AME 331, Final Exam Study Guide
May 13, 2008

Format of the exam

The final exam will be similar to the midterms - unlimited open book. You may use any reference materials you want.

Material covered

The exam may cover any material covered during the course, which includes book chapters 1 - 8 and 10. The exam will place about 50% of the emphasis on radiation and heat exchangers since these subjects have not been covered on an exam yet, with about 25% devoted to conduction and 25% to convection.

The material on heat exchangers includes:

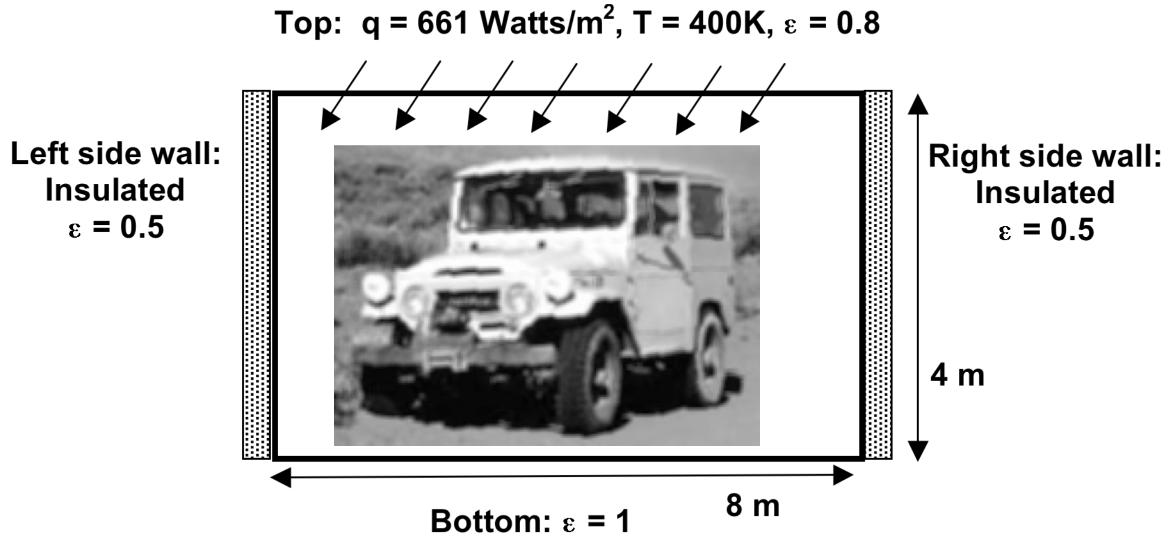
- Types of heat exchangers – concentric tube, shell-and-tube, cross-flow
- Overall heat transfer coefficient
- Log mean temperature difference & modifications for non-parallel flow
- Effectiveness-NTU method

The material on radiation includes:

- Planck's law
- Black and gray bodies
- Absorption, reflection, transmission, emission
- Radiation from surfaces
    - Radiation shape factor
    - Reciprocity and other relations between shape factors
    - Radiation to/from non-black surfaces (those radiative resistance networks)

Last year's final exam

<u>Problem #1 (radiation) (35 points total)</u>



A paint-drying oven for cars is 4 m tall and 8 m wide. It is very deep in the 3rd dimension (into the page), so the radiative transfer in the 3rd dimension is negligible. It has a radiant heater on top with emissivity 0.8, a temperature of 400K and transfers 661 Watts/m^2 to the interior of the oven. The side walls are insulated with emissivity 0.5. The bottom (floor) has an emissivity of 1. **If there is no car in the oven**:

(a) (4 points) Draw a radiative network for this system

(b) (4 points) What are the radiation shape factors $F_{top\text{-}bottom}$ and $F_{top\text{-}side}$? ($F_{top\text{-}side}$ meaning 1 side, not both sides).

