_2^2\right)^2}{M_2^2\left(1+\frac{\gamma-1}{2}M_2^2\right)}$$

$$\text{Let }\frac{\left(1+\gamma M_1^2\right)^2}{M_1^2\left(1+\frac{\gamma-1}{2}M_1^2\right)}=C, M_2^2=x\Rightarrow C=\frac{1+2\gamma x+\gamma^2x^2}{x\left(1+\frac{\gamma-1}{2}x\right)}\Rightarrow\left(\gamma^2-\frac{\gamma-1}{2}C\right)x^2+\left(2\gamma-C\right)x+1=0$$

Quadratic equation for x, 2 solutions: $M_2=M_1$ (nothing happens) or $M_2^2=\dfrac{M_1^2+\frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2-1}$

Constant everything except S (shock)

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- Complete results

$$M_2^2=\frac{M_1^2+\frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2-1}\qquad\frac{P_2}{P_1}=\frac{2\gamma}{\gamma+1}M_1^2-\frac{\gamma-1}{\gamma+1}\qquad\frac{T_{2t}}{T_{1t}}=1$$

$$\frac{P_{2t}}{P_{1t}}=\frac{\left[\left(\frac{\gamma+1}{2}M_1^2\right)\Big/\left(1+\frac{\gamma-1}{2}M_1^2\right)\right]^{\gamma/\gamma-1}}{\left(\frac{2\gamma}{\gamma+1}M_1^2-\frac{\gamma-1}{\gamma+1}\right)^{1/\gamma-1}}\qquad\frac{T_2}{T_1}=\frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)\left(\frac{2\gamma}{\gamma-1}M_1^2-1\right)}{\frac{\left(\gamma+1\right)^2}{2\left(\gamma-1\right)}M_1^2}$$

- Implications
  - $M_2 = M_1 = 1$ is a solution (acoustic wave, $P_2 \approx P_1$)
  - If $M_1 > 1$ then $M_2 < 1$ and vice versa - equations don't show a preferred direction
  - Only $M_1 > 1$, $M_2 < 1$ results in $dS > 0$, thus $M_1 < 1$, $M_2 > 1$ is impossible
  - P, T increase across shock which sounds good BUT…
  - T_t constant (no change in total enthalpy) but P_t decreases across shock (a lot if $M_1 >> 1$!);
  - Note there are only 2 states, $(\,)_1$ and $(\,)_2$ - no continuum of states

Constant everything except S (shock)

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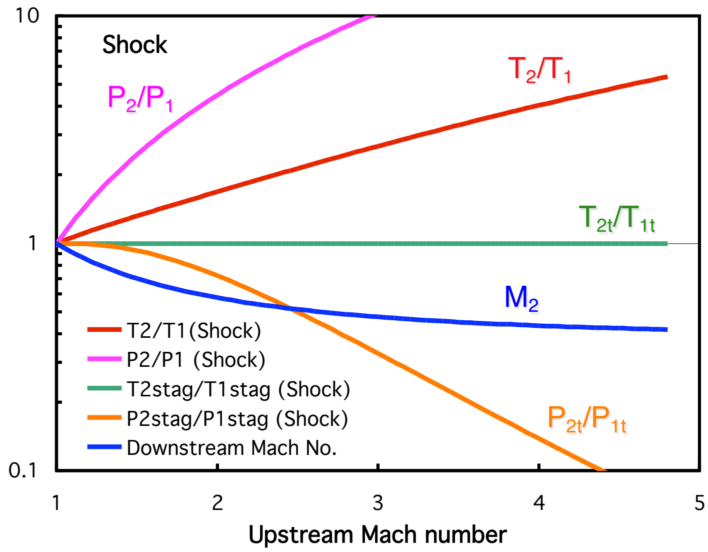

Everything const. but momentum (Fanno flow)

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- Since friction loss is path dependent, need to use differential form of momentum equation (constant A by assumption)
- Combine and integrate with differential forms of mass, energy, eqn. of state from Mach = M to reference state ( )* at M = 1 (not a throat in this case since constant area!)

$$(C/A)C_f L^* = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M^2}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right) \qquad \frac{T}{T^*} = \frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}$$

$$\frac{T_t}{T_t^*} = 1 \qquad \frac{P}{P^*} = \frac{1}{M}\left(\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right)^{1/2} \qquad \frac{P_t}{P_t^*} = \frac{1}{M}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right)^{(\gamma+1)/2(\gamma-1)}$$

- Implications
  - Stagnation pressure always decreases towards M = 1
  - Can't cross M = 1 with constant area with friction!
  - M = 1 corresponds to the maximum length (L^*) of duct that can transmit the flow for the given inlet conditions (P_t, T_t) and duct properties (C/A, C_f)
  - Note C/A = Circumference/Area = 4/diameter for round duct

Everything const. but momentum (Fanno flow)

