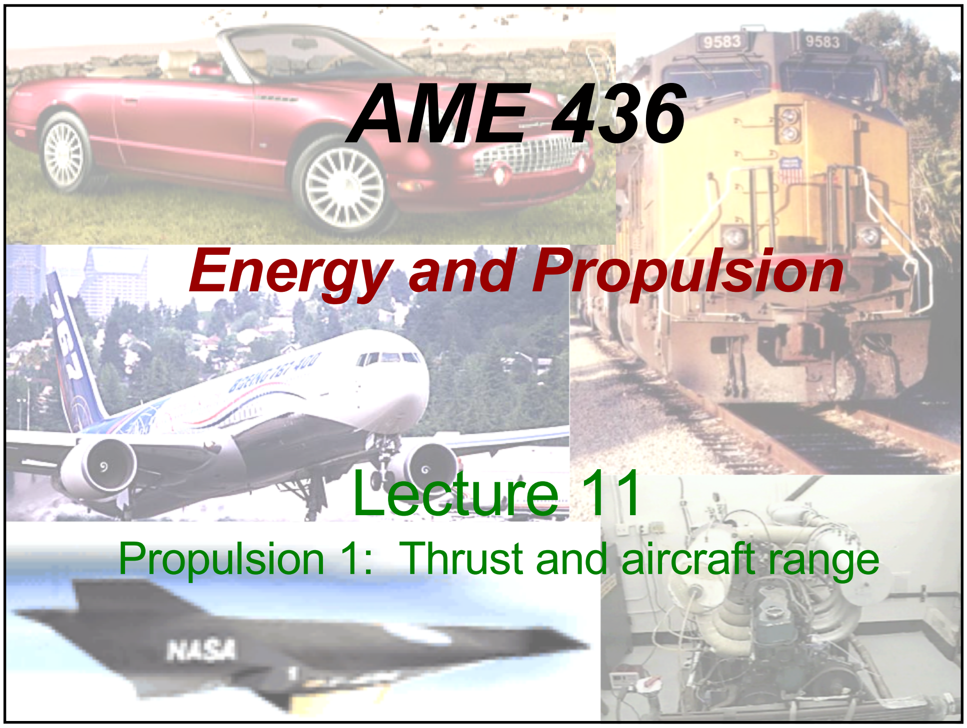

AME 436

Energy and Propulsion

Lecture 11

Propulsion 1: Thrust and aircraft range

Outline

- Why gas turbines?
- Computation of thrust
- Propulsive, thermal and overall efficiency
- Specific thrust, thrust specific fuel consumption, specific impulse
- Breguet range equation

Why gas turbines?

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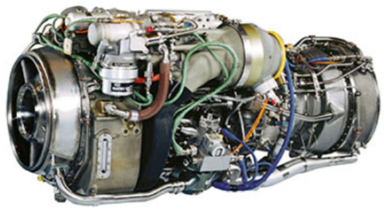

- GE CT7-8 turboshaft (used in helicopters)
- https://www.geaviation.com/commercial/engines/ct7-engine
- Compressor/turbine stages: 6/4
- Diameter 26", Length 48.8" = 426 liters = 5.9 hp/liter
- Dry Weight 537 lb, max. power 2,634 hp (power/wt = 4.7 hp/lb)
- Pressure ratio at max. power: 21 (ratio per stage = $21^{1/6}$ = 1.66)
- Specific fuel consumption at max. power: 0.452 (units not given; if lb/hp-hr then corresponds to 29.3% efficiency)

- Cummins QSK60-2850 4-stroke 60.0 liter (3,672 in^3) V-16 2-stage turbocharged diesel (used in mining trucks)
- https://mart.cummins.com/imagelibrary/data/assetfiles/0032422.pdf
- 2.93 m long x 1.58 m wide x 2.31 m high = 10,700 liters = 0.27 hp/liter
- Dry weight 21,207 lb, 2850 hp at 1900 RPM (power/wt = 0.134 hp/lb = 35x lower than gas turbine)
- BMEP = 22.1 atm
- Volume compression ratio ??? (not given)

Why gas turbines?