Figure 8-12: $X = \infty$, $Y = 8$ m, $D = 4$ m: $X/D = \infty$, $Y/D = 2$: $F_{ji} = F_{top\text{-}bottom} = F_{bottom\text{-}top} = 0.60$
Note that since the side walls are symmetrical we can treat them as identical.
Summation: $F_{top\text{-}top} + F_{top\text{-}side} + F_{top\text{-}side} + F_{top\text{-}bottom} = 1$, $F_{top\text{-}top} = 0$, $F_{bottom\text{-}top} = 0.60$,
Thus $F_{top\text{-}side} = F_{bottom\text{-}side} = 0.20$. Also $F_{top\text{-}sides} = F_{bottom\text{-}sides} = 2(0.20) = 0.4$ (both sides here)

(c) (6 points) What is the radiosity (J) of the top surface?

$$q_{top} = \frac{E_{b,top} - J_{top}}{\dfrac{1-\varepsilon_{top}}{\varepsilon_{top} A_{top}}} \Rightarrow J_{top} = \sigma T_{top}^4 - \frac{q_{top}}{A_{top}} \frac{1-\varepsilon_{top}}{\varepsilon_{top}}$$

$$= (5.67 \times 10^{-8} W/m^2K^4)(400K)^4 - (661 W/m^2)\left[(1-.8)/.8\right] = 1286 W/m^2$$

(d) (10 points) What is the temperature of the bottom?

Since side walls are insulated, no radiation is sourced/sunk on the sides, thus $q_{top} = q_{bottom}$; also $E_{b,bottom} = J_{bottom}$ since $\varepsilon_{bottom} = 1$.

$$q_{top} = q_{bottom} = \frac{J_{top} - E_{b,bottom}}{\Sigma R} \Rightarrow E_{b,bottom} = J_{top} - q_{top}\Sigma R$$

$$\Rightarrow E_{b,bottom} = J_{top} - \frac{q_{top}}{A_{top}} A_{top}\Sigma R = 1286W/m^2 - 661W/m^2(8m)(W)(\Sigma R)$$

$$\Sigma R = \left(R_{top-sides} + R_{sides-bottom}\right) \| \left(R_{top-bottom}\right) = \frac{1}{\frac{1}{R_{top-sides} + R_{sides-bottom}} + \frac{1}{R_{top-bottom}}}$$

$$R_{top-sides} = \frac{1}{F_{top-sides}A_{top}} = \frac{1}{(0.4)(8m)(W)} = R_{sides-bottom}; R_{top-bottom} = \frac{1}{F_{top-bottom}A_{top}} = \frac{1}{(0.6)(8m)(W)}$$

$$Combine: \Sigma R = \frac{1}{(6.4m)(W)} \Rightarrow E_{b,bottom} = 1286W/m^2 - 661W/m^2\frac{(8m)(W)}{(6.4m)(W)} = 460W/m^2$$

$$\Rightarrow T_{bottom} = (E_{b,bottom}/\sigma)^{1/4} = (460W/m^2/5.67\times10^{-8}W/m^2K^4)^{1/4} = 300K$$

(e) (7 points) What is the temperature of the side walls?

Since the $R_{top\text{-}sides} = R_{sides\text{-}bottom}$, $J_{sides} = (J_{top} + J_{bottom})/2 = (1286 + 460)/2 = 873\ W/m^2 = E_{b,sides}$
$\Rightarrow T_{sides} = [(873\ W/m^2)/(5.67 \times 10^{-8}\ W/m^2K^4)]^{1/4} = 352$ K

(f) (4 points) If the side wall emissivity was increased, would the side wall temperature increase, decrease or remain the same? Explain.

Since the side walls are insulated, there is no radiation ("current") sourced or sunk there. If there is no "current", there is no "voltage drop" ($E_{b,sides} - J_{sides} = 0$), and thus the surface resistance $= (1-\varepsilon_{sides})/\varepsilon_{sides}A_{sides}$ does not affect $E_{b,sides}$ or J_{sides}. Thus, the emissivity ε_{sides} does not affect $E_{b,sides} = \sigma T_{sides}^4$, and thus ε_{sides} does not affect T_{sides}. Basically this is because a higher emissivity will mean more emission (obviously) but also more absorption, in the same proportion.

(g) (Bonus point) What type of car is being painted? (Hint: it's not a Jeep.)