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- What if neither the initial state (1) nor final state (2) is the choked (*) state? Again use $P_2/P_1 = (P_2/P^*)/(P_1/P^*)$ etc., except for L, where we subtract to get net length ΔL

$$(C/A)C_f(L_2^* - L_1^*) = (C/A)C_f(\Delta L) = \frac{1-M_2^2}{\gamma M_2^2} - \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_2^2}{2\left(1+\frac{\gamma-1}{2}M_2^2\right)}\right) - \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2\left(1+\frac{\gamma-1}{2}M_1^2\right)}\right)$$

$$\Rightarrow (C/A)C_f(\Delta L) = \frac{M_2^2 - M_1^2}{\gamma M_1^2 M_2^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{M_2^2}{M_1^2}\frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)}{\left(1+\frac{\gamma-1}{2}M_2^2\right)}\right)$$

$$\frac{P_2}{P_1} = \frac{M_1}{M_2}\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{1/2} \quad \frac{P_{2t}}{P_{1t}} = \frac{M_1}{M_2}\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{(\gamma+1)/2(\gamma-1)} \quad \frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$$

Everything constant but momentum

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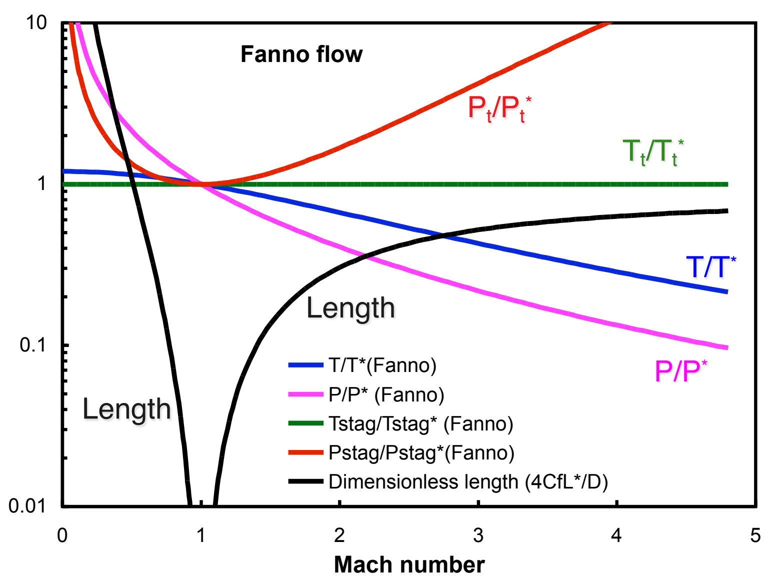

Everything constant but momentum

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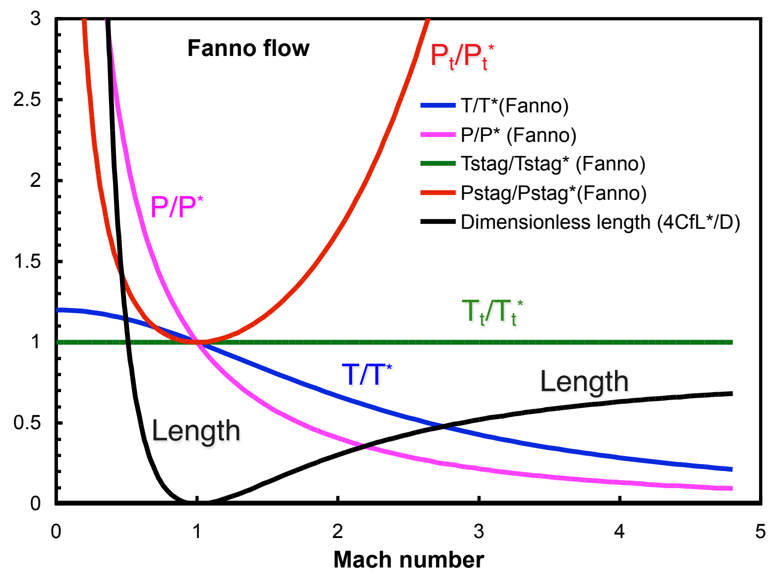

Heat addition at const. Area (Rayleigh flow)

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- Mass, momentum, energy, equation of state all apply
- Reference state $( )^*$: use M = 1 (not a throat in this case!)
- Energy equation not useful except to calculate heat input ($q = C_p(T_{2t} - T_{1t})$)

$$\frac{T}{T^*} = \left(\frac{1+\gamma}{1+\gamma M^2}\right)^2 M^2 \qquad \frac{P}{P^*} = \frac{1+\gamma}{1+\gamma M^2} \qquad \frac{T_t}{T_t^*} = \frac{2(\gamma+1)M^2\left(1+\frac{\gamma-1}{2}M^2\right)}{\left(1+\gamma M^2\right)^2}$$

$$\frac{P_t}{P_t^*} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}\left(\frac{1+\gamma}{1+\gamma M^2}\right)\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1}$$

- Implications
  - Stagnation temperature always increases towards M = 1
  - Stagnation pressure always decreases towards M = 1 (stagnation temperature increasing, more heat addition)
  - Can't cross M = 1 with constant area heat addition!
  - M = 1 corresponds to the maximum possible heat addition
  - …but there's no particular reason we have to keep area (A) constant when we add heat!