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- Pratt & Whitney R-2800 46 liter (2800 in^3) 18-cyl. 4-stroke supercharged gasoline engine (used in WWII aircraft)
- Total volume 53" dia x 81" length = 2927 liters = 0.72 hp/liter
- 2100 hp @ 2700 RPM
- Dry weight 2360 lb. (power/wt = 0.89 hp/lb = 5.3x lower than gas turbine)
- BMEP = 14.9 atm
- Volume compression ratio 6.8:1 (= pressure ratio 14.6 if isentropic)

- NuCellSys HY-80 "Fuel cell engine" (specs no longer on-line)
- Volume 220 liters = 0.41 hp/liter
- 91 hp, 485 lb. (power/wt = 0.19 hp/lb)
- 41% efficiency (fuel to electricity) at max. power; up to 58% at lower power
- Uses hydrogen - NOT hydrocarbons
- Does NOT include electric drive system (≈ 0.40 hp/lb) at ≈ 90% electrical to mechanical efficiency
- Fuel cell + motor overall 0.13 hp/lb at 37% efficiency, not including H_2 storage

Why gas turbines?

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- Why do gas turbines have much higher power/weight & power/volume than reciprocating engines? More air can be processed since steady flow, not start/stop
  - More air ⇒ more fuel can be burned
  - More fuel ⇒ more heat release
  - More heat ⇒ more work (if thermal efficiency similar)

Why gas turbines?

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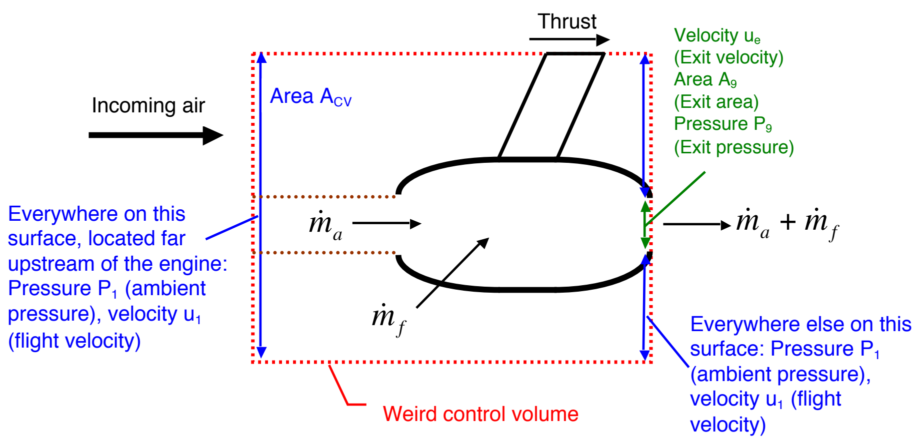
- Disadvantages
  - Compressor is a dynamic device that pushes gas from low pressure (P) to high P without positive sealing like piston/cylinder
    - » Requires very precise aerodynamics
    - » Requires blade speeds ≈ sound speed, otherwise gas flows back to low P faster than compressor can push it to high P
    - » Each stage can provide only 2:1 or 3:1 pressure ratio - need many stages for large pressure ratio
  - Since steady flow, each component sees a constant temperature - turbine stays hot continuously and must rotate at high speeds (high stress)
    - » Severe materials and cooling engineering required (unlike reciprocating engine where components feel only average gas temperature during cycle)
    - » Turbine inlet temperature limit typically 1400˚C - limits fuel input
  - As a result, turbines require more maintenance & are more expensive for same power