1968 Toyota Land Cruiser (my first car)

<u>Problem #2 (heat exchangers) (25 points total).</u>

The specifications in the Ronney Heat Exchangers, Inc. catalog aren't very clear. All the catalog states for its PDR® heat exchanger is that one fluid is liquid Freon-12 with a temperature of 10˚C at one end, 30˚C at the other end, and a mass flow rate of 0.01 kg/s. The other fluid is liquid mercury at 0˚C at one end and 10˚C at the other end, but the mass flow is not specified. The heat exchanger area is 0.01 m^2. Fluid properties are given below.

Fluid	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^2$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^7$ (m^2/s)	Pr	$\beta \cdot 10^3$ (K^{-1})
Mercury	13,529	0.1393	0.1523	0.1125	8540	45.30	0.0248	0.181
Freon-12	1305.8	0.9781	0.0254	0.195	72	0.564	3.5	2.75

a) (3 points) For the Freon-12 stream, which temperature, 10˚C or 30˚C, is at the inlet and which temperature is at the outlet?

30˚C is the hottest temperature in the device, so must be the hot-side inlet temperature. Then 10˚C must be the outlet temperature.

b) (3 points) For the mercury stream, which temperature, 0˚C or 10˚C, is at the inlet and which temperature is at the outlet?

0˚C is the coldest temperature in the device, so must be the cold-side inlet temperature. Then 10˚C must be the outlet temperature.

c) (4 points) Is this a co-flow or a counterflow exchanger, or could it be either one?

Since the outlet temperatures are the same, for it to be a co-flow exchanger, the area or the heat transfer coefficient would have to be infinite. Thus, it must be a counter-flow exchanger.

d) (5 points) What is the log mean temperature difference for this exchanger? (If your answer to part (c) was either, assume for this part that it's a co-flow exchanger.)

For a counterflow exchanger:
$\Delta T_1 = T_{H,in} - T_{C,out} = 30^\circ C - 10^\circ C = 20^\circ C$
$\Delta T_2 = T_{H,out} - T_{C,in} = 10^\circ C - 0^\circ C = 10^\circ C$
$LMTD = (\Delta T_2 - \Delta T_1)/\ln(\Delta T_2 - \Delta T_1) = (20 - 10)/\ln(20/10) = 14.4^\circ C$

e) (5 points) Which fluid has the higher mass flow rate?

$q_H = mdot_H C_{P,H}(T_{H,in} - T_{H,out}) = (0.01\ kg/s)(978.1\ J/kgK)(30^\circ C - 10^\circ C) = 195.6$ Watts
By conservation of energy, $q_H = q_C = mdot_C C_{P,C}(T_{C,out} - T_{C,in})$
Thus $mdot_C = q_H/C_{P,C}(T_{C,out} - T_{C,in}) = (195.6\ \text{Watts})/(139.3\ J/kgK)(10^\circ C - 0^\circ C) = 0.140\ kg/s$
Thus cold stream (mercury) has higher mass flow rate

f) (5 points) What is the overall heat transfer coefficient (U) for this exchanger? (Again, if your answer to part c) was either, assume for this part that it's a co-flow exchanger.)

$q = UA\Delta T_{LM}$, thus $U = q/A\Delta T_{LM} = (195.6\ \text{Watts})/(0.01\ m^2)(14.4^\circ C) = 1358\ W/m^2{}^\circ C$

Problem #3 (Miscellaneous conduction / convection) (40 points total, 5 points each part)

Ronney Chemicals, Inc. claims to have invented an additive for water, called PDR®, that decreases the thermal conductivity (k) of water or ice by 10%. **All other properties of water, in particular density (ρ), kinematic viscosity (ν), dynamic viscosity (μ), specific heats (C_P and C_v), and thermal expansion coefficient (β) are unchanged by adding PDR®.** Assume radiative transfer is negligible in water. How would each of the following be affected by adding PDR® to water? *In particular state whether the entity would increase, decrease or remain the same, and if it changes, would the change be less than, more than, or exactly 10%.*

a) Prandtl number (Pr)

$Pr = \nu/\alpha = (\mu/\rho)/(k/\rho C_P) = \mu C_P/k$; if k decreases 10%, **Pr increases 10%.**

b) Time for center of a slab of ice 5 cm thick, initially at a uniform temperature of O˚C, to reach -10˚C after the slab surface temperature is lowered to -20˚C.