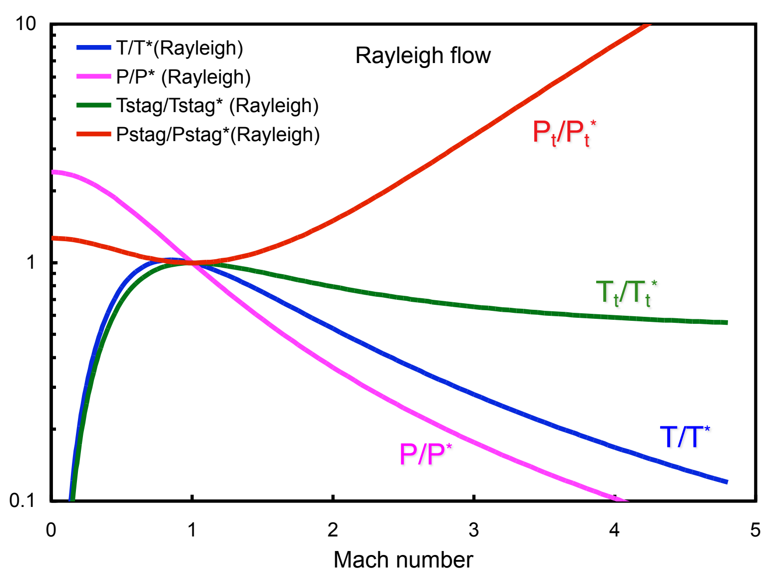
Heat addition at const. Area (Rayleigh flow)

➢ What if neither the initial state (1) nor final state (2) is the choked (*) state? Again use $P_2/P_1 = (P_2/P^*)/(P_1/P^*)$ etc.

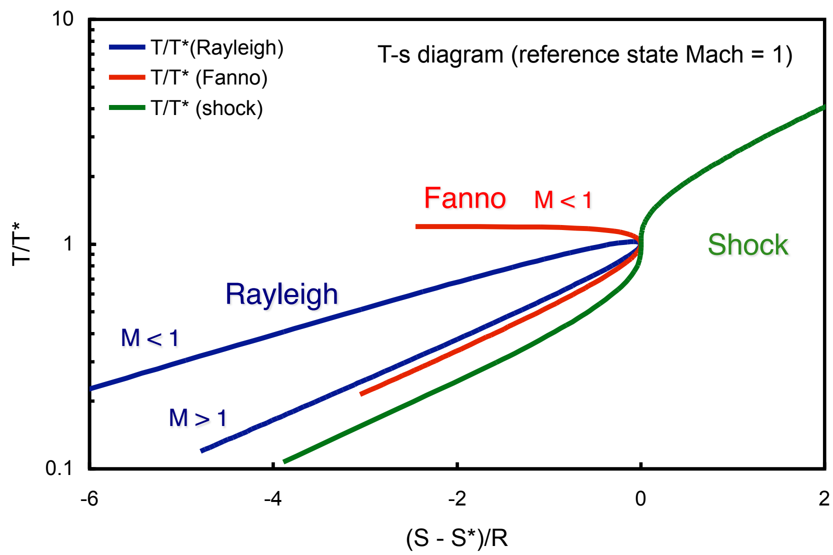
$$\frac{T_2}{T_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \frac{M_2^2}{M_1^2} \qquad \frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2} \qquad \frac{T_{2t}}{T_{1t}} = \frac{M_2^2}{M_1^2}\frac{\left(1+\gamma M_1^2\right)^2}{\left(1+\gamma M_2^2\right)^2}\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}$$

$$\frac{P_{2t}}{P_{1t}} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\gamma/\gamma-1}$$

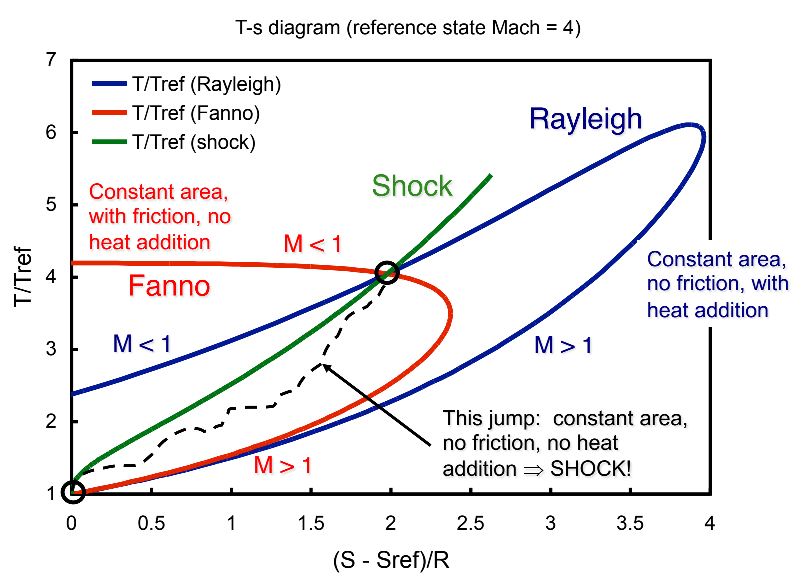
Heat addition at constant area



T-s diagram - reference state M = 1



T-s diagram - Fanno, Rayleigh, shock



Heat addition at constant pressure

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- Relevant for hypersonic propulsion if maximum allowable pressure (i.e. structural limitation) is the reason we can't decelerate the ambient air to M = 0)
- Momentum equation: AdP + $\dot{m}$du = 0 $\Rightarrow$ u = constant
- Reference state ( )*: use M = 1 again but nothing special happens there
- Again energy equation not useful except to calculate q

$$\frac{T}{T^*} = \frac{A}{A^*} = \frac{1}{M^2} \qquad \frac{P}{P^*} = 1 \qquad \frac{T_t}{T_t^*} = \frac{2}{(\gamma+1)M^2}\left(1 + \frac{\gamma-1}{2}M^2\right)$$

$$\frac{P_t}{P_t^*} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}\left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1}$$