Thrust computation

- In gas turbine and rocket propulsion we need THRUST (force acting on vehicle)
- How much thrust can we get from a given amount of fuel?
- Force = d(momentum)/d(time)
  - Force = pressure x area; momentum = mass flow x velocity
- Goal of propulsion analysis is to compute exhaust velocity and pressure for a given thermodynamic cycle

Thrust computation

- Control volume for thrust computation - in frame of reference moving with the engine



Thrust computation - steady flight

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- Newton's 2nd law: Force = rate of change of momentum

$$\sum Forces = \sum \frac{d(mu)}{dt}$$

$$\sum Forces = Thrust + P_1 A_{CV} - \left[P_1(A_{CV} - A_9) + P_9 A_9\right] = T + (P_1 - P_9)A_9$$

$$\sum \frac{d(mu)}{dt} = \sum\left(\frac{dm}{dt}u + m\frac{du}{dt}\right) = \sum(\dot{m}u + 0) \text{ (if steady, } \frac{du}{dt} = 0)$$

$$\sum \dot{m}u = (\dot{m}_f + \dot{m}_a)u_9 - (\dot{m}_a)u_1 \quad (\mathrm{u_{fuel}} = 0 \text{ in moving reference frame})$$

$$\text{Combine: } Thrust = \left(\dot{m}_a + \dot{m}_f\right)u_9 - \dot{m}_a u_1 + \left(P_9 - P_1\right)A_9$$

$$\Rightarrow \boxed{Thrust = \dot{m}_a\left[\left(1 + FAR\right)u_9 - u_1\right] + \left(P_9 - P_1\right)A_9;}$$

$$FAR = \dot{m}_f / \dot{m}_a = \text{ Fuel to air mass ratio} = f/(1-f) \ (f = \text{fuel mass fraction})$$

- At takeoff u_1 = 0; for rocket no inlet so u_1 = 0 always
- For hydrocarbon-air combustion FAR << 1; typically 0.06 at stoichiometric, but maximum allowable FAR ≈ 0.03 due to turbine inlet temperature limitations (discussed later…)

Thrust computation

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- But how to compute exit velocity (u_9) and exit pressure (P_9) as a function of ambient pressure (P_1), flight velocity (u_1)? Need compressible flow analysis, next lecture …
- Also - one can obtain a given thrust with large $(P_9 - P_1)A_9$ and a small $\dot{m}_a[(1+FAR)u_9 - u_1]$ or vice versa - which is better, i.e. for given u_1, P_1, $\dot{m}_a$ and FAR, what P_9 will give most thrust? Differentiate thrust equation and set = 0

$$\frac{d(Thrust)}{d(P_9)} = \dot{m}_a\left[\left(1 + FAR\right)\frac{d(u_9)}{d(P_9)} - 0\right] + \left(1 - 0\right)A_9 + (P_9 - P_1)\frac{d(A_9)}{d(P_9)} = 0$$

- Momentum balance at exit (see next slide)

$$AdP + \dot{m}du = 0 \Rightarrow A_9 + \dot{m}_a(1 + FAR)\frac{d(u_9)}{d(P_9)} = 0$$

- Combine

$$\frac{d(Thrust)}{d(P_9)} = (P_9 - P_1)\frac{d(A_9)}{d(P_9)} = 0 \Rightarrow P_9 = P_1$$

⇒ Optimal performance occurs for exit pressure = ambient pressure

⇒ Valid for any 1-D steady cycle (ideal or not), any material

1D momentum balance - constant-area duct

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Coefficient of friction (C_f)