Use Heisler chart (Fig. 4-7) with $h \to \infty$, thus $Bi^{-1} \to 0$.
At $\theta_o^* = (T_o - T_\infty)/(T_i - T_\infty) = (10\text{-}0)/(20\text{-}0) = 0.5$, $Fo \approx 0.3$ = constant (not a function of k).
Since $Fo = \alpha t/L^2 = kt/\rho CL^2$, if k decreases 10%, then t must increase 10% to keep the same Fo. So **time increases 10%.**

c) Average value of $T_s - T_\infty$ for a vertical flat plate submerged in water with constant heat flux subject to laminar buoyant convection

Buoyant, laminar, vertical flat plate, q" = constant: $Nu \sim (Gr^*Pr)^{1/5}$, where $Gr^* = g\beta q"L^4/k\nu^2 \sim 1/k$ and $Pr \sim 1/k$, thus $Nu \sim (1/k^2)^{1/5} \sim k^{-0.4}$.
Then $h = Nu\ k/L \sim (k^{-0.4})k = k^{0.6}$
Then $q/A = h\Delta T$ or $\Delta T \sim 1/h \sim k^{-0.6}$ (since q/A is constant)
If k decreases 10%, then **ΔT increases by less than 10%** (actually about 6%)

d) Heat transfer coefficient (h) for fully developed turbulent flow of water in a pipe

$Nu = 0.023\ Re^{0.8}\ Pr^n$, where n = 0.4 (heating) 0.4 (cooling) but whether it's heating or cooling won't change the answer. Re does not depend on k, but $Pr \sim 1/k$, thus $Nu \sim 1/k^n$ and $h \sim Nu\ k/L \sim k^1\ k^{-n} \sim k^{1-n}$.
For either n = 0.3 or n = 0.4, if k decreases by 10%, then **h decreases by less than 10%.**

e) Rate of heat transfer (q) from an isothermal solid horizontal cylinder submerged in liquid water with no forced flow.

$q = hA\Delta T = (Nu)(k/d)(A)(\Delta T)$; in this case A, d and ΔT are fixed, so $q \sim (Nu)(k)$
For buoyant heat transfer from a horizontal cylinder, $Nu \sim Ra^n$ with $0.058 < n < 0.333$ depending on Ra. And $Ra \sim GrPr \sim 1/k$. Thus $Nu \sim k^{-n}$, hence $q \sim (Nu)(k) \sim k^{1-n}$. So for all values of n, if k decreases by 10%, **q decreases by less than 10%.**

f) Thermal boundary layer thickness (δ_T) for laminar forced flow over a flat plate at a fixed distance from the leading edge

For forced laminar flow, $\delta_T \sim \delta\ Pr^{-1/3}$ (Eq. 5-38); since the velocity boundary layer thickness δ is not affected by k, and because $Pr \sim 1/k$, $\delta_T \sim k^{1/3}$.
So if k decreases by 10%, **δ_T decreases by less than 10%.**

g) Heat transfer rate (q) in an enclosure between two very long horizontal plates, lower plate heated from below, at $Ra_L = 10^7$.

$q = hA\Delta T = (Nu)(k)(A)(\Delta T)$; in this case A and ΔT are fixed, so $q \sim (Nu)(k)$
For buoyant heat transfer between flat plates with a buoyant fluid in between, $Nu \sim Ra_L^{1/3}Pr^{0.074}$ (Eq. 9.49). And $Ra \sim GrPr$, and again $Pr \sim 1/k$.
Thus $Nu \sim k^{-1/3}k^{-0.074} \sim k^{-0.41}$
Hence $q \sim (Nu)(k) \sim k^{0.59}$. So if k decreases by 10%, **q decreases by less than 10%.**

h) Temperature midway between the horizontal plates of part g, but with $Ra_L = 10^3$.

At $Ra_L = 10^3$, there is no buoyant flow, so there is just conduction. The temperature profile between two plates at temperatures T_1 and T_2 is just $T(x) = T_1 + (T_2 - T_1)x/L$, which does not depend on k at all. So **no change.**