- Implications
  - Stagnation temperature increases as M decreases, i.e. heat addition corresponds to decreasing M
  - Stagnation pressure decreases as M decreases, i.e. heat addition decreases stagnation P
  - Area increases as M decreases, i.e. as heat is added

Heat addition at constant pressure

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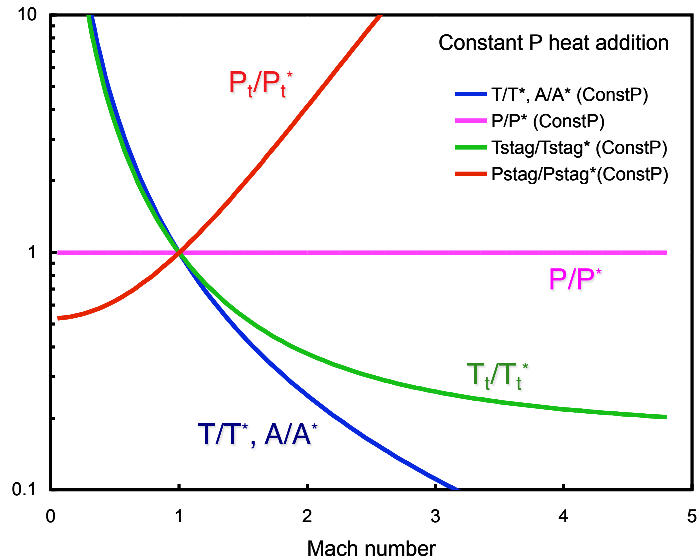
- What if neither the initial state (1) nor final state (2) is the reference (*) state? Again use $P_2/P_1 = (P_2/P^*)/(P_1/P^*)$ etc.

$$\frac{T_2}{T_1} = \frac{A_2}{A_1} = \frac{M_1^2}{M_2^2} \qquad \frac{T_{2t}}{T_{1t}} = \frac{M_1^2}{M_2^2}\frac{1 + \frac{\gamma-1}{2}M_2^2}{1 + \frac{\gamma-1}{2}M_1^2} \qquad \frac{P_{2t}}{P_{1t}} = \left(\frac{1 + \frac{\gamma-1}{2}M_2^2}{1 + \frac{\gamma-1}{2}M_1^2}\right)^{\gamma/\gamma-1}$$

Heat addition at constant P



Heat addition at constant temperature

- Probably most appropriate case for hypersonic propulsion since temperature (materials) limits is usually the reason we can't decelerate the ambient air to M = 0
- T = constant ⇒ a (sound speed) = constant
- Momentum: AdP + $\dot{m}$du = 0 ⇒ dP/P + γMdM = 0
- Reference state ( )*: use M = 1 again

$$\frac{T}{T^*} = 1 \qquad \frac{P}{P^*} = \exp\left[\frac{\gamma}{2}\left(1 - M^2\right)\right] \qquad \frac{T_t}{T_t^*} = \frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)$$

$$\frac{A}{A^*} = \frac{1}{M}\exp\left[\frac{-\gamma}{2}\left(1 - M^2\right)\right] \qquad \frac{P_t}{P_t^*} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}\exp\left[\frac{\gamma}{2}\left(1 - M^2\right)\right]\left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1}$$

- Implications
  - Stagnation temperature increases as M increases
  - Stagnation pressure decreases as M increases, i.e. heat addition decreases stagnation P
  - Minimum area (i.e. throat) at M = $\gamma^{-1/2}$
  - Large area ratios needed due to exp[ ] term

Heat addition at constant temperature

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- What if neither the initial state (1) nor final state (2) is the reference (*) state? Again use $P_2/P_1 = (P_2/P^*)/(P_1/P^*)$ etc.

$$\frac{P_2}{P_1} = \frac{\exp\left[\frac{\gamma}{2}\left(1-M_2^2\right)\right]}{\exp\left[\frac{\gamma}{2}\left(1-M_1^2\right)\right]} = \exp\left[\frac{\gamma}{2}\left(M_1^2-M_2^2\right)\right] \qquad \frac{T_{2t}}{T_{1t}} = \frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}$$

$$\frac{A_2}{A_1} = \frac{M_1}{M_2}\frac{\exp\left[\frac{-\gamma}{2}\left(1-M_2^2\right)\right]}{\exp\left[\frac{-\gamma}{2}\left(1-M_1^2\right)\right]} = \frac{M_1}{M_2}\exp\left[\frac{\gamma}{2}\left(M_2^2-M_1^2\right)\right]$$

$$\frac{P_{2t}}{P_{1t}} = \frac{\exp\left[\frac{\gamma}{2}\left(1-M_2^2\right)\right]}{\exp\left[\frac{\gamma}{2}\left(1-M_1^2\right)\right]}\frac{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1}}{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1}} = \exp\left[\frac{\gamma}{2}\left(M_1^2-M_2^2\right)\right]\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\gamma/\gamma-1}$$