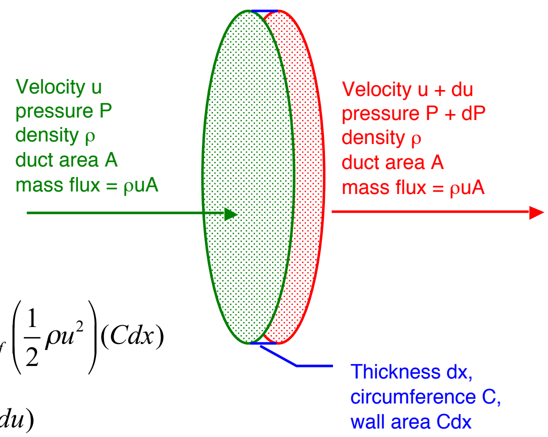
$$C_f \equiv \frac{\text{Wall drag force}}{\frac{1}{2}\rho u^2 \cdot (\text{Wall area})}$$

$$\sum Forces = \sum \frac{d(mu)}{dt}$$

$$\sum Forces = PA - (P + dP)A - C_f\left(\frac{1}{2}\rho u^2\right)(Cdx)$$

$$\sum \frac{d(mu)}{dt} = \sum \dot{m}u = \dot{m}u - \dot{m}(u + du)$$

$$\text{Combine: AdP} + \dot{m}du + C_f\left(\frac{1}{2}\rho u^2\right)Cdx = 0$$



Thrust computation

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- Wait – this says $P_9 = P_1$ is an extremum –but is it a minimum or maximum?

$$\frac{d(Thrust)}{d(P_9)} = (P_9 - P_1)\frac{d(A_9)}{d(P_9)} \Rightarrow \frac{d^2(Thrust)}{d(P_9)^2} = (P_9 - P_1)\frac{d^2(A_9)}{d(P_9)^2} + \frac{d(A_9)}{d(P_9)}(1)$$

but $P_9 = P_1$ at the extremum cases so

$$\frac{d^2(Thrust)}{d(P_9)^2} = \frac{d(A_9)}{d(P_9)}$$

- Maximum thrust if $d^2(Thrust)/d(P_9)^2 < 0 \Rightarrow dA_9/dP_9 < 0$ - we will show this is true for supersonic exit conditions
- Minimum thrust if $d^2(Thrust)/d(P_9)^2 > 0 \Rightarrow dA_9/dP_9 > 0$ - we will show this is true for subsonic exit conditions, but for subsonic, $P_9 = P_1$ always since acoustic (pressure) waves can travel up the nozzle, equalizing the pressure to P_9, so it's a moot point

Thrust computation

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- Turbofan: same as turbojet except that there are two streams, one hot (combusting) and one cold (non-combusting, fan only, use prime (') superscript):

$$Thrust = \dot{m}_a\left[\left(1+FAR\right)u_9 - u_1\right] + \left(P_9 - P_1\right)A_9 + \dot{m}_a^{'}\left[u_9^{'} - u_1\right]$$

- Note (1 + FAR) term applies only to combusting stream
- Note we assumed $P_9 = P_1$ for fan stream; for any sane fan design u_9' will be subsonic so $P_9 = P_1$ will be true

Propulsive, thermal, overall efficiency

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- Thermal efficiency (η_{th})

$$\eta_{th} \equiv \frac{\Delta(\text{Kinetic energy})}{\text{Heat input}} = \frac{(\dot{m}_a + \dot{m}_f)u_9^2/2 - (\dot{m}_a)u_1^2/2}{\dot{m}_f Q_R}$$

$$\text{If } \dot{m}_f << \dot{m}_a \ (FAR << 1) \text{ then } \eta_{th} \approx \frac{(u_9^2 - u_1^2)/2}{FAR \cdot Q_R}$$

- Propulsive efficiency (η_p)

$$\eta_p \equiv \frac{\text{Thrust power}}{\Delta(\text{Kinetic energy})} = \frac{Thrust \cdot u_1}{(\dot{m}_a + \dot{m}_f)u_9^2/2 - (\dot{m}_a)u_1^2/2}$$

$$\text{If } \dot{m}_f << \dot{m}_a \ (FAR << 1) \text{ and } P_9 = P_1 \text{ then } \eta_p \approx \frac{\dot{m}_a(u_9 - u_1)\cdot u_1}{\dot{m}_a(u_9^2 - u_1^2)/2} = \frac{2u_1/u_9}{1 + u_1/u_9}$$

