Problem #1 (10 points) (from a previous year's first midterm exam)

A quantity called BMEP is often used to quantify the performance of internal combustion engines. Its definition is

\[ BMEP = \frac{2P}{VN} \]

where \( P \) is the engine power, \( V \) the displacement volume (units length\(^3\)) and \( N \) is the rotation rate (units 1/time).

(a) What are the units of BMEP in the SI system?

(b) If \( P = 200 \) horsepower, \( V = 350 \text{ in}^3 \) and \( N = 3600/\text{min} \), what is BMEP in SI units?

Problem #2 (15 points) (from a previous midterm)

I calculated the power production from a new type of steam turbine as

\[ \text{Power} = \left( \frac{U_{\text{out}}^2}{2} - \frac{U_{\text{in}}^2}{2} \right) + \dot{m}C_P(T_{\text{out}} + T_{\text{in}}) - \frac{\dot{m}}{\rho}(P_{\text{out}} - P_{\text{in}}) - \frac{\dot{m}}{\mu}(Nd)^2 + 1 \]

where \( T_{\text{in}}, P_{\text{in}} \) and \( U_{\text{in}} \) are the temperature, pressure and velocity of the steam going into the turbine, \( T_{\text{out}}, P_{\text{out}} \) and \( U_{\text{out}} \) are the temperature, pressure and velocity of the steam leaving the turbine, \( C_P \) is the heat capacity of the steam (units Joules/kg˚C), \( \dot{m} \) the mass flow rate of steam through the turbine (units kg/s), \( \rho \) the steam density (kg/m\(^3\)), \( \mu \) the coefficient of friction in the rotating shaft (dimensionless), \( N \) the rotation rate of the shaft (units 1/s) and \( d \) the shaft diameter.

Using “engineering scrutiny,” what “obvious” mistakes can you find with this formula? There are at least 5, but list only the 4 of which you are most certain.

Problem #3 (20 points)

The power transmitted by a rotating shaft is given by

\[ P = 2\pi N\tau \]

where \( P \) is the power, \( N \) is the number of revolutions per unit time and \( \tau \) is the torque.
(a) Verify that the units are consistent, *i.e.*, show that the units on the left side of the equation are the same as on the right side of the equation.

(b) If \( N \) is in units of revolutions per minute (RPM) and \( \tau \) in units of ft lbf, what conversion factor is needed to obtain \( P \) in units of horsepower? In other words, find \( ???? \) in the following relation

\[
\text{Power (horsepower)} = \frac{\text{Torque (ft lbf)} \times \text{RPM (rev/min)}}{????}
\]

(c) Look up the horsepower and torque of any automobile engine. Is your formula consistent with these specifications? (The answer will most likely be NO).

(d) Can you explain this discrepancy? (You need to think carefully about how to interpret the manufacturer's specifications.)

**Problem #4 (10 points)**

The solar power transmitted to the ground at high noon at the equator is about 1000 Watts per square meter. How many square feet of solar collector area would be required to power your car, assuming a 100 horsepower motor and a conversion efficiency of solar power to motor shaft power of 15%?

**Problem #5 (15 points)**

You probably found that the solar collector area was prohibitive (which is why we’re not driving around in solar powered cars.) So let’s try batteries instead. Look up the specifications (volts, amp-hours and weight or mass) for any type of rechargeable battery.

(a) The energy delivered by the battery is volts \( \times \) amp-hours. Compute this energy and convert to units of Joules.

(b) Divide by the mass of the battery and convert the result to units of Joules/kg. Compare this to the heating value of gasoline, about \( 4.3 \times 10^7 \) J/kg. (This comparison shows you why most of us don’t drive around in battery powered cars.)

**Problem #6 (15 points)**

What about fuel cells? A typical Proton Exchange Membrane (PEM) fuel cell requires 1 milligram of platinum per cm\(^2\) of fuel cell area. Each cm\(^2\) of fuel cell area can generate about 0.4 amp of current. The fuel cell voltage is about 0.8 Volts, independent of the area.

(a) How much fuel cell area (in cm\(^2\)) is required for 100 horsepower? (Recall power = volts \( \times \) amps)

(b) Look up the cost of platinum online. How much will the platinum cost for a 100 hp fuel cell? I get about $7,500. Note that the cost of platinum and other precious metals is reported in dollars per troy ounce, which is different from the more common avoirdupois ounce so you’ll also need to look up the conversion of troy ounces to grams. (Yes I could just give it to you but I want you to get used to looking things up for yourself.) (And yes I could have just asked you to look up the price in dollars per gram, but how could I resist a homework problem that used troy ounces...
Problem #7 (15 points)

One last try … how about compressed air? The energy ($E$) stored in a compressed gas of volume $V_1$ and pressure $P_1$ that can be extracted by expanding the gas to a lower pressure $P_2$ is

$$E = \frac{P_1 V_1}{\gamma - 1} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\gamma - 1} \right]$$

where $\gamma$ is the specific heat ratio of the gas (dimensionless), whose value is about 1.4 for air.

(a) What volume ($V_1$) (in gallons) of air would be required to have the same energy content as one gallon of gasoline ($1.25 \times 10^8$ Joules) if $P_1 = 200$ atmospheres and $P_2 = 1$ atmosphere?

(b) What would the mass of this air be? To determine this, recall that the ideal gas law can be written in the form $m = \frac{P_1 V_1}{R T_1}$, where $m$ is the mass of gas and $R = \mathcal{R}/M$ is the mass-based ideal gas constant ($\mathcal{R}$ = universal gas constant, $M$ = molecular mass) and $T_1$ = ambient temperature (say 70˚F).