Heat addition at constant T

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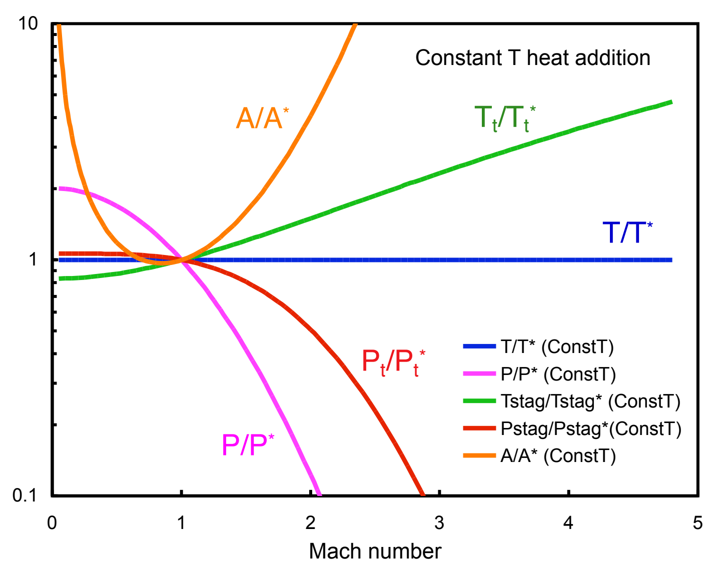

T-s diagram for diabatic flows

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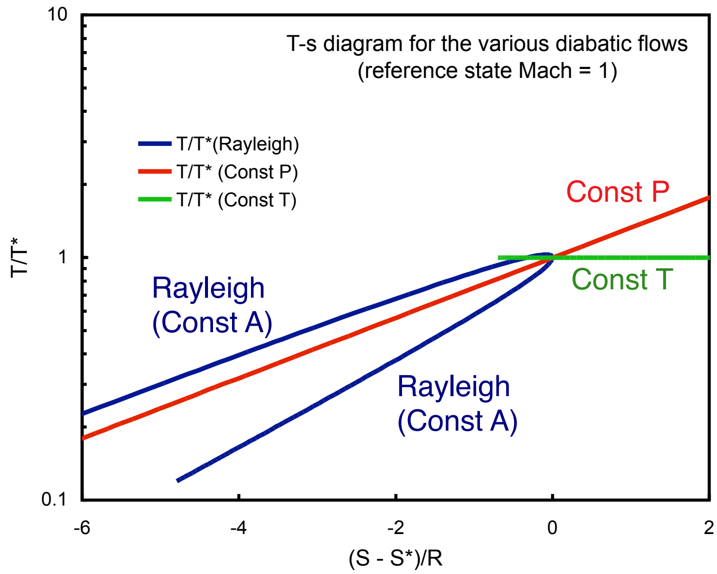

T-s diagram for diabatic flows

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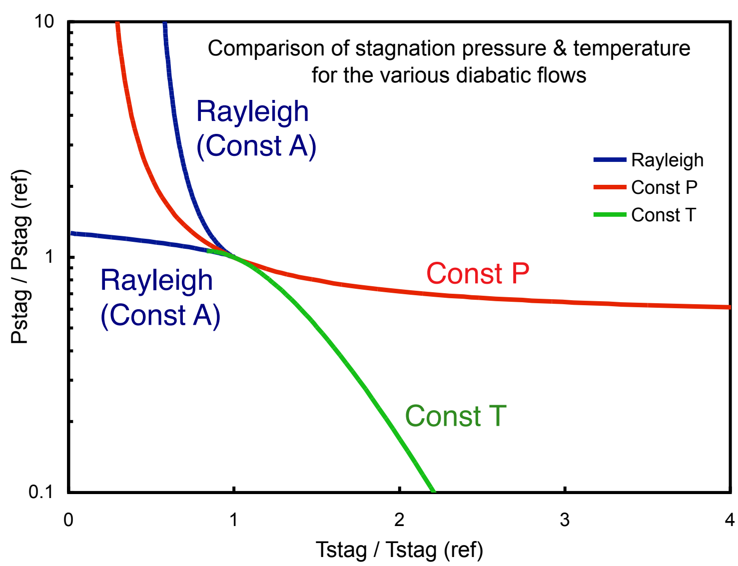

Area ratios for diabatic flows

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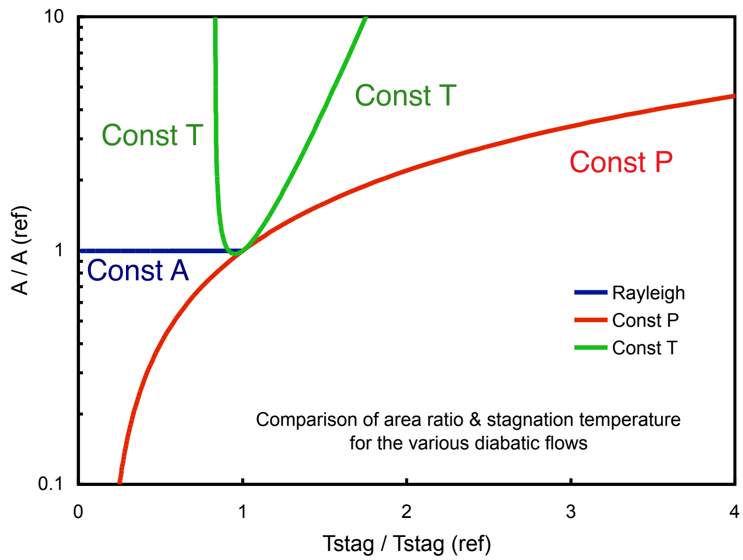

What is the best way to add heat?