- Overall efficiency (η_o)

$$\eta_o \equiv \frac{\text{Thrust power}}{\text{Heat input}} = \frac{\text{Thrust power}}{\Delta(\text{Kinetic energy})}\frac{\Delta(\text{Kinetic energy})}{\text{Heat input}} = \eta_{th}\eta_p$$

this is the most important efficiency in determining aircraft performance (see Breguet range equation, coming up…)

Propulsive, thermal, overall efficiency

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- Note on propulsive efficiency

$$\eta_p = \frac{\dot{m}_a(u_9-u_1)u_1}{\dot{m}_a(u_9^2-u_1^2)/2} = \frac{2(u_9-u_1)u_1}{\left[u_1+(u_9-u_1)\right]^2-\left[u_1\right]^2} = \frac{1}{1+\Delta u/2u_1}; \Delta u \equiv u_9-u_1$$

  - $\eta_p \to 1$ as $\Delta u \to 0 \Rightarrow u_9$ is only slightly larger than u_1
  - But then you need large mass flow rate ($\dot{m}_a$) to get the required Thrust ~ $\dot{m}_a\Delta u$ - but this is how turbofan engines work!
  - In other words, the best propulsion system accelerates an infinite mass of air by an infinitesimal Δu
  - Fundamentally this is because Thrust ~ $\Delta u = u_9 - u_1$, but energy required to get that thrust ~ $(u_9^2 - u_1^2)/2$
  - This issue will come up a lot in the next few weeks!

Other performance parameters

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- Specific thrust – thrust per unit mass flow rate, non-dimensionalized by sound speed at ambient conditions (c_1)

$$\text{Specific Thrust (ST)} \equiv \frac{Thrust}{\dot{m}_a c_1}$$

$$Thrust = \dot{m}_a[(1+FAR)u_9 - u_1] + (P_9-P_1)A_9$$ For any 1D steady propulsion system

$$ST \equiv \frac{Thrust}{\dot{m}_a c_1} = (1+FAR)\frac{u_9}{c_1} - \frac{u_1}{c_1} + \frac{(P_9-P_1)A_9}{\dot{m}_a c_1} = (1+FAR)\frac{u_9}{c_9}\frac{c_9}{c_1} - M_1 + \frac{(P_9-P_1)A_9}{\frac{\rho_9 u_9 A_9}{1+FAR}c_1}$$

$$= (1+FAR)M_9\frac{\sqrt{\gamma RT_9}}{\sqrt{\gamma RT_1}} - M_1 + \frac{(P_9-P_1)(1+FAR)}{(P_9/RT_9)u_9 c_1};\ M \equiv \frac{u}{c}\ \text{(Mach number)}$$

$$= (1+FAR)M_9\sqrt{\frac{T_9}{T_1}} - M_1 + \left(1-\frac{P_1}{P_9}\right)\frac{RT_9(1+FAR)}{\sqrt{\gamma RT_9}M_9\sqrt{\gamma RT_1}};\ \left[c=\sqrt{\gamma RT}\ \text{for ideal gas}\right]$$

$$= (1+FAR)M_9\sqrt{\frac{T_9}{T_1}} - M_1 + \left(1-\frac{P_1}{P_9}\right)\sqrt{\frac{T_9}{T_1}}\frac{1+FAR}{\gamma M_9}$$

For any 1D steady propulsion system if working fluid is an ideal gas with constant C_P, γ

Other performance parameters

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- Specific thrust (ST) continued… if $P_9 = P_1$ and FAR << 1 then

$$ST \equiv \frac{Thrust}{\dot{m}_a c_1} = M_9\sqrt{\frac{T_9}{T_1}} - M_1\left(if\ \ FAR << 1, P_1 = P_9\right)$$

- Thrust Specific Fuel Consumption (TSFC) (PDR's definition)

$$TSFC \equiv \frac{\dot{m}_f}{Thrust}\frac{Q_R}{c_1} = \left(\frac{\dot{m}_a c_1}{Thrust}\right)\frac{FAR\cdot Q_R}{c_1^2} = \frac{FAR\cdot Q_R}{ST\cdot c_1^2}$$

$$Note\ TSFC = \frac{\dot{m}_f}{Thrust}\frac{Q_R}{u_1}\frac{u_1}{c_1} = \frac{M_1}{\eta_o}$$

- Usual definition of TSFC is just $\dot{m}_f / Thrust$, but this is not dimensionless; use Q_R to convert $\dot{m}_f$ to heat input, one can use either u_1 or c_1 to convert the denominator to a quantity with units of power, but using u_1 would make TSFC blow up at $u_1 = 0$
- Specific impulse (I_{sp}) = thrust per weight (on earth) flow rate of fuel (plus oxidant if two reactants carried, e.g. rocket) (units of seconds)

$$I_{sp} = \frac{Thrust}{\dot{m}_{fuel} g_{earth}}; I_{sp} = \frac{(Thrust)u_1}{\dot{m}_{fuel}Q_R}\frac{Q_R}{g_{earth}c_1 M_1} = \frac{\eta_o Q_R}{M_1 c_1 g_{earth}} = \frac{Q_R}{(TSFC)c_1 g_{earth}}$$

Breguet range equation

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- Consider aircraft in level flight (Lift = Weight) at constant flight velocity u_1 (Thrust = Drag)

$$L = m_{vehicle} g;$$

$$D = Thrust = \frac{\eta_o \dot{m}_{fuel} Q_R}{u_1}$$

$$= \frac{\eta_o Q_R}{u_1}\frac{dm_{fuel}}{dt} = \frac{\eta_o Q_R}{u_1}\frac{-dm_{vehicle}}{dt}$$

- Combine expressions for lift & drag and integrate from time t = 0 to t = R/u_1 (R = range = distance traveled), i.e. travel time, to obtain Breguet Range Equation

$$\frac{D}{L}\frac{u_1 g}{\eta_o Q_R}dt = -\frac{dm_{vehicle}}{m_{vehicle}} \Rightarrow \int_0^{R/u_1}\frac{D}{L}\frac{u_1 g}{\eta_o Q_R}dt = -\int_{initial}^{final}\frac{dm_{vehicle}}{m_{vehicle}}$$

$$\Rightarrow \frac{D}{L}\frac{u_1 g}{\eta_o Q_R}\frac{R}{u_1} = -\ln\frac{m_{final}}{m_{initial}} \Rightarrow \boxed{R = \frac{L}{D}\frac{\eta_o Q_R}{g}\ln\frac{m_{initial}}{m_{final}}}$$

Rocket equation

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- If acceleration (Δu) rather than range in steady flight is desired [neglecting drag (D) and gravitational pull (W)], Force = mass x acceleration or Thrust = $m_{vehicle}du/dt$
- Since flight velocity u_1 is not constant, overall efficiency is not useful; instead use I_{sp}, leading to Rocket Equation:

$$Thrust = \dot{m}_{fuel} g_{earth} I_{sp} = g_{earth} I_{sp} \frac{dm_{fuel}}{dt} = -g_{earth} I_{sp} \frac{dm_{vehicle}}{dt}$$

$$Thrust = m_{vehicle} \frac{du}{dt} \Rightarrow -g_{earth} I_{sp} \frac{dm_{vehicle}}{dt} = m_{vehicle} \frac{du}{dt}$$

$$\Rightarrow du = -g_{earth} I_{sp} \frac{dm_{vehicle}}{m_{vehicle}} \Rightarrow \Delta u \equiv u_{final} - u_{initial} = -g_{earth} I_{sp} \ln\left(\frac{m_{final}}{m_{initial}}\right)$$

$$\Rightarrow \boxed{\Delta u = g_{earth} I_{sp} \ln\left(\frac{m_{initial}}{m_{final}}\right)}$$