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- If maximum T or P is limitation, obviously use that case
- What case gives least P_t loss for given increase in T_t?
  - Minimize $d(P_t)/d(T_t)$ subject to mass, momentum, energy conservation, eqn. of state
  - Result (lots of algebra - many trees died to bring you this result)

$$\frac{dP_t}{dT_t} = -\frac{\gamma M^2}{2}\frac{P_t}{T_t} \quad or \quad \frac{d(\ln P_t)}{d(\ln T_t)} = -\frac{\gamma M^2}{2}$$

- Adding heat (increasing T_t) always decreases P_t
- Least decrease in P_t occurs at lowest possible M – doesn't really matter if it's at constant A, P, T, etc.

Summary - 1D compressible flow

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	Const. A?	Adia-batic?	Friction-less?	T_t const.?	P_t const.?
Isentropic	No	Yes	Yes	Yes	Yes
Fanno	Yes	Yes	No	Yes	No
Shock	Yes	Yes	Yes	Yes	No
Rayleigh	Yes	No	Yes	No	No
Const. T heat addition	No	No	Yes	No	No
Const. P heat addition	No	No	Yes	No	No

Summary of heat addition processes

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	Const. A	Const. P	Const. T
M	Goes to M = 1	Decreases	Increases
Area	Constant	Increases	Min. at M = $\gamma^{-1/2}$
P	Decreases for M < 1 Increases for M > 1	Constant	Decreases
P_t	Decreases	Decreases	Decreases
T	Increases except for a small region at M < 1	Increases	Constant
T_t	Increases	Increases	Increases

Example #1

Helium ($\gamma = 5/3$, molecular mass 4 g/mole) is used in a simple propulsion system. It is heated to T = 1500K, P = 10 atm and Mach number 0.3 (T and P are the static temperature and pressure, that is, not T_t and P_t), then expanded isentropically through a nozzle to P = 2 atm. (1 atm = 101325 N/m^2).

a) Compute the stagnation temperature T_t and stagnation pressure P_t.

$$T_t = T\left(1+\frac{\gamma-1}{2}M^2\right) = 1500K\left(1+\frac{5/3-1}{2}0.3^2\right) = 1545K; P_t = 10atm\left(1+\frac{5/3-1}{2}0.3^2\right)^{\frac{5/3}{5/3-1}} = 10.767atm$$

b) Compute the exit velocity of the gases after expansion.

$$u_{exit} = \sqrt{\frac{2\gamma}{\gamma-1}RT_t\left(1-\left(\frac{P_{exit}}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right)} = \sqrt{\frac{2(5/3)}{5/3-1}\frac{8.314\frac{J}{moleK}}{\frac{0.004kg}{mole}}1545K\left(1-\left(\frac{2atm}{10.767atm}\right)^{\frac{5/3-1}{5/3}}\right)} = 2804.9\sqrt{\frac{J}{kg}} = 2804.9\frac{m}{s}$$

c) Compute the ratio of exit area to nozzle throat area (A/A^*).

First we need to compute the exit Mach number M_9:

$$P_t = 10.767atm = P_9\left(1+\frac{\gamma-1}{2}M_9^2\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow M_9 = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P_t}{P_9}\right)^{\frac{\gamma-1}{\gamma}}-1\right]} = \sqrt{\frac{2}{5/3-1}\left[\left(\frac{10.767atm}{2atm}\right)^{\frac{5/3-1}{5/3}}-1\right]} = 1.698$$

Example #1

d) Compute the Specific Thrust if the ambient pressure and temperature are 1 atm and 300K. Note that $u_1 = 0$ and FAR = 0 in this case (i.e. this is effectively a rocket motor not an airbreathing propulsion device) and that the exit pressure does not equal ambient pressure.

First we need to compute the exit temperature T_9:

$$T_t = T_9\left(1+\frac{\gamma-1}{2}M_9^2\right) = 1545K \Rightarrow T_9 = \frac{T_t}{1+\frac{\gamma-1}{2}M_9^2} = \frac{1545K}{1+\frac{5/3-1}{2}1.698^2} = 787.8K$$

Then using the thrust equation for $P_9 \neq P_1$ (Lecture 11, page 16):

$$ST = (1+FAR)M_9\sqrt{\frac{T_9}{T_1}} - M_1 + \left(1-\frac{P_1}{P_9}\right)\sqrt{\frac{T_9}{T_1}}\frac{1+FAR}{\gamma M_9}$$

$$= (1+0)(1.698)\sqrt{\frac{787.8K}{300K}} - 0 + \left(1-\frac{1atm}{2atm}\right)\sqrt{\frac{787.8K}{300K}}\frac{1+0}{(5/3)(1.698)} = 3.038$$

Note that this is the Specific Thrust based on the sound speed at ambient conditions (300K). If you chose to calculate Specific Thrust using the sound speed at condition 1 (1500K) that's OK too. For airbreathing propulsion systems state 1 **is** the ambient condition but in this case since there's no inlet one needs to specify the reference condition.

Example #2

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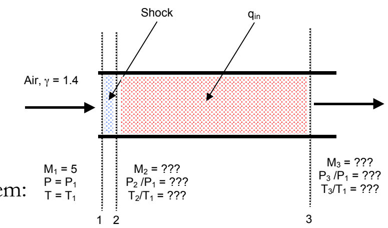
Consider a very simple propulsion system operating at a flight Mach number of 5 that consists of 2 processes:



Process 1: Shock at entrance to duct
Process 2: Heat addition in a constant-area duct until thermal choking occurs

a) Compute all of the following properties of this system:

(i) Static (not stagnation) temperature relative to T_1 after the shock

$$\frac{T_2}{T_1} = \frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)\left(\frac{2\gamma}{\gamma-1}M_1^2-1\right)}{\frac{(\gamma+1)^2}{2(\gamma-1)}M_1^2} = \frac{\left(1+\frac{1.4-1}{2}M_1^2\right)\left(\frac{2(1.4)}{1.5-1}5^2-1\right)}{\frac{(1.4+1)^2}{2(1.4-1)}5^2} = 5.8$$

(ii) Static (not stagnation) pressure relative to P_1 after the shock

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1} = \frac{2(1.4)}{1.4+1}5^2 - \frac{1.4-1}{1.4+1} = 29$$

Example #2

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(iii) Static (not stagnation) temperature and pressure relative to T_1 at the exit

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2 - 1} = \frac{5^2 + \frac{2}{1.4-1}}{\frac{2(1.4)}{1.4-1}5^2 - 1} = 0.1724 \Rightarrow M_2 = 0.4152$$

$$\frac{T}{T^*} = \left(\frac{1+\gamma}{1+\gamma M^2}\right)^2 M^2 \Rightarrow \frac{T_2}{T_3} = \left(\frac{1+\gamma}{1+\gamma M_2^2}\right)^2 M_2^2 \Rightarrow \frac{T_3}{T_1} = \frac{T_3}{T_2}\frac{T_2}{T_1} = \left(\frac{1+\gamma M_2^2}{1+\gamma}\right)^2 \frac{1}{M_2^2}\frac{T_2}{T_1}$$

$$\Rightarrow \frac{T_3}{T_1} = \left(\frac{1+1.4(0.4152)^2}{1+1.4}\right)^2 \frac{1}{0.4152^2}(5.8) = 9$$

$$\frac{P}{P^*} = \frac{1+\gamma}{1+\gamma M^2} \Rightarrow \frac{P_2}{P_3} = \frac{1+\gamma}{1+\gamma M_2^2} \Rightarrow \frac{P_3}{P_1} = \frac{P_3}{P_2}\frac{P_2}{P_1} = \frac{1+\gamma M_2^2}{1+\gamma}\frac{P_2}{P_1} = \frac{1+1.4(0.4152)^2}{1+1.4}(29) = 15$$

(iv) Dimensionless heat addition $\{q_{in}/RT_1 = C_P(T_{3t}-T_{2t})/RT_1 = [\gamma/(\gamma-1)](T_{3t}-T_{2t})/T_1\}$

$$\frac{q_{in}}{RT_1} = \frac{\gamma}{\gamma-1}\frac{T_{3t}-T_{2t}}{T_1} = \frac{\gamma}{\gamma-1}\frac{T_{2t}(T_{3t}/T_{2t}-1)}{T_1} = \frac{\gamma}{\gamma-1}\frac{T_{1t}(T_{3t}/T_{2t}-1)}{T_1} = \frac{\gamma}{\gamma-1}\frac{T_{1t}}{T_1}\left(\frac{T_{3t}}{T_{2t}}-1\right) = \frac{\gamma}{\gamma-1}\left(1+\frac{\gamma-1}{2}M_1^2\right)\left(\frac{T_{3t}}{T_{2t}}-1\right)$$

$$\frac{T_{2t}}{T_{3t}} = \frac{2(\gamma+1)M_2^2\left(1+\frac{\gamma-1}{2}M_2^2\right)}{\left(1+\gamma M_2^2\right)^2} = \frac{2(1.4+1)0.4152^2\left(1+\frac{1.4-1}{2}0.4152^2\right)}{\left(1+1.4(0.4152)^2\right)^2} = 0.5555$$

$$\frac{T_{1t}}{T_1} = 1+\frac{\gamma-1}{2}M_1^2 = 1+\frac{1.4-1}{2}5^2 = 6; T_{2t} = T_{1t} = 6T_1; T_{3t} = 1.8T_{2t} = 10.8T_1$$

$$\Rightarrow \frac{q_{in}}{RT_1} = \frac{1.4}{1.4-1}\left(1+\frac{1.4-1}{2}5^2\right)\left(\frac{1}{0.5555}-1\right) = 16.8$$

Example #2

(v) Specific thrust (assume FAR << 1 in the thrust calculation)

$$\text{Specific Thrust (ST)} \equiv Thrust / \dot{m}_a c_1$$

$$FAR << 1: Thrust = \dot{m}_a[u_9 - u_1] + (P_9 - P_1)A_9$$

$$ST \equiv \frac{Thrust}{\dot{m}_a c_1} = \frac{u_9}{c_1} - \frac{u_1}{c_1} + \frac{(P_9 - P_1)A_9}{\dot{m}_a c_1} = \frac{u_9}{c_9}\frac{c_9}{c_1} - M_1 + \frac{(P_9 - P_1)A_9}{(\rho_9 u_9 A_9)c_1}$$

$$= M_9 \frac{\sqrt{\gamma R T_9}}{\sqrt{\gamma R T_1}} - M_1 + \frac{(P_9 - P_9)}{(P_9 / RT_9)u_9 c_1} = M_9\sqrt{\frac{T_9}{T_1}} - M_1 + \left(1 - \frac{P_1}{P_9}\right)\frac{RT_9}{c_9 M_9 c_1}$$