- Gravity and aerodynamic drag will increase Δu requirement for a given mission above that required by orbital mechanics alone

Breguet & rocket equations - comments

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- Range (R) for aircraft depends on
  - η_o (propulsion system)
  - Q_R (fuel)
  - L/D (lift to drag ratio of airframe)
  - g (gravity)
  - Fuel consumption ($m_{initial}/m_{final}$); $m_{initial}$ - m_{final} = fuel mass burned (or fuel + oxidizer, if not airbreathing)
- R does not consider extra fuel mass required for taxi, takeoff, climb, decent, landing, fuel reserve, etc.
- Note (irritating) ln( ) or exp( ) term in both Breguet and Rocket

$$\frac{m_{initial}}{m_{final}} = \exp\left(\frac{R \cdot g}{\eta_o Q_R L/D}\right) \text{ (Breguet)}; \quad \frac{m_{initial}}{m_{final}} = \exp\left(\frac{\Delta u}{g_{earth} I_{sp}}\right) \text{ (rocket)}$$

that occurs because more thrust is required at the beginning of the flight to carry fuel that won't be used until the end of the flight - if not for ln( ) term it would be easy to fly around the world without refueling and the Chinese would have sent skyrockets into orbit thousands of years ago!

Examples

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- What initial to final mass ratio is needed to fly around the world without refueling?

  Assume distance traveled (R) = 40,000 km, g = 9.8 m/s^2; hydrocarbon fuel (Q_R = 4.3 x 10^7 J/kg); good propulsion system (η_o = 0.25), good airframe (L/D = 25),

$$\frac{m_{initial}}{m_{final}} = \exp\left(\frac{R \cdot g}{\eta_o Q_R \, L/D}\right) = \exp\left(\frac{(40\times 10^6 m)(9.81 m/s^2)}{(0.25)(4.3\times 10^7 J/kg)(25)}\right) = 4.31$$

  So the aircraft takeoff mass has to be mostly fuel, i.e. $m_{fuel}/m_{initial} = (m_{initial} - m_{final})/m_{initial} = 1 - m_{final}/m_{initial} = 1 - 1/4.31 = 0.768$! – that's why no one flew around with world without refueling until 1986 (solo flight 2005)

- What initial to final mass ratio is needed to get into orbit from the earth's surface with a single-stage rocket propulsion system?

  For this mission Δu = 8000 m/s; using a good rocket propulsion system (e.g. Space Shuttle main engines, I_{SP} ≈ 400 sec

$$\frac{m_{initial}}{m_{final}} = \exp\left(\frac{\Delta u}{g_{earth} I_{sp}}\right) = \exp\left(\frac{(8000 m/s)}{(9.81 m/s^2)(400 s)}\right) = 7.68$$

  It's practically impossible to obtain this large a mass ratio in a single stage, thus *staging* is needed where you jettison larger, heavier stages as fuel mass is consumed – that's why no one put an object into earth orbit until 1957, and no one has <u>ever</u> built a *single stage to orbit* vehicle.

Summary

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- Steady flow (e.g. gas turbine) engines have much higher power-to-weight ratios than unsteady flow (e.g. reciprocating piston) engines
- A simple momentum balance on a steady-flow propulsion system shows that the best performance is obtained when
  - Exit pressure = ambient pressure
  - A large mass of gas is accelerated by a small Δu
- Two types of efficiencies for propulsion systems - thermal efficiency and propulsive efficiency (product of the two = overall efficiency, which is the most important figure of merit)
- Definitions - specific thrust, thrust specific fuel consumption, specific impulse
- Range of an aircraft depends critically on overall efficiency - effect more severe than in ground vehicles, because aircraft must generate enough lift (thus thrust, thus required fuel flow) to carry entire fuel load at first part of flight