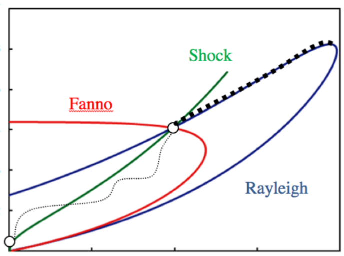
$$= M_9\sqrt{\frac{T_9}{T_1}} - M_1 + \left(1 - \frac{P_1}{P_9}\right)\frac{RT_9}{\sqrt{\gamma R T_9} M_9 \sqrt{\gamma R T_1}} = M_9\sqrt{\frac{T_9}{T_1}} - M_1 + \left(1 - \frac{P_1}{P_9}\right)\sqrt{\frac{T_9}{T_1}}\frac{1}{\gamma M_9}$$

$$= 1\sqrt{9} - 5 + \left(1 - \frac{1}{15}\right)\sqrt{9}\frac{1}{1.4(1)} = 0(???)$$

(vi) Overall efficiency

$$\eta_o = \left(\frac{Thrust}{m_a c_1}\right)\frac{(\gamma - 1)T_1 M_1}{(T_{3t} - T_{2t})} = (0)\frac{(1.4-1)(5)}{(10.8-6)} = 0$$

(vii) Draw this cycle on a T - s diagram.
Include appropriate Rayleigh and Fanno curves.



Example #2

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b) Repeat (a) if a nozzle is added after station 3 to expand the flow isentropically back to P = P_1.

Everything is the same up to state 3, but now we have a state 4, i.e. isentropic expansion ($P_{4t} = P_{3t}$, $T_{4t} = T_{3t}$ until $P_4 = P_1$.

$$P_{4t} = P_{3t} \Rightarrow P_4\left(1 + \frac{\gamma-1}{2}M_4^2\right)^{\gamma/\gamma-1} = P_3\left(1 + \frac{\gamma-1}{2}M_3^2\right)^{\gamma/\gamma-1} \Rightarrow M_4 = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P_3}{P_4}\right)^{\gamma-1/\gamma}\left(1 + \frac{\gamma-1}{2}M_3^2\right) - 1\right]}$$

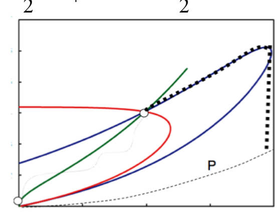
$$M_3 = 1, P_4 = P_1, \frac{P_3}{P_1} = 15 \Rightarrow M_4 = \sqrt{\frac{2}{\gamma-1}\left[(15)^{\gamma-1/\gamma}\left(1 + \frac{1.4-1}{2}1^2\right) - 1\right]} = 2.829$$

$$T_{4t} = T_{3t} \Rightarrow T_4\left(1 + \frac{\gamma-1}{2}M_4^2\right) = T_3\left(1 + \frac{\gamma-1}{2}M_3^2\right) \Rightarrow \frac{T_4}{T_1} = \frac{T_3}{T_1}\frac{1 + \frac{\gamma-1}{2}M_3^2}{1 + \frac{\gamma-1}{2}M_4^2} = (9)\frac{1 + \frac{1.4-1}{2}1^2}{1 + \frac{1.4-1}{2}2.829^2} = 4.152$$

$$P_9 = P_1: ST = M_9\sqrt{\frac{T_9}{T_1}} - M_1 = 2.829\sqrt{4.152} - 5 = 0.764$$

Heat addition is the same as before, so

$$\eta_o = \left(\frac{Thrust}{m_a c_1}\right)\frac{(\gamma - 1)T_1 M_1}{(T_{3t} - T_{2t})} = (0.764)\frac{(1.4-1)(5)}{(10.8-6)} = 0.3183$$



Example #2

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c) Why was thrust generated in part (b) but not part (a)?

In part (a) there is no area change and no friction, so there is no mechanism for the gas pressure to exert a force in the x-direction on the walls of the combustion device, so there is no way to generate thrust (net force in the x-direction). In part (b) a nozzle was added, so the area changes, so some part of the wall is not oriented parallel to the x-direction, so the gas pressure can exert a force in the x-direction on the walls of the combustion device and thus generate thrust.

Summary - compressible flow

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- The 1D conservation equations for energy, mass and momentum along with the ideal gas equations of state yield a number of unusual phenomenon
  - Choking - isentropic, diabatic (Rayleigh), friction (Fanno), all at M = 1; for heat addition at constant T, choking at $M = 1/\gamma^{1/2}$
  - Garden hose in reverse (rule of thumb: for supersonic flow, all of your intuitions about flow should be reversed)
  - If no friction, no heat addition, no area change - it's a shock!
- Stagnation conditions
  - Temperature - a measure of the total energy (thermal + kinetic) contained by a flow
  - Pressure - a measure of the "usefulness" (ability to expand) of a flow